Hoang Thu Minh

CV1011 19-20 S1

Date No.

1.

(a) $\vec{F_A} = -400\hat{k}$ (N) ; $\vec{OA} = 2\hat{i} + 5\hat{j}$

$\vec{F_B} = -600\hat{k}$ (N) ; $\vec{OB} = 10\hat{i} + 10\hat{j}$

$\vec{F_C} = -300\hat{k}$ (N) ; $\vec{OC} = 16\hat{i} + 6\hat{j}$

$\vec{F_R} = \vec{F_A} + \vec{F_B} + \vec{F_C} = -1300\hat{k}$ (N)

Let $M(x, y, z)$ be the point of application of $\vec{F_R}$

Consider moment about O:

$\vec{OM} \times \vec{F_R} = \vec{OA} \times \vec{F_A} + \vec{OB} \times \vec{F_B} + \vec{OC} \times \vec{F_C}$

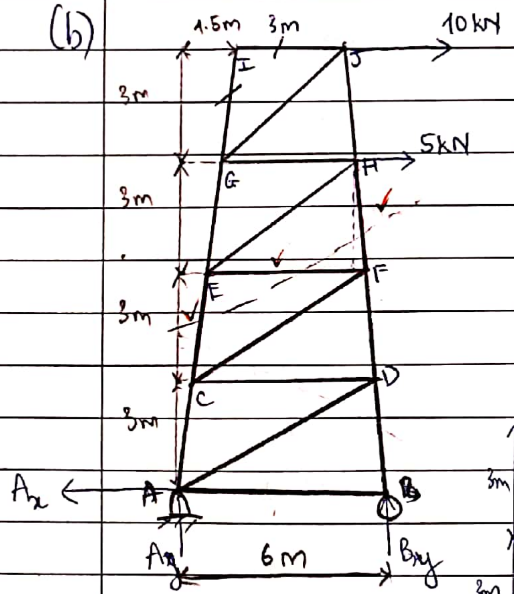
$(x\hat{i} + y\hat{j} + z\hat{k}) \times (-1300\hat{k}) = (2\hat{i} + 5\hat{j}) \times (-400\hat{k}) + (10\hat{i} + 10\hat{j}) \times (-600\hat{k})$
$+ (16\hat{i} + 6\hat{j}) \times (-300\hat{k})$

$-1300y\,\hat{i} + 1300x\,\hat{j} = -2000\hat{i} + 800\hat{j} - 6000\hat{i} + 6000\hat{j} + 4800\hat{j} - 1800\hat{i}$

$\Rightarrow \begin{cases} -1300y = -2000 - 6000 - 1800 = -9800 \\ +1300x = 800 + 6000 + 4800 = 11600 \end{cases}$

$\Rightarrow \begin{cases} x = 8.92 \\ y = 7.54 \end{cases}$

(b)



Zero-force members: IJ, IG

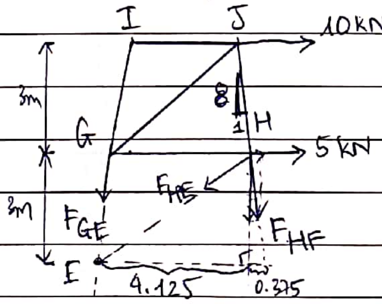
FBD: the whole structure:

$\xrightarrow{+} \Sigma F_x = 0: \quad -A_x + 10 + 5 = 0 \Rightarrow A_x = 15\text{kN}$

$\circlearrowleft \Sigma M_B = 0: \quad -A_y(6) - 5(9) - 10(12) = 0$

$\Rightarrow A_y = -27.5$ kN

$+\uparrow \Sigma F_y = 0: \quad A_y + B_y = 0 \Rightarrow B_y = 27.5\text{kN}$

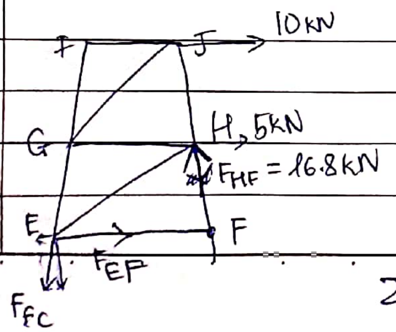


$\circlearrowleft \Sigma M_E = 0:$

$0 = -10(6) - 5(3) - \frac{1}{\sqrt{65}}F_{HF}(3) - \frac{8}{\sqrt{65}}F_{HF}(4.125)$

$\Rightarrow F_{HF} = -16.8$ kN

$= 16.8$ kN (C) (ans)



$\circlearrowleft \Sigma M_F = 0: -5(3) - 10(6) + \frac{8}{\sqrt{65}} F_{EC}(4.5) = 0$

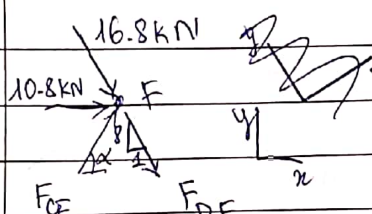
$\Rightarrow F_{EC} = 16.8$ kN (T) (ans)

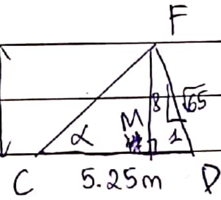
$\Sigma F_x = 0: \; 10 + 5 + F_{EF} - \frac{1}{\sqrt{65}}(16.8) - \frac{1}{\sqrt{65}}(16.8) = 0$

$\Rightarrow F_{EF} = -10.8$ kN (C) (ans)

Date No.

Joint F





$DF = \frac{\sqrt{65}}{8} \times 3 = 3.02\ m$

~~Area of $\Delta CFD = \frac{1}{2}(5.25)(3) =$~~

$MF = 3m$; $MD = \frac{1}{8}(3) = 0.375\ m$

$MC = CD - MD = 4.875\ m$

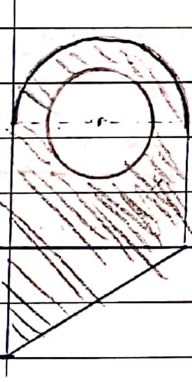
$\alpha = \tan^{-1}(3/4.875) = 31.6°$

$\Sigma F_x = 0: \quad 10.8 + \frac{1}{\sqrt{65}}(16.8) + \frac{1}{\sqrt{65}} F_{DF} + F_{CF}\cos\alpha = 0$

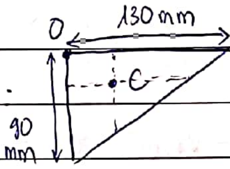
$\Sigma F_y = 0: \quad F_{CF}\sin\alpha - \frac{8}{\sqrt{65}} F_{DF} - \frac{8}{\sqrt{65}}(16.8) = 0$

Solving for $F_{CF} \Rightarrow F_{CF} = -11.8$ kN (C)

2.
(a) The measurements in the problem are wrong!



Part 1:

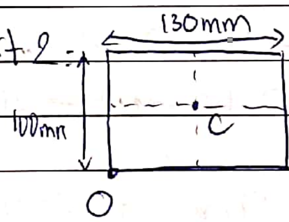


$x_1 = \frac{130}{3}$ mm

$y_1 = -30$ mm

$A_1 = \frac{1}{2}(90)(130) = 5850\ mm^2$

Part 2:

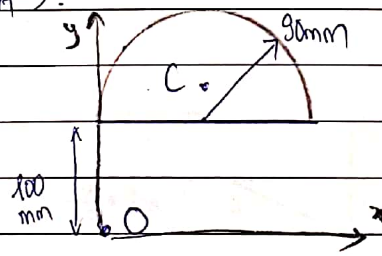


$x_2 = \frac{130}{2} = 65$ mm

$y_2 = 100/2 = 50$ mm

$A_2 = (100)(130) = 13000\ mm^2$

Part 3:

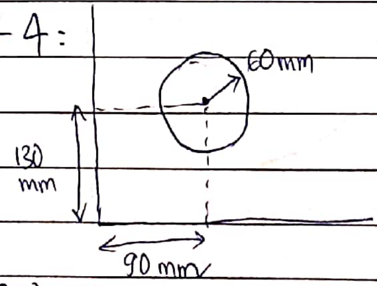


$x_3 = 90$ mm

$y_3 = 100 + \frac{4(90)}{3\pi} = 138.2$ mm

$A_3 = \frac{1}{2}(\pi \times 90^2) = 12723\ mm^2$

Part 4:



$x_4 = 90$ mm, $y_4 = 130$ mm

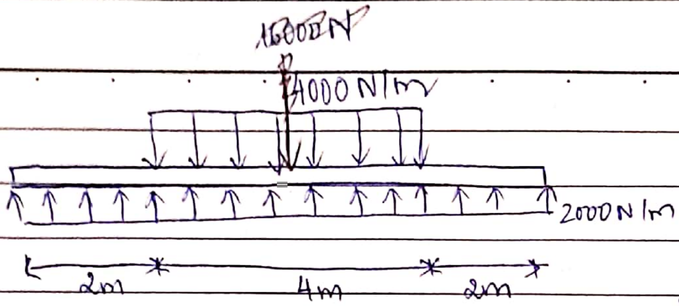
$A_4 = \pi(60^2) = 11310\ mm^2$

$$\bar{x} = \frac{\frac{130}{3} \times 5850 + 65 \times 13000 + 90 \times 12723 - 90 \times 11310}{5850 + 13000 + 12723 - 11310} = 60.5\ mm$$

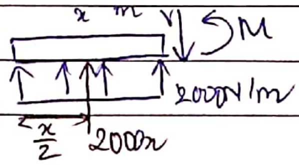
$$\bar{y} = \frac{-30 \times 5850 + 50 \times 13000 + 138.2 \times 12723 - 130 \times 11310}{5850 + 13000 + 12723 - 11310} = -40.8\ mm$$

Besform®

(b)



- For $0 < x < 2$ m:

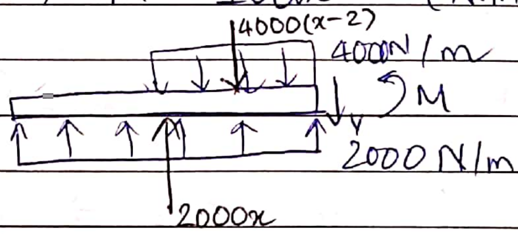


$\uparrow \sum F_y = 0: \; 2000x - V = 0$

$\Rightarrow V = 2000x \quad (N)$

$\circlearrowleft + \sum M = 0: \; M - 200x\left(\frac{x}{2}\right) = 0 \Rightarrow M = 1000x^2 \quad (N.m)$

- For $2 \le x \le 6$ m:



$\uparrow \sum F_y = 0:$

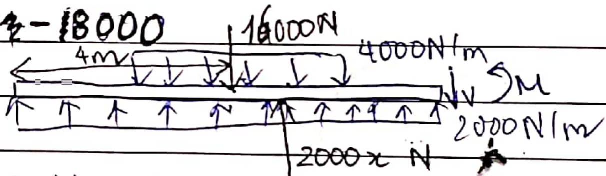
$2000x - 4000(x-2) - V = 0$

$\Rightarrow V = -2000x + 8000 \quad (N)$

$\circlearrowleft + \sum M = 0: \; -2000x\left(\frac{x}{2}\right) + 4000(x-2)\left(2 + \frac{x-2}{2}\right) + M = 0$

$\Rightarrow M = -1000x^2 + 8000x \; \cancel{2} - 18000$

- For $6 < x \le 8$ m:

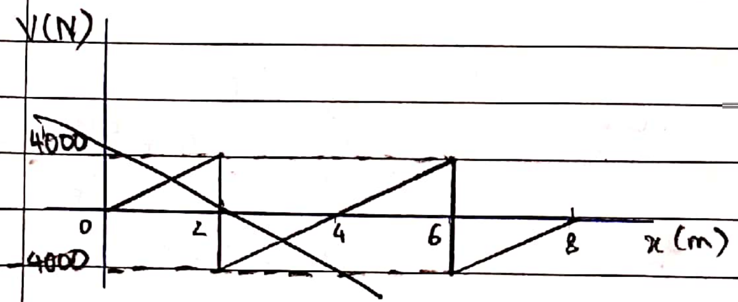


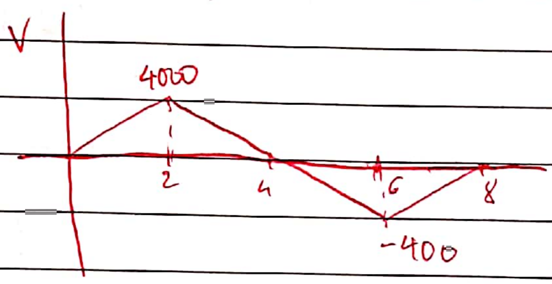
$\uparrow \sum F_y = 0:$

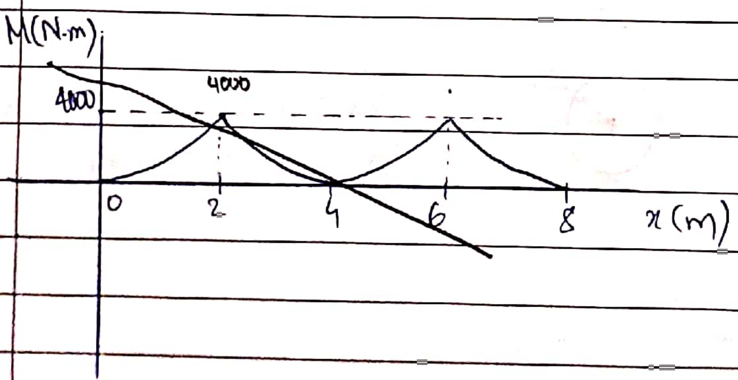
$2000x - 16000 - V = 0 \rightarrow V = 2000x - 16000 \quad (N)$

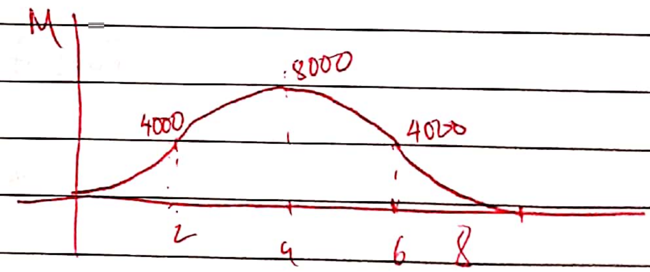
$\circlearrowleft + \sum M = 0: \; +16000(x-4) - 2000x\left(\frac{x}{2}\right) + M = 0$

$\Rightarrow M = 1000x^2 - 16000x + 64000 \quad (N.m)$



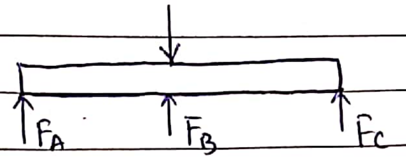






Date No.

3(a)



$\overset{+}{\circlearrowleft} \sum M_B = 0: -F_A(3000) + F_C(3000) = 0$

$\Rightarrow F_A = F_C$

$\overset{+}{\uparrow} \sum F_y = 0: \; F_A + F_B + F_C = 1000(10^3) \Rightarrow F_B = 1000(10^3) - 2F_A$

Compatibility: $\delta_A = \delta_C = \delta_B + 0.001$

$\Rightarrow \frac{F_A L_A}{A_A E_A} = \frac{F_B L_B}{A_B E_B} + 0.001$

$\Rightarrow \frac{F_A(1)}{\frac{\pi}{4}(0.02^2)(70\times10^9)} = \frac{[1000(10^3) - 2F_A](0.999)}{\frac{\pi}{4}(0.02^2)(70\times10^9)} + 0.001$

$\Rightarrow F_A = 340557\,N$

$F_B = 1000(10^3) - 340557\times2 = 318886\,N$

For post B: $\varepsilon_{axial} = \frac{\delta_B}{L_B} = \frac{F_B}{A_B E} = \frac{-318886}{\frac{\pi}{4}(0.02^2)(70\times10^9)} = \cancel{-0.02999}$ -0.0145

$\varepsilon_{lateral} = -\nu\varepsilon_{axial} = -0.35(\cancel{-0.02999}\; -0.0145) = \cancel{0.0105}\; 0.0051\; \cancel{mm}$

$\Delta d = \varepsilon_{lateral}\, d = 0.0051\times20 = \cancel{0.21\,mm}\; 0.102\,mm$

New diameter $d' = d + \Delta d = \cancel{20.21\,mm}\; 20.102\,mm$

(b)

$T_{AB} + T_{BC} = 2t_0$ (1)

Compatibility: $\phi_{AB} = 0 \Rightarrow \frac{-T_{AB}L_{AB}}{GJ_{AB}} + \int_0^{0.2}\frac{t_0 x\,dx}{GJ_{AB}} - \frac{T_{BC}L_{BC}}{GJ_{BC}} = 0$

$\Rightarrow \frac{-T_{AB}(0.2)}{\frac{\pi}{2}(10^4)} + \int_0^{0.2}\frac{t_0 x\,dx}{\frac{\pi}{2}(10^4)} - \frac{T_{BC}(0.5)}{\frac{\pi}{2}(20^4)} = 0$

$-0.2T_{AB} + 0.02t_0 - T_{BC}(0.03125) = 0$ (2)

From (1) & (2) $\Rightarrow T_{AB} = \frac{11t_0}{135}$; $T_{BC} = \frac{16t_0}{135}$

On AB $(\tau_{AB})_{max} = \frac{\frac{11t_0}{135}(0.01)}{\frac{\pi}{2}(0.01^4)} = t_0(51.9\times10^3) = 60(10^6)$

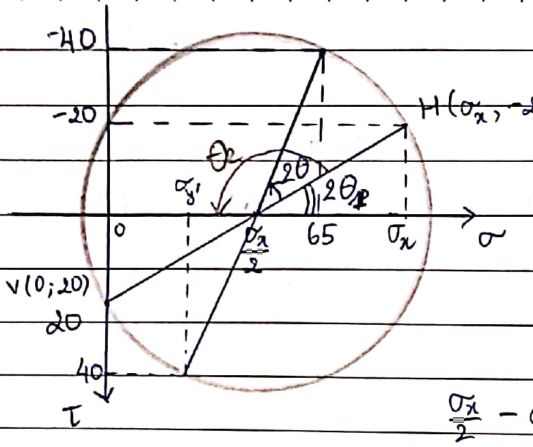
$\Rightarrow t_0 = 1157\,Pa/m$

On BC $(\tau_{BC})_{max} = \frac{\frac{16t_0}{135}(0.02)}{\frac{\pi}{2}(0.02^4)} = t_0(9431.4) = 60(10^6)$

$\Rightarrow t_0 = 6.36(10^3)\,Pa/m.$

Hence, $t_0 = 1157\,Pa/m$

4(a)



$\sigma_y = 0$, $\tau_{xy} = -20$

$\sigma_{avg} = \frac{\sigma_x}{2}$; $\Delta\sigma = \frac{\sigma_x}{2}$

$$R^2 = \left(\frac{\sigma_x}{2}\right)^2 + 20^2 = \left(65 - \frac{\sigma_x}{2}\right)^2 + 40^2$$

$$\Rightarrow \frac{\sigma_x^2}{4} + 400 = 65^2 - 65\sigma_x + \frac{\sigma_x^2}{4} + 40^2$$

$$\Rightarrow \sigma_x = 83.46 \text{ MPa}$$

$$\frac{\sigma_x}{2} - \sigma_y' = 65 - \frac{\sigma_x}{2} \Rightarrow \sigma_y' = 83.46 - 65 = 18.46 \text{ Pa}$$

$$2\theta = \tan^{-1}\left(\frac{40}{65 - \frac{83.46}{2}}\right) - \tan^{-1}\left(\frac{20}{83.46/2}\right) \Rightarrow \theta = \cancel{23.17}\ 17.10^\circ$$

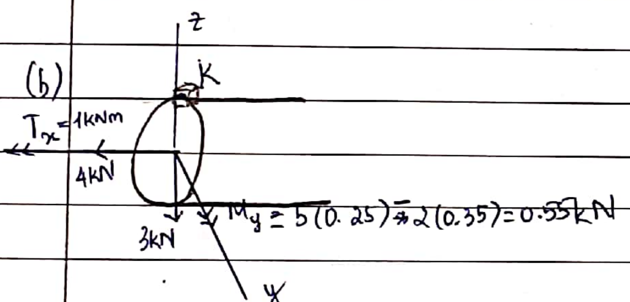
$$R = \sqrt{\left(\frac{83.46}{2}\right)^2 + 20^2} = 46.28$$

Principal stresses: $\sigma_1 = \frac{\sigma_x}{2} + R = \frac{83.46}{2} + 46.28 = 88.01$ MPa with $\theta_1 = -12.8^\circ$

$\sigma_2 = \frac{\sigma_x}{2} - R = -4.55$ MPa with $\theta_2 = 77.2^\circ$

~~$\tau_{max} = 46.28$ MPa~~

(b)



$$A = \pi(20^2) = 1257 \text{ mm}^2$$

$$J = \frac{\pi}{2}(20^4) = 251.3(10^3) \text{ mm}^4$$

$$I = \frac{\pi}{4}(20^4) = 125.7 \times 10^3 \text{ mm}^4$$

Shear stress : – Due to shear force : $\tau = 0$

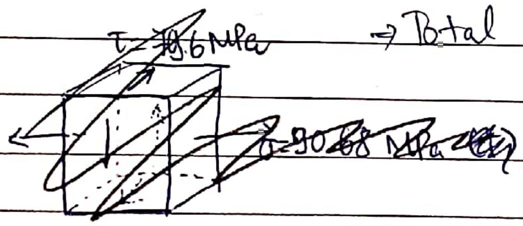
– Due to torsion : $\tau = \frac{T_x \rho}{J} = \frac{(1\text{kN})(20\text{mm})}{251.3(10^3) \text{ mm}^4} = 79.6$ MPa

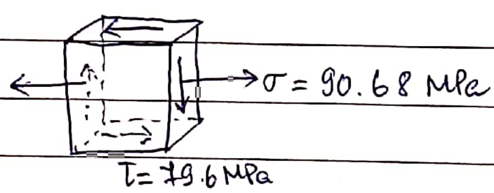
$\Rightarrow$ Total $\tau = 79.6$ MPa

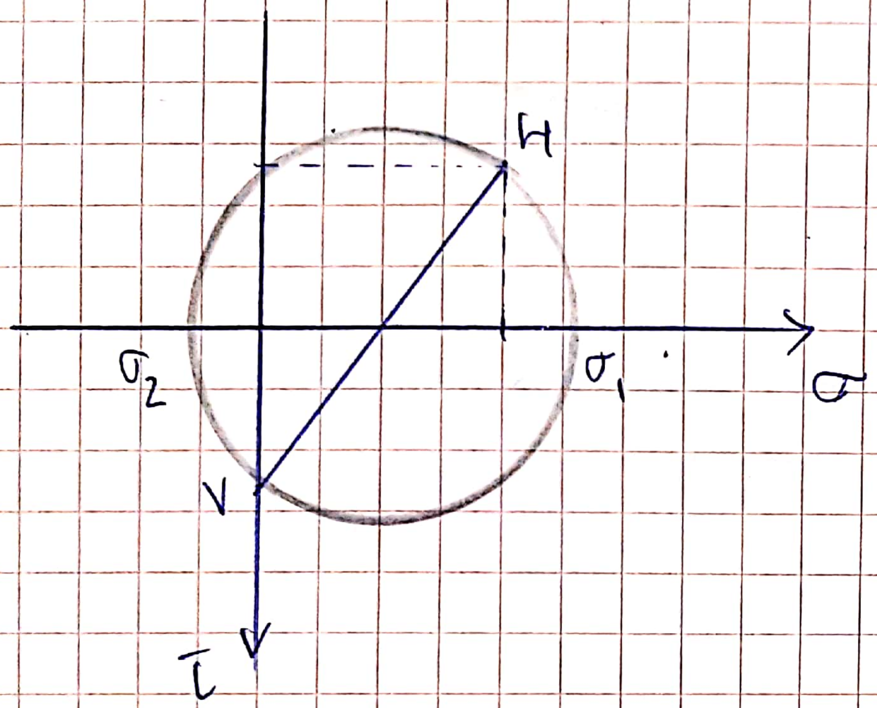
Normal stress : – Due to axial force : $\sigma = \frac{N}{A} = \frac{4\text{kN}}{1257 \text{ mm}^2} = 3.18$ MPa (T)

– Due to bending : $\sigma = \frac{My}{I} = \frac{(0.55\text{kN})(20\text{mm})}{125.7(10^3)\text{mm}^4} = 87.5$ MPa (T)

$\rightarrow$ Total $\sigma = 3.18 + 87.5 = 90.68$ MPa (T)







$\sigma_{avg} = \frac{90.68}{2} = 45.34$ MPa

$\Delta\sigma = 45.34$ MPa

$R = \sqrt{\Delta\sigma^2 + \tau_{xy}^2} = \sqrt{45.34^2 + 79.6^2} = 91.61$

$\sigma_1 = \sigma_{avg} + R = 136.95$ MPa

$\sigma_2 = \sigma_{avg} - R = -46.27$ MPa

Hence maximum tensile stress = 136.95 MPa
maximum compressive stress = 46.27 MPa
maximum inplane shear ~~str~~ stress = 91.61 MPa