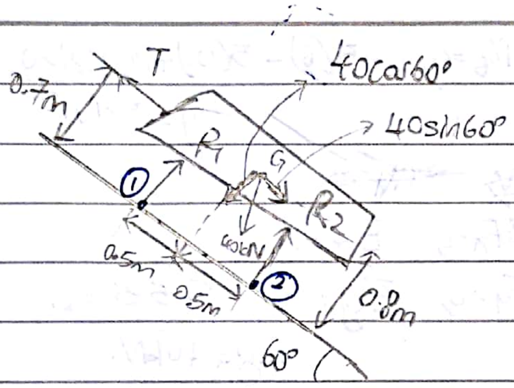
Low Khai Jer CV1011 April/May 2018

1(a).



$\searrow$ $\Sigma F = 0$, $T - 40\sin 60° = 0$

$T = 34.64\,kN$

$\nearrow$ $\Sigma F = 0$, $R_1 + R_2 - 40\cos 60° = 0$

$R_1 + R_2 = \cancel{20kN}\ 20$ —— ①

At ①, $\Sigma M_1 = 0$, $34.64(0.7) + R_2(1.0) - 40\sin 60°(0.8) - 40\cos 60°(0.5) = 0$

$R_2 = 13.46\,kN$

From ①, $R_1 = 6.536\,kN$

$\therefore$ tension is $34.64\,kN$, R_1 is $6.536\,kN$, $R_2 = 13.46\,kN$.

b). For whole system, $\circlearrowleft +\ \Sigma M_E = 0,\quad -5(6) - 5(12) + D_y(18) = 0$

$D_y = 5\text{kN}.$

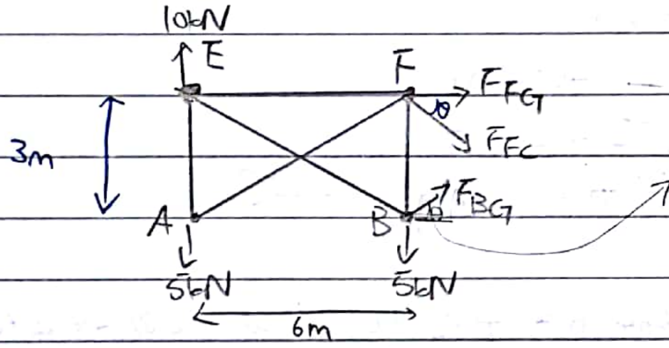
~~$F_x = 5\text{kN}$~~ ~~$\rightarrow + \Sigma F$~~

$\rightarrow + \ \Sigma F_x = 0,\quad E_x = 0,$

$\uparrow + \ \Sigma F_y = 0,\quad E_y + 5 - 5 - 5 - 5 = 0$

$E_y = 10\text{kN}.$

Cut the truss system,



$\sin\theta = \frac{1}{\sqrt{5}}$

$\cos\theta = \frac{2}{\sqrt{5}}$

$\circlearrowleft +,\ \Sigma M_F = 0,\quad F_{BG}\cos\theta(3) - 10(6) + 5(6) = 0$

$F_{BG} = 5\sqrt{5}$

$= 11.18\text{ kN (T)}$

$\circlearrowleft +,\ \Sigma M_G = 0,\quad F_{FC}\sin\theta(6) - 10(12) + 5(12) + 5(6) = 0$

$F_{FC} = \cancel{10\sqrt{5}}\ 5\sqrt{5}$

$= \cancel{22.36\ (T)}\ 11.18\text{kN (T)}$

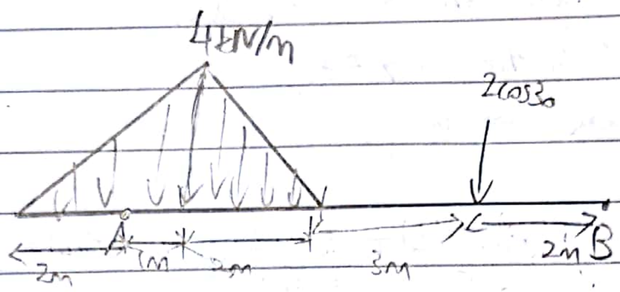
$\rightarrow + \ \Sigma F_x = 0,\quad F_{FG} + F_{FC}\cos\theta + F_{BG}\cos\theta = 0$

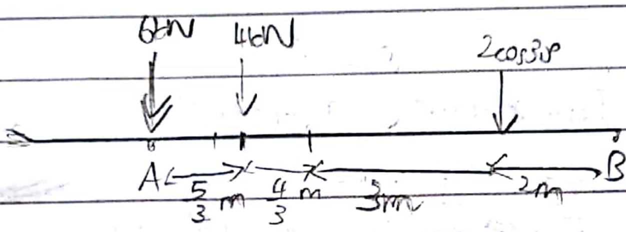
$F_{FG} = -2(5\sqrt{5})\left(\frac{2}{\sqrt{5}}\right)$

$= -20$

$= 20\text{kN (C)}$

2(a).





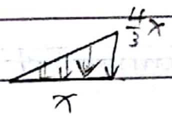
$G+, \Sigma M_A = 0, \quad -4\left(\frac{5}{3}\right) - 2\cos 30°(6) + B_y(8) = 0$

$B_y = 2.132 \text{ kN}$

$A_y = 6 + 4 + 2\cos 30° - 2.132$

$= 9.6 \text{ kN}.$

For $0 < x < 2m$,

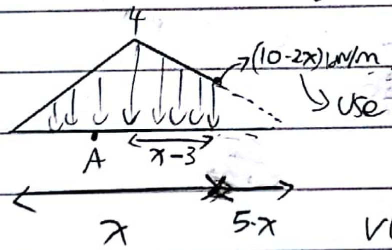


$v(x) = -\frac{1}{2}\left(\frac{4}{3}x\right)(x)$

$= -\frac{2}{3}x^2$

$2 < x < 3$, $\quad v(x) = 9.6 - \frac{2}{3}x^2$

$3 < x < 5$,



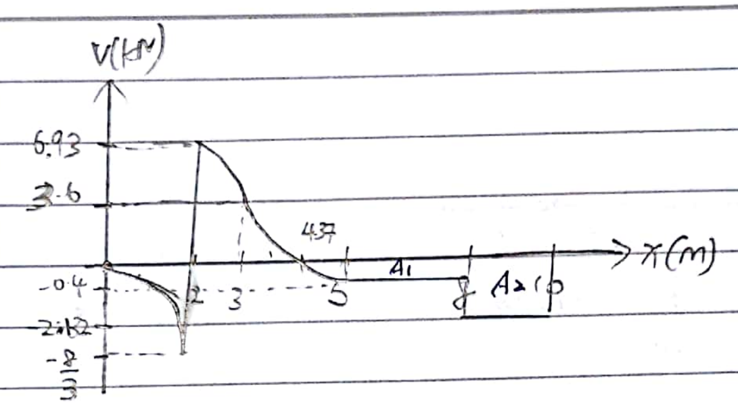
$v(x) = V(3) - \frac{1}{2}(4 + 10 - 2x)(x-3)$

$= 3.6 - (7 - x)(x-3)$

$= x^2 - 10x + 24.6$

$5 < x < 8$, $\quad v(x) = v(5) = -0.4$

2(a)



$M = \int v(x)dx$, $0<x<2$, $m(x) = \int_0^x -\frac{2}{3}x^2\, dx$

$= -\frac{2}{9}x^3$

$2<x<3$, $M(x) = M(2) + \int_2^x \left(9.6 - \frac{2}{3}x^2\right) dx$

$= -\frac{16}{9} + \left[9.6x - \frac{2}{9}x^3\right]_2^x$

$= -\frac{16}{9} + 9.6x - \frac{2}{9}x^3 - 19.2 + \frac{16}{9}$

$= -\frac{2}{9}x^3 + 9.6x - 19.2$

$3<x<5$, $M(x) = m(3) + \int_3^x (x^2 - 10x + 24.6)\, dx$

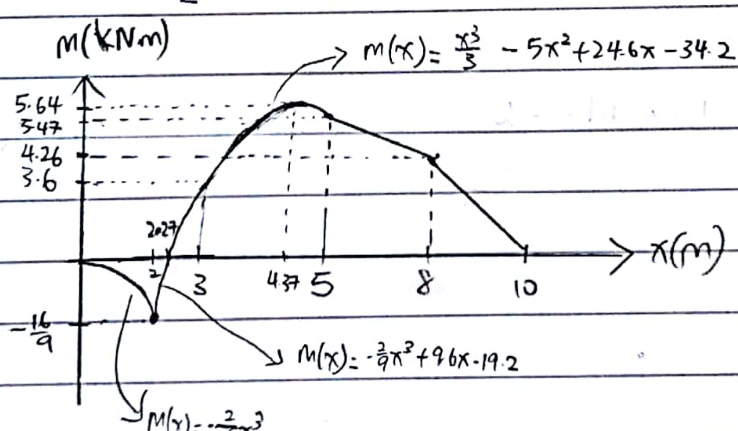
$= 3.6 + \cancel{8}\left[\frac{x^3}{3} - 5x^2 + 24.6x\right]_3^x$

$= 3.6 + \frac{x^3}{3} - 5x^2 + 24.6x - 9 + 45 - 73.8$

$= \frac{x^3}{3} - 5x^2 + 24.6x - 34.2$

$A_1 = -(8-5)(0.4)$, $A_2 = -(10-8)(2.132)$

$= -1.2$ $\quad = \cancel{4.624}\ -4.264$



2(b) Aluminium expands more than steel as $\alpha_a > \alpha_s$.

Since they are bonded together, $\delta_a = \delta_s$

Let the force between them be P.

length be L.

$$\alpha_a(\Delta T)(L) + \frac{(-P)(L)}{(E_a)(A_a)} = \alpha_s(\Delta T)(L) + \frac{P(L)}{(E_s)(A_s)} \quad — (1)$$

Aluminium experiences compression force as it expands more , steel experiences tension as it expands less.

$A_a = \pi(0.03^2 - 0.01^2)$, $E_a = 70 \times 10^9$ Pa , $\alpha_a = 23.6 \times 10^{-6}/°C$

$= 2.513 \times 10^{-3}\ m^2$

$A_s = \pi(0.01^2)$, $E_s = 200 \times 10^9$ Pa , $\alpha_s = 11.7 \times 10^{-6}/°C$

$= 3.142 \times 10^{-4}\ m^2$

From (1), (Both sides divide L)

$$23.6 \times 10^{-6}(200-20) - \frac{P}{(70 \times 10^9)(2.513 \times 10^{-3})} = 11.7 \times 10^{-6}(200-20) + \frac{P}{(200 \times 10^9)(3.142 \times 10^{-4})}$$

$$2.142 \times 10^{-3} = P(2.1598 \times 10^{-8})$$

$$P = 99.175\ kN$$

~~Contact area between steel core and aluminium shell , $A_c = 2\pi(0.01) \times L$~~

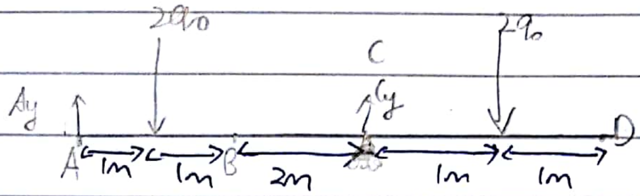
Stress in steel $= \frac{99.175 \times 10^3}{3.142 \times 10^{-4}}$

$= 0.316$ GPa.

Stress in aluminum $= \frac{99.175 \times 10^3}{2.513 \times 10^{-3}}$

$= 39.5$ MPa.

3(a)

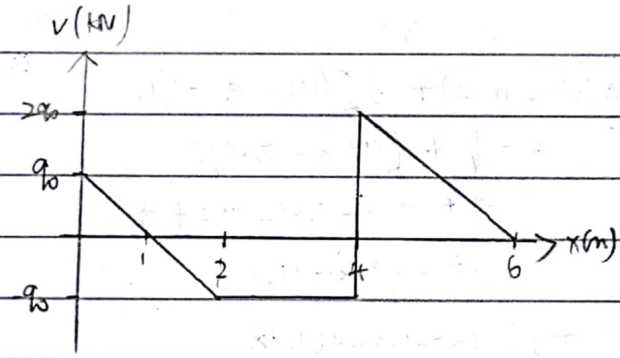


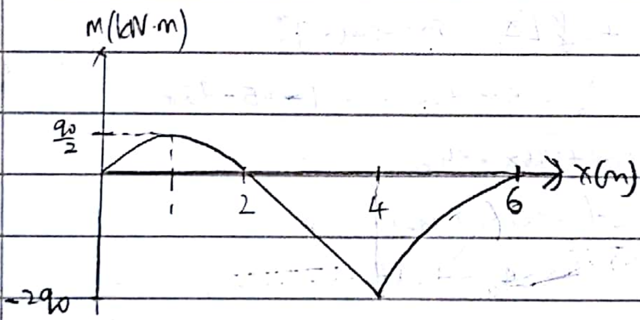
$\circlearrowleft + \Sigma M_A = 0, \quad -2q_0(1) + C_y(4) - 2q_0(5) = 0$

$C_y = 3q_0$

$\uparrow + \Sigma F_y = 0, \quad C_y + A_y = 4q_0$

$A_y = q_0.$





Max $V = 2q_0$, max $M = 2q_0$

$J = \frac{\pi}{2}(0.05)^4$

$= 9.817 \times 10^{-6}\,m^4$

$I = \frac{J}{2} = 4.909 \times 10^{-6}\,m^4$

$Q = \frac{2}{3}(0.05)^3$

$= 8.33 \times 10^{-5}\,m^3$

$\tau_{max} = 80 \times 10^6 \geq \frac{2q_0(8.33 \times 10^{-5})}{4.909 \times 10^{-6}(0.1)}$

$q_0 \leq 236\,kN$

$\sigma_{max} = 140 \times 10^6 \geq \frac{2q_0(0.05)}{4.909 \times 10^{-6}}$ $\quad \therefore q_{0\,max} = 6873\,N$

$q_0 \leq 6873\,N$

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3b)(i) $\tau = \frac{T\rho}{J}$ $\quad J_{AB} = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$, $J_{BC\,steel} = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$, $J_{BD\,aluminium} = \frac{\pi}{2}\left[\left(\frac{3}{4}d\right)^4 - \left(\frac{1}{2}d\right)^4\right] = \frac{65}{512}\pi d^4$

$\phi = \sum \frac{TL}{GJ}$ $\quad J_{BC} = \frac{\pi}{2}\left(\frac{3}{4}d\right)^4 = \frac{81}{512}\pi d^4$

For section AB, $\tau = \frac{T\left(\frac{d}{2}\right)}{\frac{\pi d^4}{32}} = \frac{16T}{\pi d^3}$

Section BC, $\tau = \frac{2T\left(\frac{3}{4}d\right)}{\frac{81}{512}\pi d^4} = 9.48\frac{T}{\pi d^3}$

Section CD, $\tau = \frac{2T\left(\frac{3}{4}d\right)}{\frac{65}{512}\pi d^4} = 11.82\frac{T}{\pi d^3}$ $\quad \therefore$ Max shear stress $= \frac{16T}{\pi d^3}$

ii) For BC, $\phi_{BC\,steel} = \phi_{BC\,aluminium}$

$$\frac{T_s L}{G_s J_s} = \frac{T_a L}{G_a J_a} \quad \text{— ①}$$

$$T_s = T_a\left(\frac{G_s}{G_a}\right)\left(\frac{J_s}{J_a}\right) = T_a\left(\frac{G_s}{G_a}\right)\left(\frac{\frac{\pi d^4}{32}}{\frac{65}{512}\pi d^4}\right) = \frac{16}{65}\frac{G_s}{G_a}T_a$$

$T_s + T_a = 2T \implies \left(\frac{65G_a + 16G_s}{65G_a}\right)T_s = 2T$

$T_s = \frac{130\,TG_a}{65G_a + 16G_s}$

$\therefore \sum\phi = \phi_{AB} + \phi_{BC} + \phi_{CD}$

$$= \frac{TL}{G_s\left(\frac{\pi d^4}{32}\right)} + \left(\frac{130\,TG_a}{65G_a + 16G_s}\right)\frac{L}{G_s\left(\frac{\pi d^4}{32}\right)} + \frac{2TL}{G_a\left(\frac{65}{512}\pi d^4\right)}$$

$$= \frac{32\,TL}{\pi G_s d^4} + \frac{4160\,TG_a L}{\pi d^4 G_s(65G_a + 16G_s)} + \frac{1024\,TL}{65\,G_a \pi d^4}$$

$\Downarrow$

May have more than 1 form.

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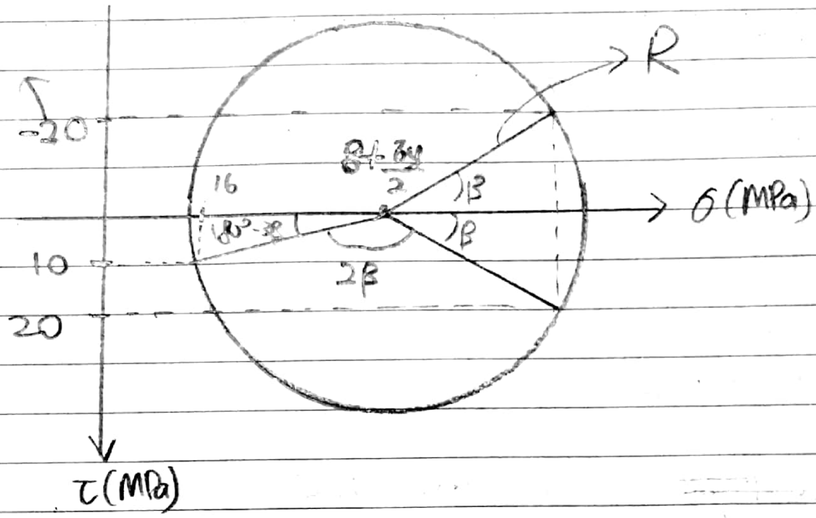
4 (a)

Given by Prof. during finals!



$$\sin(180°-3\beta) = \sin 3\beta = \frac{10}{R} \quad — ①$$

$$\sin\beta = \frac{20}{R} = 2\sin 3\beta \quad — ②$$

$$\sin\beta = 6\sin\beta - 8\sin^3\beta$$

$$\sin\beta\,(8\sin^2\beta - 5) = 0$$

$$\sin\beta \neq 0, \quad \sin\beta = \sqrt{\tfrac{5}{8}} > 0 \ (0 < \beta < 90°)$$

$$\beta = 52.24°$$

From ②, $\sin 52.24 = \frac{20}{R}$, $R = 25.3$ MPa.

$$\cos[180 - 3(52.24)] = \frac{\frac{8+\sigma_y}{2} - 16}{25.3}$$

$$\sigma_y = 62.48 \text{ MPa}$$ #

Low Khai Jer

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4(b) $A = \pi(0.04^2)$
$= 5.0265 \times 10^{-3} m^2$
$Q = \frac{2}{3}(0.04)^2$
$= 1.067 \times 10^{-3} m^3$
$J = \frac{\pi}{2}(0.04)^4$
$= 4.021 \times 10^{-6} m^4$
$I = \frac{J}{2}$
$= 2.011 \times 10^{-6} m^4$

~~Mx~~ $T = M_x = 300(0.9 \cos 45°)$
$= 191$ Nm

$M_y = -300(\cancel{0.1+0.2}\ 0.1+0.3-0.1) + 200(0.9\cos 45°)$
$= \cancel{90 Nm}$ 37.28 Nm

$M_z = 200(0.9 \sin 45°) + 300(0.3 - 0.1)$
$= 187.28$ Nm.

$$\sigma_H = \frac{-200}{5.0265 \times 10^{-3}} + \frac{(-37.28)(0.04)}{2.011 \times 10^{-6}}$$

$= -0.781$ MPa

$$\tau_H = \frac{VQ_A}{It} = \frac{300(1.067 \times 10^{-3})}{(2.011 \times 10^{-6})(0.08)}$$
$= 1.99$ MPa

in-plane shear stress, $R = \sqrt{\left(\frac{0.781-0}{2}\right)^2 + 1.99^2}$
$= 2.028$ MPa

Max tensile stress $= -0.781 + 2.028$
$= 1.247$ MPa

Max compressive stress $= -0.781 - 2.028$
$=$ ~~-1.562 M~~ -2.81 MPa

Aaw Khai Jo

~~Aahalte~~

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