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1 a) $M_x = (3F_2 + 1500)\ Nm$ $M_y = 0$ $M_x = 600\ Nm$

$M_z = -(6F_1 + 10F_2 + 3000)\ Nm$ $M_z = -1200\ Nm$

$F_R = -(F_1 + F_2 + 300)$ $F_R = -200\ N$

$R_O = F_1 + F_2 + 300$ $R_O = -200\ N$

$R_{MO} = -M_x - M_z$ $R_m = (-600\,i + 1200\,k)\ Nm$

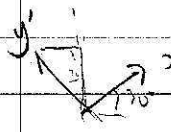
M_x caused by F_R $= (F_1 + F_2 + 300)(3) = 3F_2 + 1500$ $3F_1 = 600N$ $F_1 = 200N$

M_z caused by F_R $(F_1 + F_2 + 300)(6) = 6F_1 + 10F_2 + 3000$

$6F_2 + 1800 = 10F_2 + 3000$ $F_2 = -300N$

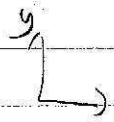
b) $\circlearrowleft + \Sigma M_A = 0$ $-5(\sin 70°)(4) + T_{CD}\sin(10°)(1.5) = 0$

$T_{CD} = 72.15\ kN$



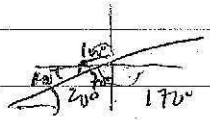
$\Sigma F_x = 0$ $A_x - 5\cos(70°) - 72.15(\cos 10°) = 0$ $A_x' = 72.766$

$A_y - 5\sin(70°) + 72.15\sin(10°) =$ $A_y' = -7.826$



$A_x = 72.76\cos 20° = 63\ kN$

$A_y = -7.826\cos 20° = -6.78\ kN$



2. a) $\Sigma M_D = 0$ $500(4)(12) - F_B(10) + 200(8) + 300(8)(4) - (300)(4)(2) =$

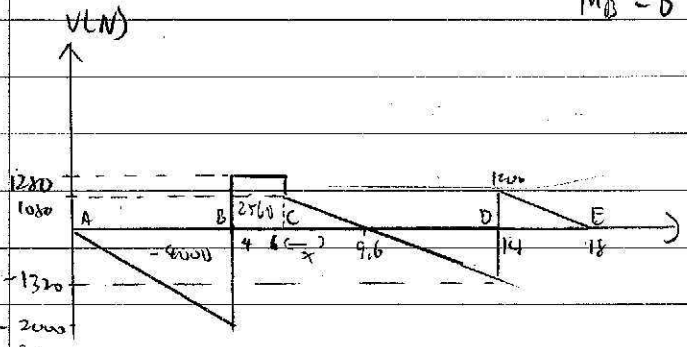
$F_B = 3280\ kN$

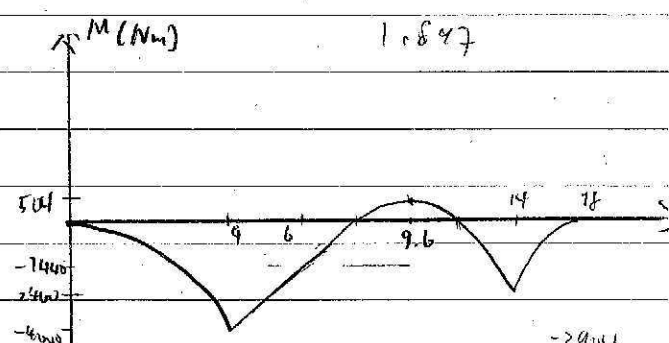
$\uparrow +$ F_D $-500(4) - 200 - 300(12) + 3280 = 0$

$F_D = 2520$

$\circlearrowleft + \Sigma M_B = 0$ $M_B + 500(4)(2) - 200(2) - 300(12)(8) + 2520(10) =$

$M_B = 0$





$-540 = \frac{x}{2}(-300x)$

$= 150x^2$

$1440 = \frac{1080 + (1080 - 300x)}{2}x$ $1440 = 1080x - 150x^2$

$6 + 1.76 = 7.76$ 11.44 $x = 5.43$ $x = 1.76$

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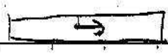
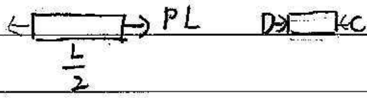


b)  PL $D=C$

$\delta_p - \delta_c = \delta$

$$\frac{PL(\frac{L}{2})}{EA} - \frac{F_c L}{EA} = \delta$$

$$\frac{PL^2}{2EA} - \frac{F_c L}{EA} = \delta$$

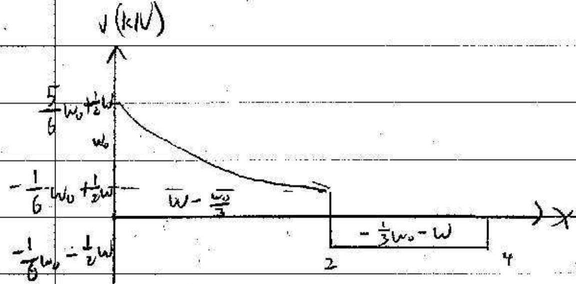
3. a) $\tau_{\text{breal}} = 140\ MPa$

$\circlearrowleft + \Sigma M_A = 0 \quad -\frac{2w_0}{2}\left(\frac{2}{3}\right) - 2W + 4F_B = 0$

$F_B = \frac{w_0}{6} + \frac{W}{2}$

$0 = \frac{w_0}{6} + \frac{W}{2} - W - w_0 + F_A$

$F_A = \frac{5}{6}w_0 + \frac{1}{2}W$



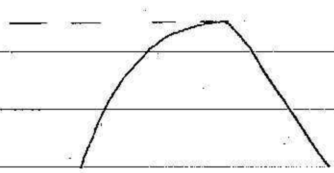
$\Sigma M_B = 0 \qquad 0 = 2W + \frac{10}{3}(w_0) - \left(\frac{5}{6}w_0 + \frac{1}{2}W\right)4 +$

$M_B = 0$

$I = \frac{\pi}{4}(45 \times 10^{-3})^4$

$= 3.22 \times 10^{-6}\ m^4$

$\frac{1}{3}w_0 + W$



$$140 \times 10^6 = \frac{\left(\frac{1}{3}w_0 + W\right)(45 \times 10^{-3})}{3.22 \times 10^{-6}}$$

$10\ kN = \frac{1}{3}w_0 + W$

$30 - w_0 = 3W \qquad \text{let } 3W = w_0 \qquad w_0 = 15$

$$\tau = \frac{VQ}{IB} = \frac{\left(\frac{5}{6}w_0 + \frac{1}{2}W\right)\left(\frac{\pi}{2}(45 \times 10^{-3})^2 \times \frac{4(45 \times 10^{-3})}{3\pi}\right)}{3.22 \times 10^{-6}(90 \times 10^{-3})} = 80 \times 10^6$$

$30 - w_0 = 3W$

$\frac{5}{6}w_0 + \frac{1}{2}W = 381.6$

$5w_0 + 3W = 2289.6 \qquad w_0 = 381.6$

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ii) $\tau = \frac{Tr}{J}$

$$\tau_{AB} = \frac{T_A (0.75d)}{\frac{81}{512}\pi d^4} = \frac{1.509}{d^3}\left(\frac{4G_S}{7G_S + 6.4815G_m}\right)T$$

$$= \frac{6.036 G_S}{7G_S + 6.4815 G_m}\frac{T}{d^3}$$

Find T_{C_S} & T_{C_A} $T_{BC,S\,max}$ $T_{DC,A\,max}$

4.

$\sigma = \sigma_{ave} + \Delta\sigma \cos 2\theta + \tau_{xy} \sin 2\theta$

$\tau = -\Delta\sigma \sin 2\theta + \tau_{xy} \cos 2\theta$

$\sigma = \sigma_{ave.} + R\cos(\phi - 2\theta)$

$\tau = R\sin(\phi - 2\theta)$

$\sigma_{AVE} = \frac{3 + 12}{2} = 10.5\ MPa$

$\Delta\sigma = \frac{-15}{2} = -7.5\ MPa$

$\tau = 0 \Rightarrow \Delta\sigma \sin 2\theta = \tau_{xy} \cos 2\theta$

$\tau_{xy} = -12.99\ MPa$

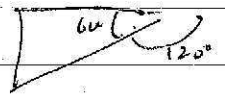
$R = \sqrt{(7.5)^2 + (12.99)^2} = 15.0\ MPa$

$\sigma = 0 = \sigma_{ave} + R\cos(\phi - 2\theta)$

$-\sigma_{ave} = R\cos(\phi - 2\theta)$

$\phi = \tan^{-1}\left(\frac{12.99}{7.5}\right) = 60$

$-10.5 = 15\cos(\phi - 2\theta)$



$\cos(\phi - 2\theta) = -0.7$

$134° = \phi - 2\theta$

$134 = 120 - 2(\theta + 90°)$

$7 = \theta + 90$

$\theta = -87°$

$\theta = 87°$

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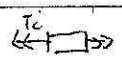
3 b) i) At BC



$T_C = T_{C,S} + T_{C,A}$

$J_S = \frac{\pi}{2}\left(\frac{3}{4}d\right)^4$

$= \frac{81}{512}\pi d^4$

$J_{AS} = \frac{\pi}{2}(d)^4$

$= \frac{1}{2}\pi d^4$

$J_A = \frac{175}{512}\pi d^4$

$\phi_{BC,S} = \phi_{BC,A}$

$$\frac{T_{C,S}(2L)}{G_S J_S} = \frac{T_{C,A}(2L)}{G_A J_A}$$

$$T_{C,A} = \frac{G_A J_A}{G_S J_S} T_{C,S}$$

$$T_C = \left(1 + \frac{G_A J_A}{G_S J_S}\right) T_{C,S}$$

$T_A + T_C = T$ $\quad T_A = T - T_C$

A relative to C

$$\phi_{AC} = \frac{T_{C,S}(2L)}{G_S J_S} - \frac{T_A(1.5L)}{G_S J_S} = 0$$

$$4T_{C,S} - 3T_A = 0$$

$$4\left(\frac{G_S J_S}{G_A J_A + G_S J_S}\right) T_C - 3T + 3T_C = 0$$

$$\frac{4G_S J_S + 3(G_A J_A + G_S J_S)}{G_A J_A + G_S J_S} T_C = 3T$$

$$T_C = \frac{3(G_A J_A + G_S J_S)}{3G_A J_A + 7G_S J_S} T \qquad T_A = \frac{4G_S J_S}{3G_A J_A + 7G_S J_S} T$$

$\frac{J_A}{J_S} = \frac{175}{512} \div \frac{81}{512} = 2.1605$ $\quad J_A = 2.1605 J_S$

$$T_C = \frac{3(G_S + 2.1605 G_A)}{7G_S + 6.4815 G_A} \qquad T_A = \frac{4G}{7G_S + 6.4815 G_A}$$

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4 b) Normal stress due to axial load

$$\sigma = \frac{15000}{\pi(0.072^2 - 0.063^2)} = 3.930 \text{ MPa}$$

torsion due to 20 kN

$T = 20000 \times 0.372 = 7440 \text{ Nm}$

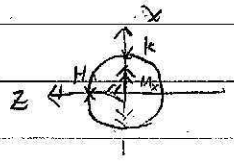
$$\tau_{Torsion} = \frac{7440 \times 0.072}{1.747 \times 10^{-5}} = 30.66 \text{ MPa}$$

$$J = \frac{\pi}{2}(0.072^4 - 0.063^4) = 1.747 \times 10^{-5} \text{ m}^4$$

$M_x = 10000 \times 0.2 = 2000 \text{ Nm}$

$M_z = 20000 \times 0.2 = 4000 \text{ Nm}$

bottom view



$$I_x = I_z = \frac{J}{2} = 8.735 \times 10^{-6} \text{ m}^4$$

$$\sigma_H = -3.930 \times 10^6 + \frac{M_x z}{I_x}$$

$$= -3.930 \times 10^6 + \frac{2000(0.072)}{8.735 \times 10^{-6}}$$

$= 12.58 \text{ MPa}$ tensile

$$\sigma_k = -3.930 + \frac{M_z y}{I_z} \times 10^{-6}$$

$= 29.0 \text{ MPa}$

$$\bar{y} = \frac{\left(\frac{4(0.072)}{3\pi}\left(\frac{\pi}{2}(0.072)^2\right) - \frac{4(0.063)}{3\pi}\left(\frac{\pi}{2}(0.063^2)\right)\right)}{\frac{\pi}{2}(0.072^2) - \frac{\pi}{2}(0.063^2)} \times$$

$$= \frac{\frac{4}{3\pi}\left((0.072)^3 - 0.063^3\right)}{0.072^2 - 0.063^2} = 0.0630 \text{ m}$$

$$Q = \frac{2}{3}(r_1^3 - r_2^3) = 8.2134 \times 10^{-5} \text{ m}^3$$

Point H $\tau_H = \frac{2000(8.2134 \times 10^{-5})}{8.735 \times 10^{-6} \times 0.018} = 10.4 \text{ MPa}$ (−)) $\tau_H = 10.4 + 30.66 \text{ MPa} = 31 \text{ MPa}$

Point k $\tau_k = 5.2 \text{ MPa}$ $\tau_k = 5.2 + 30.6 = 35.8 \text{ MPa}$

Lin Yaqiu