NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2008-2009

CV2601 - Fluid Mechanics

November 2008 **Time Allowed: 2½ hours**

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **SIX (6)** pages.
2. Answer all **FOUR (4)** questions.
3. An **Appendix** of **ONE (1)** page is attached to the paper.
4. All questions carry equal marks.

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1. (a) A piston with a mass m of 10 kg slides down a vertical cylinder as shown in Figure Q1(a). The average gap between the piston and cylinder is 0.05 mm, and the gap is filled with lubricating oil which has a dynamic viscosity μ of 0.4 N.s/m^2. Using Newton's second law of motion, F = m.a, derive a general equation relating the instantaneous velocity V (m/s) and acceleration a (m/s^2) of the piston. What is the terminal velocity V_t (m/s) of the piston, when its self weight is balanced by shear resistance on its surface?

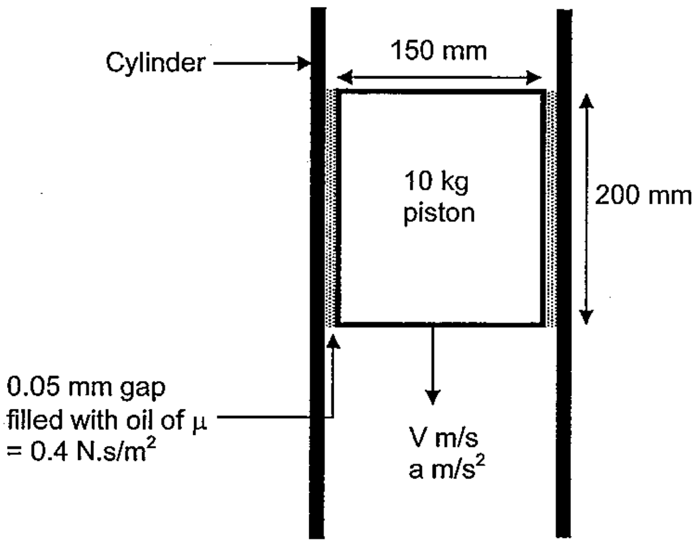


Figure Q1(a)

(8 marks)

Note: Question No. 1 continues on page 2

(b) Find the horizontal and vertical components of the hydrostatic thrust acting on the 2 m long quadrant gate in Figure Q1(b), which is hinged about the centre of the quadrant. What is the magnitude of the resultant hydrostatic thrust, and the angle which it makes with the water surface? Explain why you would expect the resultant hydrostatic thrust to have zero turning moment about the hinge.

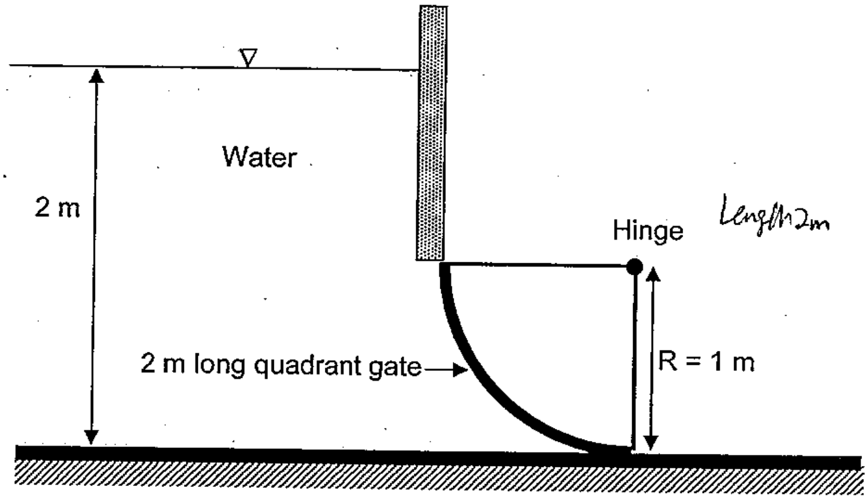


Figure Q1(b)

(12 marks)

(c) A 0.3 m solid cube floats in equilibrium in a mercury-water system as shown in Figure Q1(c). Compute the weight of the cube.

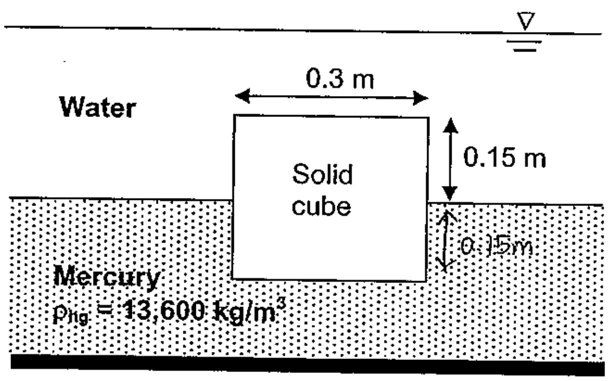
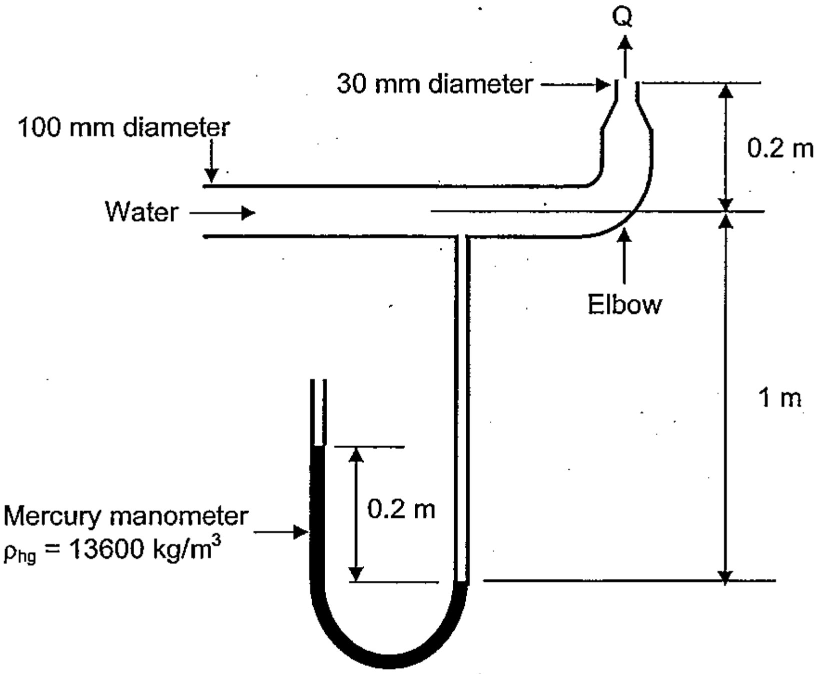


Figure Q1(c)

(5 marks)

2. (a) Water flows through a vertical pipe elbow and nozzle as shown in Figure Q2(a). A mercury manometer measures the pressure at the start of the elbow as shown in the figure. What is the volume flow rate of water Q through the nozzle? State any assumptions made in your computations.



<u>**Figure Q2(a)**</u>

(12 marks)

Note: Question No. 2 continues on page 3

(b) The converging pipe bend which lies on a horizontal plane turns water flowing at 0.05 m^3/s through an angle of 135° as shown in Figure Q2(b). Determine the magnitude of the resultant hydro-dynamic force acting on the bend, if pressure at the start of the bend is measured to be 150 kPa.

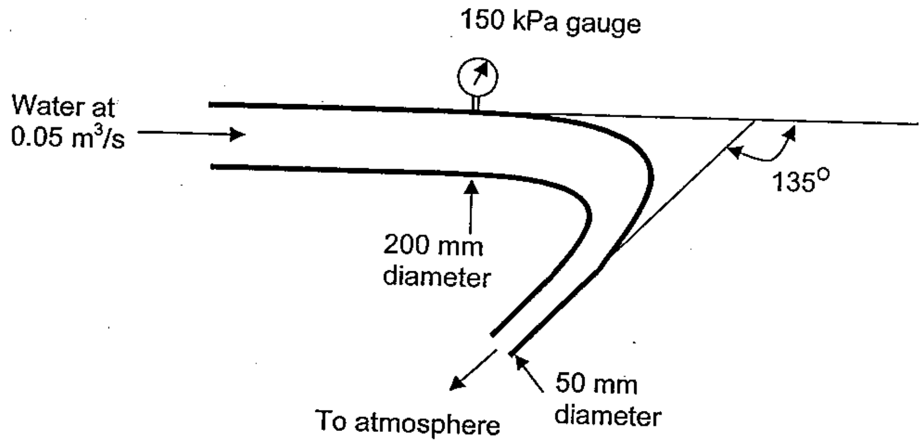


Figure Q2(b)

(13 marks)

3. (a) (i) For turbulent flows in a long, horizontal, circular pipe, the wall shear stress τ_w is a function of fluid density ρ, fluid viscosity μ, average flow velocity V, pipe diameter D, and wall roughness height ε. Using Buckingham's Π-Theorem, show that the above dimensional function may be reduced to

$$\frac{\tau_w}{\rho V^2} = g_1\left(\frac{\rho VD}{\mu}, \frac{\varepsilon}{D}\right)$$

(6 marks)

(ii) Given that the pressure drop Δp between two points along a horizontal, uniform, circular pipe is

$$\Delta p = \frac{4L\tau_w}{D}$$

where L = distance between the two points, and using the Bernoulli equation, show that the friction factor f is also related to the two Π-terms in part (i) such that

$$f = g_2\left(\frac{\rho VD}{\mu}, \frac{\varepsilon}{D}\right)$$

(5 marks)

(b) Water flows from a large tank through a smooth pipe before discharging into the atmosphere as shown in Figure Q3. The pipe consists of a 0.3-m length vertical section, which is connected to a 2-m length horizontal section through an elbow. The water level in the tank is kept at a constant of 0.8 m vertically above the horizontal section of the pipe. Calculate the pipe diameter if the total flow rate through the system is 5 L/s when the globe valve is fully opened. The entrance, elbow and globe valve loss coefficients are 0.3, 0.5 and 10, respectively. Use the Blasius equation to compute the friction factor f of the smooth pipe, where $f = \dfrac{0.316}{Re^{\frac{1}{4}}}$. The kinematic viscosity of water = 1 x 10^{-6} m^2/s.

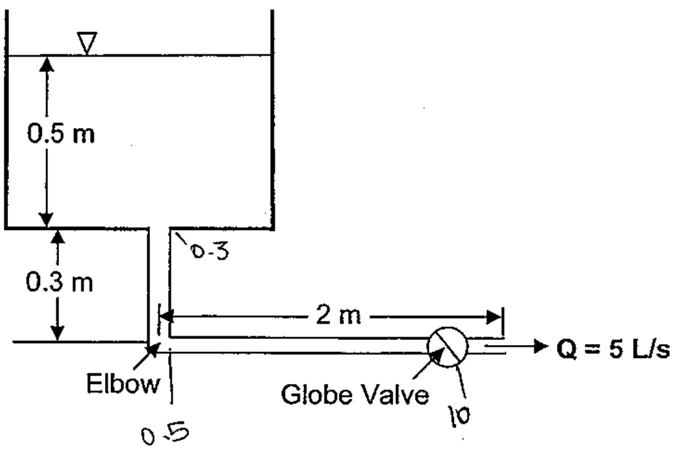


Figure Q3

(14 marks)

4. (a) Water with a kinematic viscosity of 1.0 x 10^{-6} m^2/s flows from the reservoir through a 18-cm diameter inclined pipe (inclined at 5° to the horizontal) with a length = 100 m as shown in Figure Q4. The flow emerges from the pipe as a free jet into the atmosphere. Calculate the roughness height of the pipe if the exit velocity is 2 m/s, and hence sketch the total energy line of the flow. Clearly indicate the magnitude of total head and energy loss in your sketch.

[**Hint**: Ignore all minor losses in your computation; you need to use the Moody Diagram in the Appendix for your computation.]

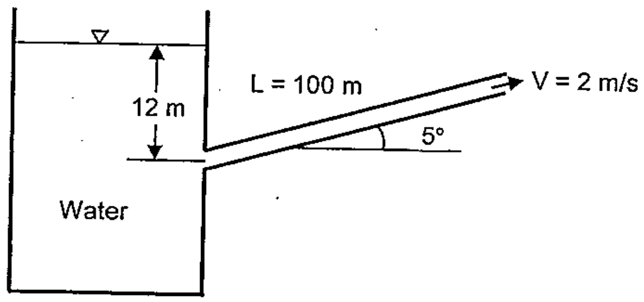


Figure Q4

(12 marks)

(b) A pump is used to deliver water with a density of 1,000 kg/m^3 from one reservoir to another. The flow rate through the pump is 60 L/s, and the diameters of the inlet and delivery sections of the pump are 100 mm and 50 mm, respectively. Calculate the power added to the flow if the pump outlet section is 1.5 m above the inlet, and the static pressure reading at the outlet section is 80 kPa higher than at the inlet section.

(7 marks)

(c) A 1:30 hydraulic model of a spillway is constructed in the laboratory. If the discharge in the prototype is 250 m^3/s, calculate the discharge in the model to ensure dynamic similarity. If the force acting on a certain part of the model is found to be 25 N, calculate the corresponding force in the prototype.

(6 marks)

END OF PAPER

(set 1)

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1.

(a)

$\tau = \mu \frac{du}{dy} = 0.4 \times \frac{V}{0.05\times10^{-3}} = 8\times10^{3}V$

$A = 2\pi r L = 2\pi \times \frac{0.15}{2} \times 0.2 = 0.0942\ m^3$

$F = \tau A = 8\times10^3 V \times 0.0942 = 753.6V$

since $Mg - F = ma$

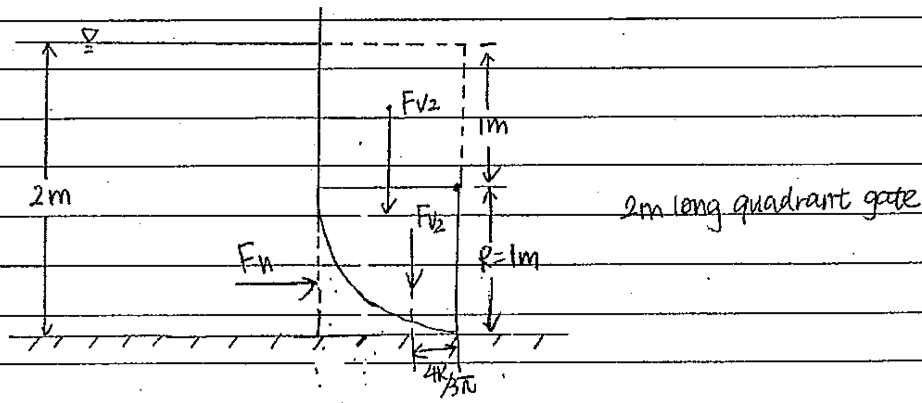
$98.1 - 753.6V = 10a$

$a = 9.81 - 75.36V$

when the piston reaches its terminal velocity, $a=0$

which means $9.81 - 75.36V_t = 0$, $V_t = 0.13\ m/s$

(b)



$F_h = \rho g A h_c = 9810 \times (1\times2)\times(1+\frac{1}{2}) = 29.43\ kN$

$F_{v1} = \rho g V_1 = 9810 \times (\frac{1}{4}\pi \times 1^2 \times 2) = 15.41\ kN$

$F_{v2} = \rho g V_2 = 9810 \times (1\times1\times2) = 19.62\ kN$

$\Delta y = I_c / A y_c = (\frac{1}{12}\times 2\times1^3)/(1\times2\times1.5) = 0.056m$

$F_v = F_{v1} + F_{v2} = 15.41 + 19.62 = 35.03\ kN$

$F = (F_h^2 + F_v^2)^{1/2} = (29.43^2 + 35.03^2)^{1/2} = 45.75\ kN$

$\theta = \tan^{-1}(F_v/F_h) = \tan^{-1}(35.03/29.43) = 50^\circ$

$M_o = F_h(\frac{1}{2}+\Delta y) - F_{v1}\times(\frac{4R}{3\pi}) - F_{v2}(\frac{1}{2})$

$= 29.43(\frac{1}{2}+0.056) - 15.41 \times (\frac{4}{3\pi}) - 19.62\times\frac{1}{2} = 0$

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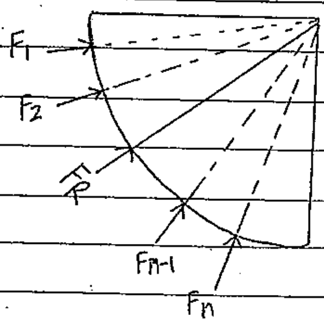
Explanation:

At every point the hydrostatic thrust is alway perpendicular to the gate, thus the thrust at each point would pass through the centre of the circle, which is the hinge. So the sum of hydrostatic thrust at each point, which is the resultant thrust, will pass through the hinge and the moment about the hinge would be zero.



(c)

$W = F_b$

$W = \rho_M g V_i = 13600 \times 9.81 \times (0.15 \times 0.3 \times 0.3) = 1.801\ kN$

$W = \rho g V_{wt}$

2.

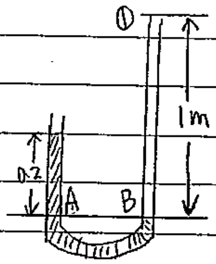
(a)

Assumption:

① Ideal Fluid Flow, which is zero viscosity and incompressible

② It's uniform normal flow, Q is constant along the streamline.

③ No energy loss during the motion of flow.



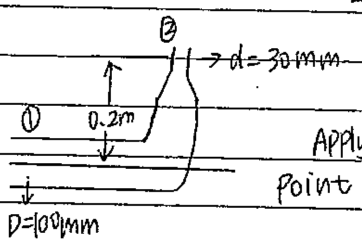
At point A and B: $P_A = P_B$

$\rho_{Hg} g (0.2) = P_1 + \rho_w g (1)$

$P_1 = \rho_{Hg} g \times 0.2 - \rho_w g \times 1$

$= 13600 \times 9.81 \times 0.2 - 1000 \times 9.81 \times 1$

$= 16.873\ kPa$



Apply Continuity Equation and Bernalli Equation to point ① and ②:

$$A_1 V_1 = A_2 V_2 \Rightarrow \pi \left(\frac{0.1}{2}\right)^2 V_1 = \pi \left(\frac{0.03}{2}\right)^2 V_2 \quad ①$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 \Rightarrow \frac{16873}{9810} + \frac{V_1^2}{2g} = \frac{V_2^2}{2g} + 0.2 \quad ②$$

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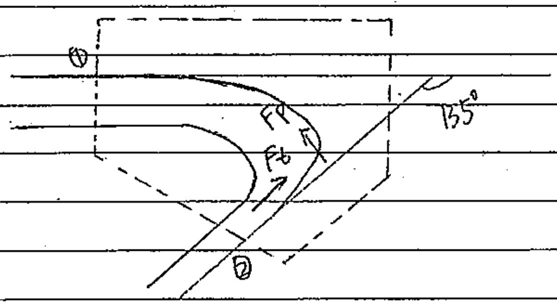
solve Equation ① & ② :

$V_1 = 0.49\ m/s,\quad V_2 = 5.48 m/s$

$Q = A_1V_1 = \pi(\frac{0.1}{2})^2 \times 0.49 = 3.85\times10^{-3}\ m^3/s$

(b)



Ft: Force along the pipe Fp: Force perpendicular to the pipe.

Since $Q = 0.05\ m^3/s$, $D = 200 mm$, $d = 50 mm$

$V_1 = \frac{Q}{\pi(\frac{D}{2})^2} = \frac{0.05}{\pi\times0.1^2} = 1.59 m/s$, $V_2 = \frac{Q}{\pi(\frac{d}{2})^2} = \frac{0.05}{\pi\times0.025^2} = 25.46 m/s$

Apply moment Equation Along plate:

$Ft = \rho QV_2 - (-\rho QV_1\cos45^\circ) = \rho Q(V_2 + V_1\cos45^\circ)$

$= 1000\times0.05\ (25.46 + 1.59\times\frac{\sqrt{2}}{2}) = 1.33 kN$

Apply moment Equation perpendicular to plate:

$Fp = 0 - (-\rho QV_1\sin45^\circ) = \rho QV_1\sin45^\circ$

$= 1000\times0.05\times1.59\times\frac{\sqrt{2}}{2} = 0.056 kN$

$F = \sqrt{Ft^2+Fp^2} = \sqrt{1.33^2+0.056^2} = 1.331\ kN$

3

(a)

(i) $[\tau_w] = ML^{-1}T^{-2}$, $[\rho] = ML^{-3}$, $[\mu] = ML^{-1}T^{-1}$

$[V] = LT^{-1}$, $[D] = L$, $[\varepsilon] = L$

$n = k - r = 6 - 3 = 3$, select ρ, V, D as repeating variables.

$\Pi_1 = \tau_w\rho^a V^b D^c$

$(ML^{-1}T^{-2})(ML^{-3})^a(LT^{-1})^b(L)^c = M^0L^0T^0$

For M^0: $1 + a = 0$

L^0: $-1 - 3a + b + c = 0$

T^0: $-2 - b = 0$

$a = -1,\ b = -2,\ c = 0,\quad \Pi_1 = \tau_w/(\rho V^2)$

Similarly,

$\Pi_2 = \mu \rho^a V^b D^c$

$[ML^{-1}T^{-1}][ML^{-3}]^a[LT^{-1}]^b[L]^c = M^0L^0T^0$

$a=-1, \quad b=-1, \quad c=-1$

$\Pi_2 = \mu/(\rho V D)$

$\Pi_3 = \varepsilon \rho^a V^b D^c$

$[L][ML^{-3}]^a[LT^{-1}]^b[L]^c = M^0L^0T^0$

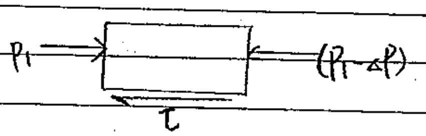
$a=0, \quad b=0, \quad c=-1$

Check Π_1, Π_2 and Π_3 are all dimensionless.

Thus, $\frac{\tau_w}{\rho V^2} = g\left(\frac{\mu}{\rho V D}, \frac{\varepsilon}{D}\right)$

Which is $\frac{\tau_w}{\rho V^2} = g_1\left(\frac{\rho V D}{\mu}, \frac{\varepsilon}{D}\right)$

(ii)



By balancing the forces acting on a fluid element:

$P_1(\pi r^2) - (P_1-\Delta P)\pi r^2 - \tau(2\pi r L) = 0$

$\Delta P = 2\tau L/r$

At the pipe wall, $\tau = \tau_w$, $r = D/2$

$\Delta P = \frac{4\tau_w L}{D}$

By Bernoulli Equation: $\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$

Along a horizontal, uniform, circular pipe: $V_1 = V_2$, $z_1 = z_2$

$h_L = \Delta P/\rho g = \frac{4L\tau_w}{\rho g D}$, since $\cancel{h_L = f\frac{L}{D}\frac{V^2}{2g}}$

~~we have~~ $\cancel{f = \frac{\Delta P D}{\frac{1}{2}\rho V^2 L} = g_2\left(\frac{\rho V D}{\mu}, \frac{\varepsilon}{D}\right)}$

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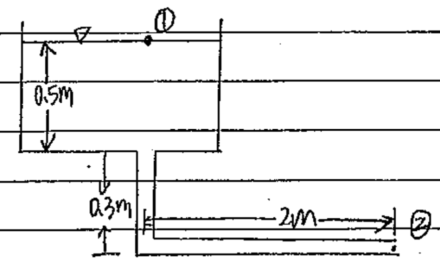
i(b)



Apply Bernoulli Equation to point ① & ②

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \Delta h$$

$$0.8 = \frac{V_2^2}{2g} + \Delta h \quad ①$$

$$\Delta h = h_L + \Sigma k_i \frac{V^2}{2g} = f\frac{L}{D}\frac{V^2}{2g} + (0.3 + 0.5 + 10)\frac{V^2}{2g} \quad ②$$

$$f = \frac{0.316}{Re^{\frac{1}{4}}} = \frac{0.316}{(\frac{\rho VD}{\mu})^{\frac{1}{4}}} \quad ③$$

since $Q = AV = \frac{\pi}{4}D^2 V = 5\times10^{-3} m^3/s$, $V = \frac{6.366}{D^2}\times10^{-3}$ ④

Rearrange the equations:

$$Re = \frac{\rho VD}{\mu} = \frac{10^3\times\frac{6.366}{D^2}\times10^{-3}\times D}{10^{-6}\times10^{3}} = \frac{6366}{D}$$

$$f = \frac{0.316}{\sqrt[4]{\frac{6366}{D}}} = 0.0354 D^{1/4}$$

$$\text{Since } 0.8 = \frac{V^2}{2g} + f\frac{L}{D}\frac{V^2}{2g} + 10.8\frac{V^2}{2g} = 11.8\frac{V^2}{2g} + f\frac{L}{D}\frac{V^2}{2g}$$

$$[11.8 + 0.0354D^{1/4}\times\frac{2}{D}](\frac{6.366\times10^{-3}}{D^2})^2 = 1.6g$$

$$478.254\times10^{-6}D^{-4} + 2.87\times10^{-6}D^{-\frac{19}{4}} = 15.7$$

$$478.254D^{-4} + 2.87D^{-19/4} = 15.7\times10^{6}$$

By trial and error,

$$D = 0.0744m$$

The pipe diameter should be 74.4mm

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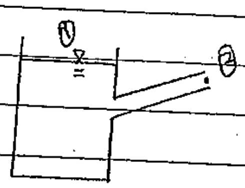
4.

(a)



Apply Bernoulli Equation to point ① & ②

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \Delta h$$

$$12 = \frac{2^2}{2g} + 100\sin 5^\circ + \Delta h$$

$$\Delta h = 3.081\,m$$

Since $\Delta h = f\frac{L}{D}\frac{V^2}{2g} = f\,\frac{100}{0.18}\,\frac{2^2}{2g}$, $f = 0.027$

$$Re = \frac{\rho V D}{\mu} = \frac{10^3 \times 2 \times 0.18}{10^{-6} \times 10^3} = 3.6 \times 10^5$$

According to the Moody Diagram, we can find $\frac{\varepsilon}{D} = 0.002$

$\varepsilon = 0.002D = 0.002 \times 0.18 = 3.6 \times 10^{-4}\,m$.

(b) Bernoulli equation applied to the inlet and outlet:

$$\left(\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1\right) + h_p = \left(\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2\right)$$

$$h_p = \frac{\Delta P}{\rho g} + \frac{V_2^2 - V_1^2}{2g} + \Delta z$$

$$= \frac{80 \times 10^3}{9810} + \frac{\left(\frac{60 \times 10^{-3}}{\pi 0.025^2}\right)^2 - \left(\frac{60 \times 10^{-3}}{\pi 0.05^2}\right)^2}{2 \times 9.81} + 1.5$$

$$= 885.96\,m$$

$$P = \rho g Q h_p = 9810 \times 60 \times 10^{-3} \times 885.96 = 521.48\,kW$$

(c)

① Froude number similarity:

$$\frac{V_m}{\sqrt{gL_m}} = \frac{V_p}{\sqrt{gL_p}} \qquad \frac{V_m}{V_p} = \left(\frac{L_m}{L_p}\right)^{\frac{1}{2}} \qquad \frac{Q_m}{Q_p} = \frac{V_m L_m^2}{V_p L_p^2} = \left(\frac{L_m}{L_p}\right)^{2.5}$$

$$Q_m = Q_p\left(\frac{L_m}{L_p}\right)^{2.5} = 250 \times \left(\frac{1}{30}\right)^{2.5} = 0.051\,m^3/s$$

② Linear Moment Equation: $F_n = \Delta \dot{M} = \Delta \rho Q V$.

$$\frac{F_m}{F_p} = \frac{Q_m V_m}{Q_p V_p} = \left(\frac{L_m}{L_p}\right)^{2.5}\left(\frac{L_m}{L_p}\right)^{\frac{1}{2}} = \left(\frac{L_m}{L_p}\right)^3$$

$$F_p = F_m / \left(\frac{1}{30}\right)^3 = 675\,KN$$

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1 (a) $m = 10\,kg$; $d = 0.05\,mm = 5\times10^{-5}\,m$; $D = 150\,mm$; $L = 200\,mm$)

$\mu = 0.4\,N\cdot s/m^2 = 0.4\,kg/s\cdot m$

$\tau = \mu \frac{v}{d} =$

$a m = mg - f \Rightarrow a = g - \frac{\mu}{md} v (DL) = 9.81\,m/s^2 - \frac{0.4\,kg/s\cdot m}{10\,kg \times 0.5\times10^{-4}\,m} \times 0.03\,m^2 \times v$

$= 9.81\,m/s^2 - 24\,s^{-1}\cdot v$

Terminal velocity occurs when $a = 0$

that is $9.81\,m/s^2 = 24\,s^{-1}\cdot v$

$v = 0.40875\,m/s$

(b) $L = 2m$; $R = 1\,m$.

$F_v = \gamma V = \gamma (A_v L) = 9.81\,kN/m^3 \times [(\frac{1}{4}\pi \times (1m)^2 + 1m\times 1m) \times 2m]$

$= 35.0217\,kN$

$F_h = \gamma \bar{h} A_h = 9.81\,kN/m^3 \times 1.5m \times (\cancel{\frac{1}{4}\times(2\pi\,m)}\ \times 1m \times 2m) = \cancel{46.2051}\ 29.43\,kN$

$\Rightarrow$ Resultant Force : $F = \sqrt{F_v^2 + F_h^2} = \cancel{57.9778\,kN}\ 45.7454\,kN$

$M_v = F_v \cdot L_v = (\frac{\pi}{4} \times (1m)^2 \times \frac{4}{3\pi} \times 1m + 1m^2 \times 0.5m) \times 2m \times 9.81\,kN/m^3 = 16.35\,kN\cdot m$ CCW

$M_F = F_h \cdot L_h = F_h \times L_h = 29.43\,kN \times \frac{(\frac{1}{2}\times 1 + \frac{2}{3}\times\frac{1}{2})}{3/2}\,m = 16.35\,kN\cdot m$ CW

$\Rightarrow$ Resultant Moment around ~~point~~ Hinge is 0

As a result of the pressure is always perpendicular to the surface

次 [illegible]

$\Delta H_1 = 1\,m$, $\Delta H_2 = 0.2m$ $\Delta H_3 = 0.2m$

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2(a). At point 1: $p_1 + \rho_W g\ \Delta H_1 = 0 + \rho_{Hg}\, g\ \Delta H_3$

$$p_1 = \rho_{Hg}\, g\, \Delta H_3 - \rho_W g \Delta H_1$$

$$= 13.6 \cdot 9.81\ kN/m^3 \times 0.2m - 9.81\,kN/m^3 \times 1\,m = 16.8732\ kPa$$

Apply B.E to ① → ②.

$$\frac{V_1^2}{2g} + z_1 + \frac{p_1}{\gamma_W} = \frac{V_2^2}{2g} + z_2 + \cancel{\frac{p_2}{\gamma}}$$

$$\frac{V_1^2 - V_2^2}{2g} = (z_2 - z_1) - \frac{P_1}{\gamma_W} \quad \cdots\cdots ①$$

Apply C.E to ① – ②.

$$V_1 \cdot A_1 = V_2 \cdot A_2 = Q$$

$$\frac{V_2}{V_1} = \frac{A_1}{A_2} = \left(\frac{D_1}{D_2}\right)^2 = \left(\frac{100mm}{30mm}\right)^2 = \frac{100}{9} \quad \cdots$$

substitude into ①

$$\Rightarrow \frac{(1 + 0.09^2)\ V_1^2}{2g} = \Delta H_2 - \frac{P_1}{\gamma_W}$$

$$V_1 = \sqrt{\left(\Delta H_2 - \frac{P_1}{\gamma_W}\right) 2g \Big/ \left(1 - \frac{100}{9}\right)} = 0.4935\,m/s$$

$$Q = V_1\left(\frac{1}{4}\pi D_1^2\right) = 0.4935\,m/s \times \left[\frac{1}{4} \times 3.14 \times (0.1\,m)^2\right] =$$
$$= 3.874 \times 10^{-3}\,m^3/s$$

① No Energy Loss in the Elbow, nozzle

② Steady Flow

③ Friction loss

③ Fluid is incompressible

2(b) Apply C.E to ① → ②

$$V_1 \cdot A_1 = V_2 \cdot A_2$$

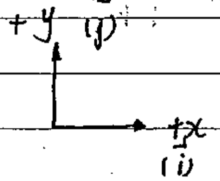
$$\frac{V_1}{V_2} = \frac{A_2}{A_1} = \left(\frac{D_2}{D_1}\right)^2 = \left(\frac{50\,mm}{200\,mm}\right)^2 = \frac{1}{16}$$

Apply B.E to ① → ②

$$\frac{V_1^2}{2g} + \cancel{z_1} + \frac{P_1}{\gamma} = \frac{V_2^2}{2g} + \cancel{z_2} + \cancel{\frac{P_2}{\gamma}}$$

$$\frac{V_2^2 - V_1^2}{2g} = \frac{P_1}{\gamma}$$

$$(16-1)\ V_1^2 = \frac{2P_1}{\rho} \Rightarrow V_1 = \sqrt{\frac{2 \times 150\ kpa}{15 \times 10^3 kg/m^3}} = 4.472\ m/s$$



$$\vec{V_1} = 4.472\ \hat{i}\ m/s$$

$$\vec{V_2} = 16 \times 4.472\ m/s \times \left(\cos(-135°)\hat{i} + \sin(-135°)\hat{j}\right)$$

$$= [-50.5949\ \hat{i} - 50.5949\ \hat{j}]\ m/s$$

$$F = \dot{\vec{M}}_{12} - \dot{\vec{M}}_1$$

$$= \rho Q \vec{V_2} - \rho Q \vec{V_1}$$

$$= 10^3 kg/m^3 (4.472 \times 3.14 \times 0.1^2\ m^3/s) \times (-55.0669\ \hat{i} - 50.5949\ \hat{j})$$

$$= [-7.7325\ \hat{i} - 7.1046\ \hat{j}]\ kN$$

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3(a) (i) τ_w, ρ, V, μ, ε, D

① $\tau_w = f(\rho, V, \mu, \varepsilon, D)$

② select reapeating variables : ρ, V, D

$\tau_w : M L^{-1} T^{-2}$ $\qquad \rho : M L^{-3}$

$\mu : M \cdot L^{-1} T^{-1}$ $\qquad V : L T^{-1}$

$D : L$ $\qquad \varepsilon : L$

$$\pi_1 = \tau_w \rho^a V^b D^c$$

$$= M^{1+a} L^{-1-3a+b+c} T^{-2-b}$$

$$\begin{cases} a = -1 \\ 3a+b+1 = c \\ b = -1 \end{cases} \Rightarrow \begin{cases} a = -1 \\ b = -2 \\ c = 0 \end{cases} \Rightarrow \pi_1 = \frac{\tau_w}{\rho V^2}$$

$$\pi_2 = \mu \rho^a V^b D^c$$

$$= (M L^{-1} T^{-2}) (M L^{-3})^a (L T^{-1})^b (L)^c$$

$$= M^{1+a} L^{-1-3a+b+c} + T^{-1-b}$$

$$\begin{cases} a+1 = 0 \\ b+c = 3a+1 \\ b = -1 \end{cases} \Rightarrow \begin{cases} a = -1 \\ b = -1 \\ c = -1 \end{cases} \qquad \pi_2 = \frac{\mu}{\rho V D}$$

$$\pi_3 = \varepsilon \rho^a V^b D^c$$

$$= \frac{\varepsilon}{D} \qquad \pi_3 = \frac{\mu}{D}.$$

$$\therefore \frac{\tau_w}{\rho V^2} = g_1 \left(\frac{\rho V D}{\mu}, \frac{\varepsilon}{D} \right)$$

(ii) $h_L = \frac{\Delta p}{\rho g} = f \frac{L}{D} \times \frac{V^2}{2g}$

$$f = \frac{\Delta p}{\rho g} \times \frac{D}{L} \times \frac{2g}{V^2} = \frac{4\cancel{L}\tau_w}{\cancel{D}} \times \frac{1}{\rho \cancel{g}} \times \frac{\cancel{D}}{\cancel{L}} \times \frac{2\cancel{g}}{V^2}$$

$$= \frac{8\tau_w}{\rho V^2}$$

substitude into (i)

$$\frac{1}{8} f = g_1 \left(\frac{\rho V D}{\mu}, \frac{\varepsilon}{D} \right)$$

$$f = 8 g_1 \left(\frac{\rho V D}{\mu}, \frac{\varepsilon}{D} \right) = g_2 \left(\frac{\rho V D}{\mu}, \frac{\varepsilon}{D} \right)$$

(b) $H_L = (0.3 + 0.5 + 10) \frac{V^2}{2g} + f \cdot \frac{L_1 + L_2}{D} \times \frac{V^2}{2g}$

$$V = \frac{Q}{\frac{\pi}{4}D^2} = \frac{4Q}{\pi D^2} = \frac{4\times5\times10^{-3}}{3.14}\times\frac{1}{D^2} = \frac{6.369\times10^{-3}}{D^2}$$

$$Re = \frac{\rho V D}{\mu} = \frac{\rho D}{\mu} \times \frac{4Q}{\pi D^2} = \frac{4\rho Q}{\pi \mu D} = \frac{4\times10^3\times5\times10^{-3}}{3.14\times1\times10^{-6}} \times \frac{1}{D} = \frac{6.369\times10^6}{D}$$

$$= \left[10.8 + \frac{0.316}{\left(\frac{6.369\times10^6}{D}\right)^{1/4}} \times \frac{23}{D} \right] \times \frac{V^2}{2g}$$

$$= \left[10.8 + \frac{0.01446762}{D^{3/4}} \right] \times \frac{2.0678\times10^{-6}}{D^4}$$

$$\cancel{\frac{V_1^2}{2g}} + \cancel{\frac{p_1}{\gamma}} + z_1 = \frac{V^2}{2g} + \cancel{\frac{p_2}{\gamma}} + z_2 + H_L$$

$$0.8m = z_1 - z_2 = \frac{V^2}{2g} + H_L = \left[11.8 + 0.01446762 \times D^{4/3} \right] \times \frac{2.0678\times10^{-6}}{D^4}$$

$D \approx 0.074$ m ≈ 74mm

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4(a) $H_L = f \cdot \frac{L}{D} \times \frac{V^2}{2g}$ $\qquad Re = \frac{\rho V D}{\mu} = \frac{10^3 \times 2 \times 0.18}{1 \times 10^{-6}} = 0.36 \times 10^9$

$$\cancel{\frac{V_1^2}{2g}} + \cancel{\frac{P_1}{\gamma}} + z_1 = \frac{V_2^2}{2g} + \cancel{\frac{P_2}{\gamma}} + z_2 + H_L \qquad = 3.6 \times 10^6$$

$$(12m - L\sin 5^\circ) = (z_1 - z_2) = \frac{V_2^2}{2g}\left(f \cdot \frac{L}{D} + 1\right)$$

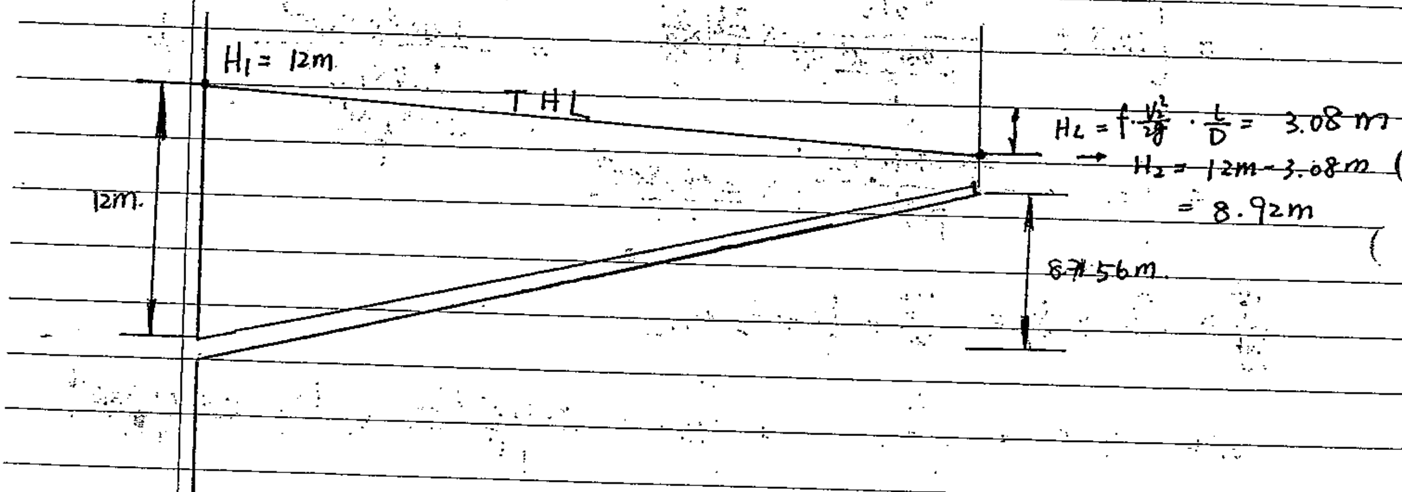
$$\cancel{V_2 = \sqrt{\frac{2g\,(12m - 100m \times \sin 5^\circ)}{f\frac{L}{D} + 1}}}$$

$$f \cdot \frac{L}{D} + 1 = \frac{12 - 100 \times \sin 5^\circ}{V_2^2/2g}$$

$$f = 0.027198$$

from Moody's Diagram $\Rightarrow$ $\frac{\varepsilon}{D} = 0.0035$

$\varepsilon = 0.063\,cm = 0.63\,mm$



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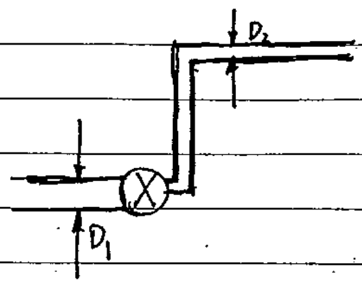
(b) $\rho = 10^3 kg/m^3$

$Q = 60 L/s = 0.06 m^3/s$

$D_1 = 100mm, \quad D_2 = 50mm.$



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

$$h_p = \frac{p_2 - p_1}{\gamma} + \frac{V_2^2 - V_1^2}{2g} + z_2 - z_1$$

$$= \frac{80 kpa}{9.81 kN/m^3} + \frac{\frac{0.06}{\pi \times 0.025^2} - \frac{0.06}{\pi \times 0.05^2}}{2 \times 9.81} + 1.5m$$

$$= 10.82 m$$

$P = h_p \times Q \times \gamma = 10.82m \times 0.06 m^3/s \times 9.81 kN/m^3 = 6.37 \times 10^3 W$

(c) $F_r = \frac{V}{\sqrt{gh}} = \frac{Q}{A \cdot \sqrt{gh}}$

$(F_r)_p = (F_r)_m \Rightarrow \frac{Q_m}{Q_p} = \frac{A_m}{A_p} \times \sqrt{\frac{h_m}{h_p}} = \frac{1}{30} \times \frac{1}{\sqrt{30}}$

$Q_m = 1.52 \; m^3/s$

$\cancel{F =} \rho \quad \frac{V_m}{V_p} = \sqrt{\frac{h_m}{h_p}} = \sqrt{\frac{1}{30}}$

$F = ma = \rho L^3 \cdot \frac{V^2}{L} = \rho L^2 V^2$

$\frac{F_p}{F_m} = \left(\frac{L_p}{L_m}\right)^2 \cdot \left(\frac{V_p}{V_m}\right)^2 = \cancel{A_r} \; 30^2 \cdot 30$

$F_p = 30^3 \times 25 N = 675 kN$