NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2009-2010

CV2601 - Fluid Mechanics

November/December 2009 **Time Allowed: 2½ hours**

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **SIX (6)** pages.
2. Answer **ALL** questions.
3. An **Appendix** of **ONE (1)** page is attached to the paper.
4. All questions carry equal marks.
5. This is a Closed-Book Examination.

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1. (a) The cylindrical weir in Figure Q1(a) has a diameter of 2 m, and length of 5 m. Compute the magnitude, direction and line of action (in terms of distance from base of weir) of the resultant **horizontal** hydrostatic thrust acting on the weir.

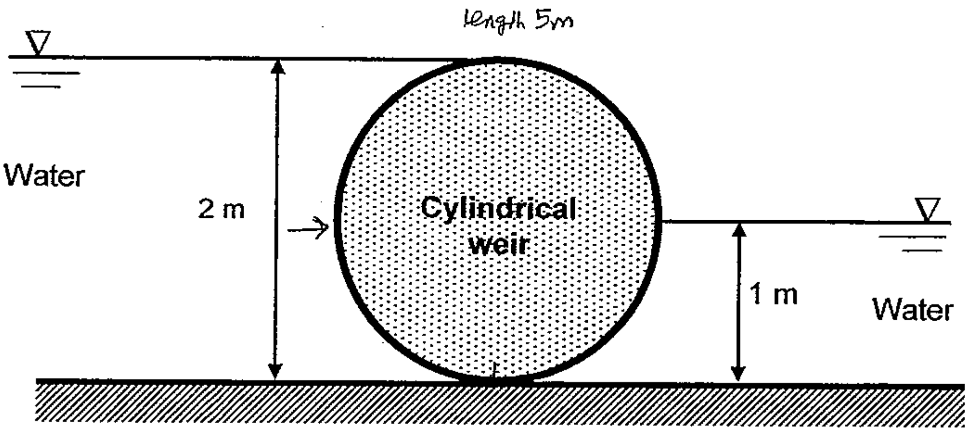


Figure Q1(a)

(10 Marks)

Note: Question No. 1 continues on page 2

(b) The steel tubing in Figure Q1(b) weighs 5 kg in air, has a uniform external diameter of 5 cm, and is closed at its base. The tubing floats upright in liquid A with its top 10 cm lower than when it is floating in water, as shown in the figure. What is the specific gravity of liquid A? What is the distance of the C.G. (centre of gravity) of the tubing from its base if it floats upright in neutral equilibrium in water? Given I_{yy} for a circle = $\pi D^4/64$.

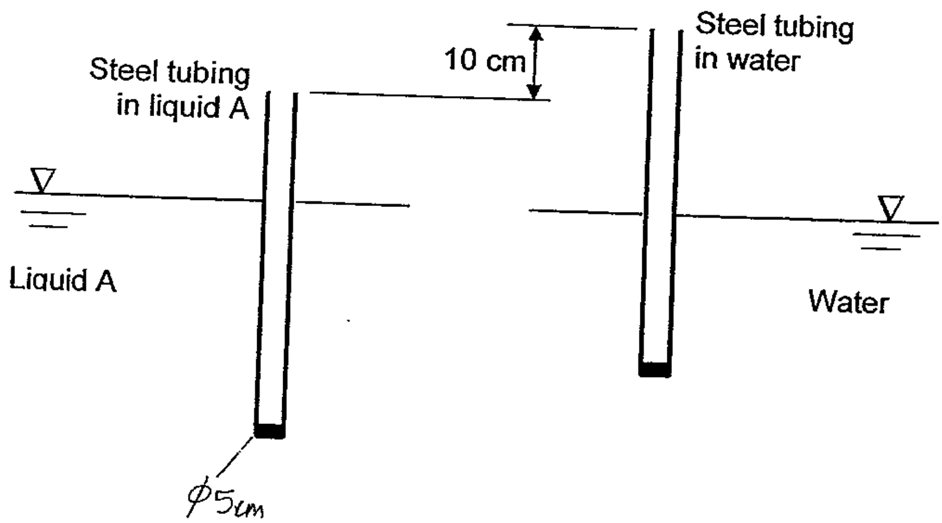


Figure Q1(b)

(15 Marks)

kg/m³x

2. (a) An ideal fluid of density 900 kg/m^3 flows through a horizontal reducer as shown in Figure Q2(a). The pressure difference at two ends of the reducer is measured by a mercury manometer as shown in the figure. What is the volume flow rate of the fluid, Q, along the pipeline? ~~Is it possible to~~ compute the pressure just downstream of the reducer? Explain your answer.

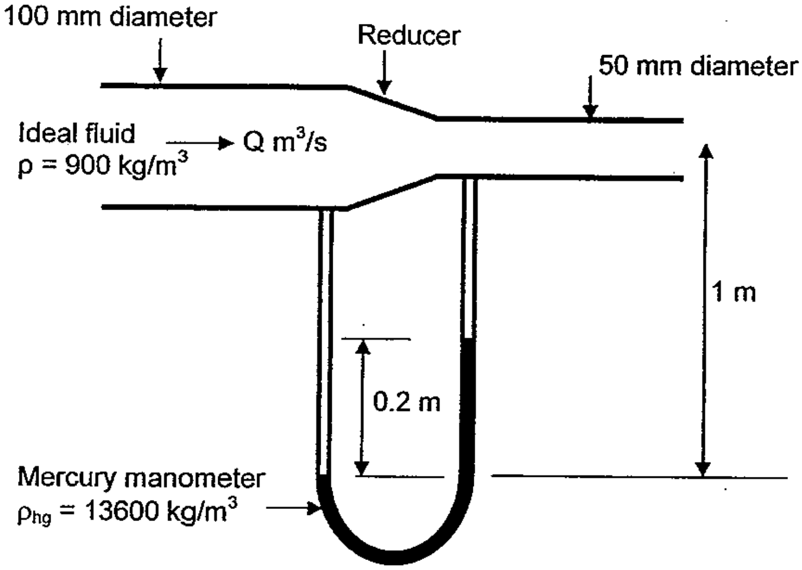


Figure Q2(a)

(13 Marks)

(b) Water flows beneath a cylindrical drum as shown in Figure Q2(b). Assuming ideal flow, determine the horizontal hydrodynamic thrust acting on 1 m length of the drum.

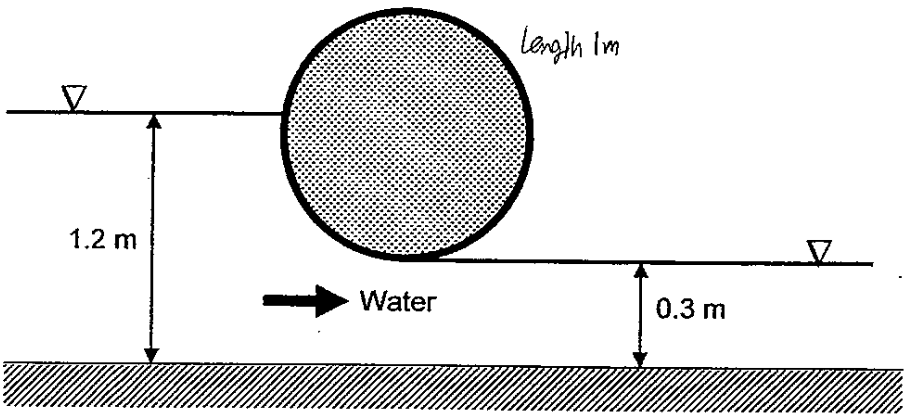


Figure Q2(b)

(12 Marks)

3. (a) (i) By applying Newton's second law of motion to a fully developed steady, uniform flow along a straight upward inclined circular pipe laid on an angle θ to the horizontal as shown in Figure Q3(a), show that the shear stress, τ is related to the change in pressure per unit length of pipe, $\frac{\Delta p}{\ell}$ as follows:

$$\tau = \frac{r}{2} \cdot \frac{(\Delta p - \gamma\, l \sin\theta)}{l}$$

$\tau = \frac{\Delta P \cdot D}{4L} = \frac{\Delta P \cdot r}{2L}$

where r = radial distance measured from the pipe centre; and γ = specific weight (ρg) of fluid.

$\frac{\Delta P}{L} = \frac{2\tau}{r}$

$\frac{\Delta P \cdot r}{2L} + \gamma L \cdot \sin\theta \stackrel{?}{=} \tau$

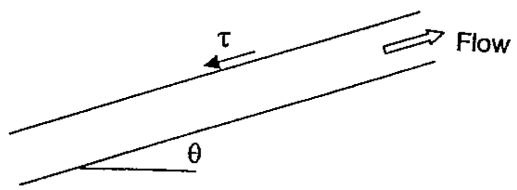


Figure Q3(a)

(ii) By applying Bernoulli's equation to a steady flow in a straight upward inclined circular pipe with radius = R and length = L, show that the wall shear stress, τ_w can be computed using the same form of equation as that shown in part a(i) above. The angle between the inclined pipe and the horizontal is θ.

(Hint: The friction factor, f is related to the wall shear stress, τ_w as follows: $f = \frac{8\tau_w}{\rho V^2}$, where ρ = fluid density; and V = flow velocity)

(11 Marks)

$H_L = f \frac{L}{D} \cdot \frac{V^2}{2g}$

$= \frac{8\tau_w}{\rho V^2} \cdot \frac{L}{2r} \cdot \frac{V^2}{2g}$ ρg

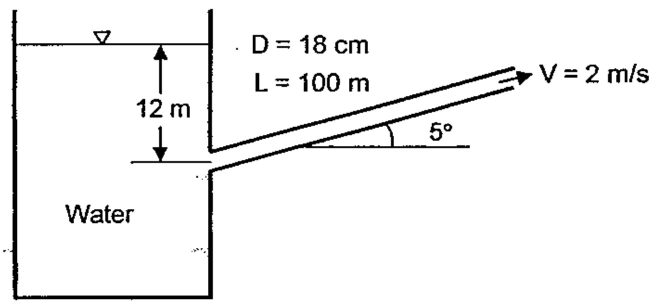
Note: Question No. 3 continues on page 5

$Re = \frac{\rho V D}{\mu} = \frac{VD}{\gamma}$

γ

(b) Water with a kinematic viscosity = 1 x 10^{-6} m^2/s flows from the reservoir through a 18-cm diameter inclined pipe of length 100 m as shown in Figure Q3(b). The flow emerges from the pipe as a free jet into the atmosphere. Calculate the roughness height of the pipe if the exit velocity is 2 m/s.

[Hint: Ignore all minor losses in your computation; you need to use the Moody Diagram in the Appendix for your computation.]



$Re = \frac{\rho V D}{\nu}$

Figure Q3(b)

(14 Marks)

$m^2/s \times$ kg =

4. (a) Reservoir A is connected to Reservoir B by a cast iron pipe (equivalent roughness height = 0.26 mm) that is 250 m long. The pipe consists of two sections as shown in Figure Q4. . The length of the upstream section is 100 m and diameter = 20 cm, while the length of the downstream section is 150 m with diameter = 15 cm.

(i) If the elevation of water in Reservoir B is 100 m, what is the elevation of water in Reservoir A when the flow rate through the pipe is 40 L/s? You may assume that the kinematic viscosity of water is 1×10^{-6} m^2/s and that the entrance, contraction and exit loss coefficients = 0.5, 0.26 and 1.0, respectively.

$\nu = 1\times10^{-6}$

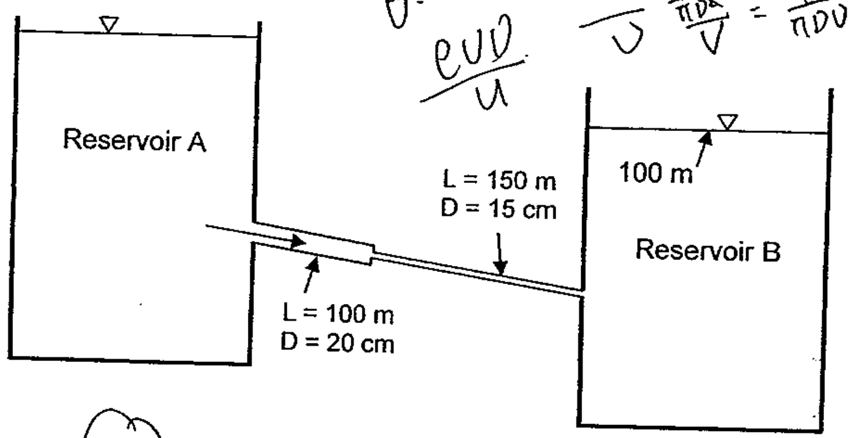
$Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

(ii) Draw the total energy line and hydraulic grade line of the pipe flow system. Indicate clearly the magnitude of the total head and all the components of energy loss in your sketch.

(iii) If the pipe in Figure Q4 is replaced by a single 22-cm diameter pipe with length and friction factor = 250 m and 0.02, respectively, calculate the total flow rate if the reservoir levels in A and B were to remain unchanged.

[Hint: You need to use the Moody Diagram in the Appendix for your computation.]

$\frac{\rho V D}{\mu}$ $\quad \frac{4Q}{\pi D V} $



2.20

6.36

V_1 $\left(\frac{4Q}{\pi D^2}\right)^2$

Figure Q4

(19 Marks)

(b) A 1:25 hydraulic model is tested in the laboratory. The model discharge and velocity are 30 L/s and 2.6 m/s, respectively. If dynamic similarity based on Reynolds number is satisfied in the study:

$\rho L^3 \frac{V^2}{L}$

(i) Calculate the prototype flow rate if the same fluid is used in both the model and prototype.

(ii) If the force acting on a certain part of the model is found to be 50 N, calculate the corresponding force in the prototype.

$F = ma$ $\quad V^2 = 2aS$ $\quad \rho L^3 \frac{V^2}{2L}$

(6 Marks)

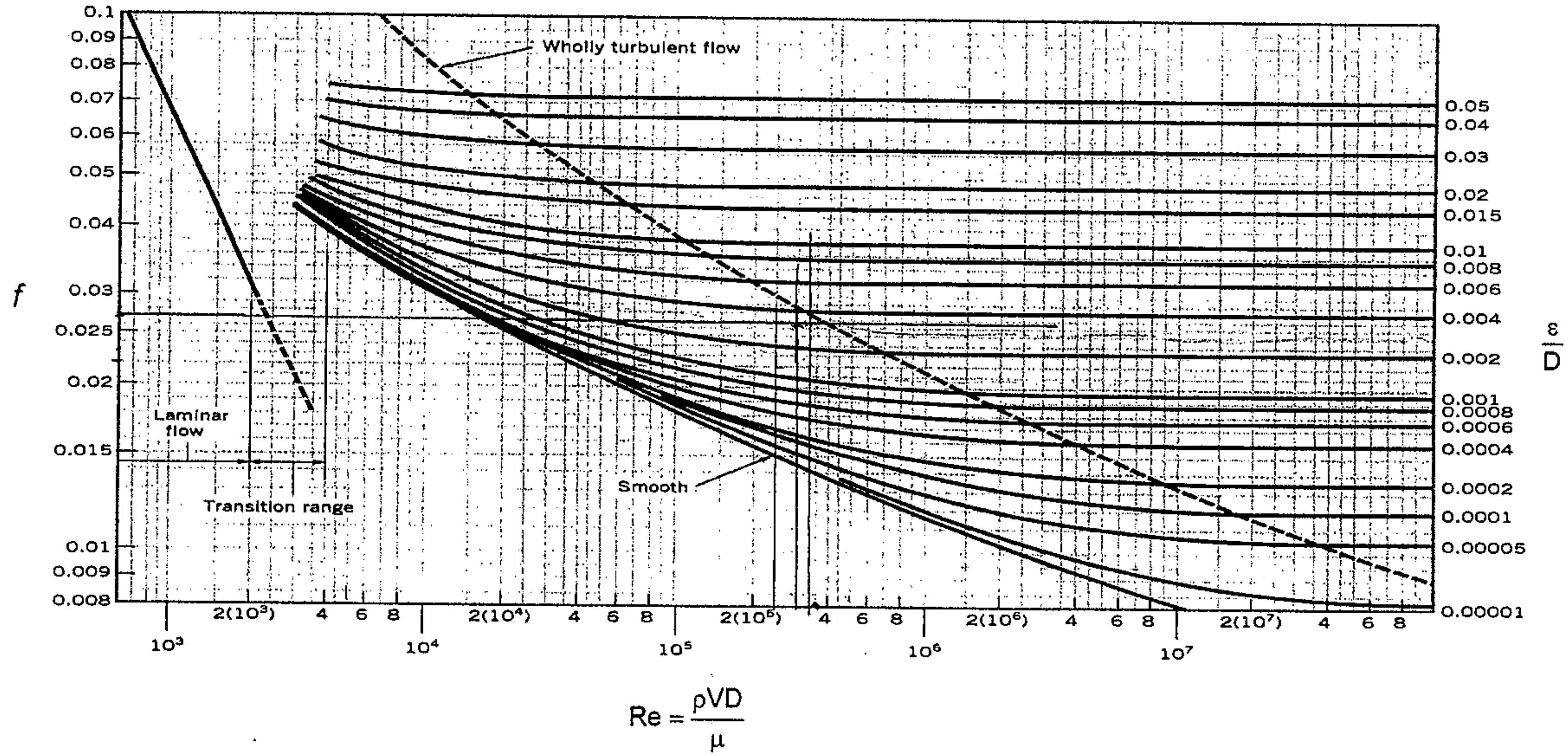
$L^3 \rho \frac{V^2 L^{-1}}{L}$ **END OF PAPER** ma $\quad \frac{V_m L_m^2}{V_p L_p^2} =$

1.3×10^{-3}

0.0013

APPENDIX

MOODY DIAGRAM



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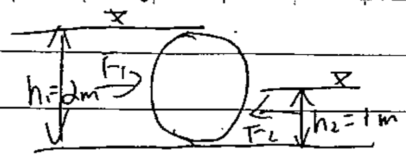
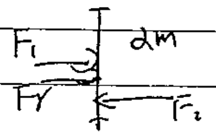
1(a)



$F_1 = \rho g A_1 h_{c1} = 1\times10^3\times9.81\times2\times5\times1 = 98100\,N$

$F_2 = \rho g A_2 h_{c2} = 1000\times9.81\times1\times5\times\frac{1}{2} = 24525\,N$



$F_r = F_1 - F_2 = 73575\,N$

$F_1\times r_1 - F_2\times r_2 = F_r\cdot r \Rightarrow 98100\times1 - 24525\times\frac{1}{2} = 73575\times r$

$\Rightarrow r = 1.167\,m$ from the base.

(b) In liquid A, $Mg = \rho_A g S\cdot h_A$.

In Water, $Mg = \rho_w g S h_w$

$\Rightarrow \rho_A g S h_A = \rho_w g S h_w$.

$h_A = (h_w + 0.1) \Rightarrow \rho_A g S(h_w + 0.1) = \rho_w g S h_w$

$\rho_A = \frac{\rho_w\cdot h_w}{h_w + 0.1}$

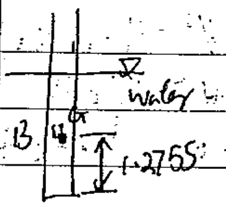
$M = 5kg$ $S = \frac{\pi}{4}D^2 = \frac{\pi\times0.05^2}{4} = 1.96\times10^{-3}\,m^2$

$h_w = 2.551\,m$ $h_A = 2.651\,m$

$\gamma_A = \rho_A g = \frac{\rho_w\cdot h_w}{h_w+0.1} = \frac{1000\times2.551}{2.651} = 962.3$

Since we know the tube floats in neutral equilibrium in water.

$BM = BG$, $BM = I_{yy}/V_i = \frac{\pi D^4/64}{S\cdot h_w} = \frac{\pi\cdot0.05^4/64}{1.96\times10^{-3}\times2.551} = 6.136\times10^{-5}$



So we know the point $G = BG + 1.2755 = 1.2755 + 6.136\times10^{-5}$ (from the base)

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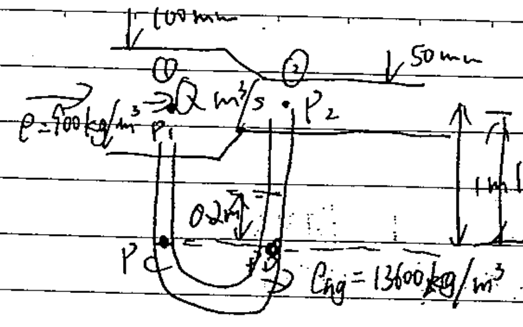
2 (a)



~~poin~~ As we know, $P_C = P_D$

$P_1 + \rho_w g H = P_2 + \rho_w g(H - 0.2) + \rho_{hg} g \times 0.2$

$\Rightarrow P_1 - P_2 = \rho_{hg} g \times 0.2 - \rho_w g \times 0.2 = \cancel{24721.2}\ 24917.4\ Pa$

By using B. E,

$P_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2$

$z_1 = z_2 \quad P_1 - P_2 = 24721.2 Pa$

$\Rightarrow \frac{1}{2}\rho V_2^2 - \frac{1}{2}\rho V_1^2 = P_1 - P_2 = \cancel{24721.2}\ 24917.4\ Pa$

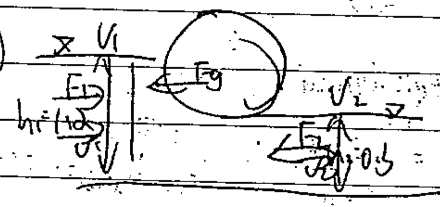
$\Rightarrow V_2^2 - V_1^2 = \cancel{49.4424}\ 49.8348$

$V_1^2 = \left(\frac{Q}{\frac{\pi 0.1^2}{4}}\right)^2 = \frac{16Q^2}{\pi^2 0.1^4} = 16211.39 Q^2 \quad V_2^2 = \frac{16Q^2}{\pi^2 0.05^4} = 259382.23 Q^2$

$V_2^2 - V_1^2 = 259382.23 Q^2 - 16211.39 Q^2 = 49.8348$

$\Rightarrow Q^2 = 5.123 \times 10^{-3} \Rightarrow Q = 0.0715 m^3/s$

(b).



by applying B. E

$\rho g h_1 + \frac{1}{2}\rho V_1^2 + P_1 = \rho_w g h_2 + \frac{1}{2}\rho V_2^2 + P_2$

$P_1 = P_2 \Rightarrow \frac{1}{2}\rho V_2^2 - \frac{1}{2}\rho V_1^2 = \rho_w g h_1 - \rho_w g h_2$

$\Rightarrow V_2^2 - V_1^2 = 17.658$

~~Apply M. E~~

$F_{net} = F_1 - F_2 - F_g = \rho g \frac{h_1^2}{2} - \rho g \frac{h_2^2}{2} - F_g = M_{out} - M_{in} = \rho A V_2^2 - \rho A V_1^2$

$\Rightarrow F_g = 11036.25\ N$ direction ($\leftarrow$)

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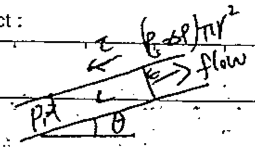
3(a)(i)



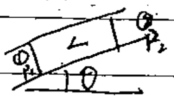
Applying Newton's second Law

$\Sigma F = 0$ $\qquad P_1 \cdot \pi r^2 - (P_1 - \Delta P)\pi r^2 - \tau \cdot 2\pi r \cdot L - \pi r^2 L \gamma \cdot \sin\theta = 0$

$$\Rightarrow \Delta P \cdot \pi r^2 - \pi r^2 L \gamma \sin\theta = \tau \, 2\pi r L$$

$$\tau = \frac{r}{2} \cdot \frac{(\Delta P - \gamma L \sin\theta)}{L}$$

(ii)



By applying B.E. at point ① and ②

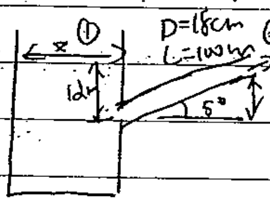
$$P_1 + \tfrac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \tfrac{1}{2}\rho V_2^2 + \rho g h_2 + H_L$$

$V_1 = V_2 \qquad h_2 - h_1 = L \sin\theta \qquad f = \frac{8\tau_w}{\rho V^2} \qquad H_L = f \cdot \frac{L}{2r} \cdot \frac{V^2}{2g} = \frac{2\tau_w \cdot L}{r}$

$$\Rightarrow P_1 - P_2 = \Delta P = \rho g (h_2 - h_1) + H_L = \rho g L \sin\theta + \frac{2\tau_w L}{r}$$

$$\Delta P = \gamma \cdot L \cdot \sin\theta + \frac{2\tau_w \cdot L}{r} \Rightarrow \tau_w = \frac{r}{2} \frac{(\Delta P - \gamma L \sin\theta)}{L}$$

(b)



$\nu = 1 \times 10^{-6}\, m^2/s$

By Applying B.E. at point ① and ②

$$P_1 + \tfrac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \tfrac{1}{2}\rho V_2^2 + \rho g h_2 + H_L \cdot \rho g$$

$P_1 = P_2 = P_{atm} \qquad V_0 = 0\, m/s \quad V_2 = 2\, m/s \qquad h_1 - h_2 = 12 - 100 \cdot \sin 5° = 3.284\, m$

$$H_L = f \cdot \frac{L}{2r} \cdot \frac{V_2^2}{2g}$$

By rearranging the equation. $\qquad \rho g h_1 - \rho g h_2 = \rho g \times 3.284 = \tfrac{1}{2}\rho V_2^2 + H_L \cdot \rho g$

$$\Rightarrow H_L = \cancel{30216.04}\ 3.08$$

$$Re = \frac{V \cdot D}{\nu} = 360000 \qquad \Rightarrow f = 0.0272$$

Now we know Re and f, we can find $\frac{\varepsilon}{D} = 0.003$ from the Moody diagram $\Rightarrow \varepsilon = 0.003 \times 0.18 = 5.4 \times 10^{-4}\, m$

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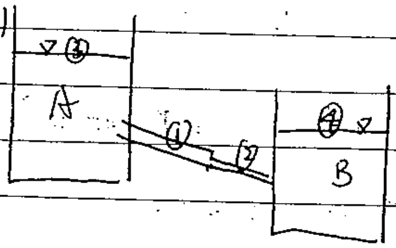
4 (a) (i)



$Re = \frac{\rho V D}{\mu} = \frac{\frac{4Q}{\pi D^2} D}{\nu} = \frac{4Q}{\pi D \nu} = \cancel{\frac{4\times 60\times 10^{-3}}{\pi\times}}$

$Q_1 = Q_2 = 60\ L/s$

$Re_1 = \frac{4Q_1}{\pi D_1 \nu} = \frac{4\times 60\times 10^{-3}}{\pi\times 0.2\times 1\times 10^{-6}} = 254648$

$Re_2 = \frac{4Q_2}{\pi D_2 \nu} = 339530$

$\frac{\varepsilon_1}{D_1} = \frac{0.26\times 10^{-3}}{0.2} = 1.3\times 10^{-3} \qquad \frac{\varepsilon_2}{D_2} = 1.73\times 10^{-3}$

Since we already their Re and roughness, we can find their f respectively

$f_1 = 0.022 \qquad f_2 = 0.023$

By apply B.E. at point ③ and ④

$$\frac{P_3}{\rho g} + \frac{1}{2}\frac{V_3^2}{g} + h_3 = \frac{P_4}{\rho g} + \frac{1}{2}\frac{V_4^2}{g} + h_4 + H_L$$

$P_3 = P_4 = P_{atm} \quad V_3 = V_4 = 0$

$\Rightarrow h_3 - h_4 = \Delta h = H_L = (0.5 + f_1 \cdot \frac{L_1}{D_1})\frac{V_1^2}{2g} + (0.26 + 1 + f_2 \cdot \frac{L_2}{D_2})\frac{V_2^2}{2g}$

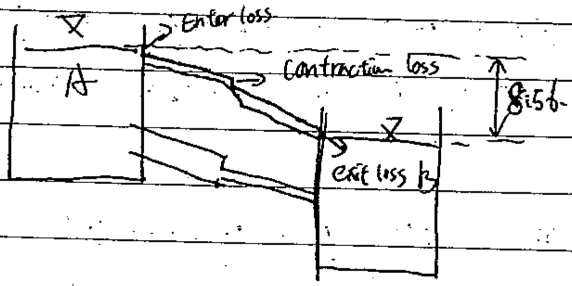
$= \frac{11.5 V_1^2}{2g} + \frac{24.26 V_2^2}{2g} = 0.586\left(\frac{4Q}{\pi D_1^2}\right)^2 + 1.2366\left(\frac{4Q}{\pi D_2^2}\right)^2$

$= 2.2 + 6.36 = 8.56\ m$

$h_A = h_B + \Delta h = 100 + 8.56 = 108.56\ m$.

(ii)

7.285 m



(iii) $H_L = \Delta h = (1.5 + f \cdot \frac{L'}{D'})\frac{V^2}{2g} = 8.56\ m$

$\Rightarrow V = \cancel{105}\ \cancel{100\ L/s}$ 2.633 m/s

$Q = 100\ L/s$

NS7

(b). the dynamic similarity based on Reynolds number

$Re_m = Re_p \Leftrightarrow \left(\frac{\rho V D}{\mu}\right)_m = \left(\frac{\rho V D}{\mu}\right)_p$

$\Rightarrow \frac{V_m}{V_p} = \frac{D_p}{D_m} = \frac{25}{1} \quad V_m = 2.6\ m/s \quad V_p = 0.104\ m$

$\frac{Q_m}{Q_p} = \frac{1}{25} \quad Q_m = 30\ L/s \Rightarrow Q_p = 750\ L/s \quad \frac{F_m}{F_p} = \frac{L_m^2 V_m^2}{L_p^2 V_p^2} = \frac{1}{\cancel{25}\,1} \quad F_p = 50\ N$