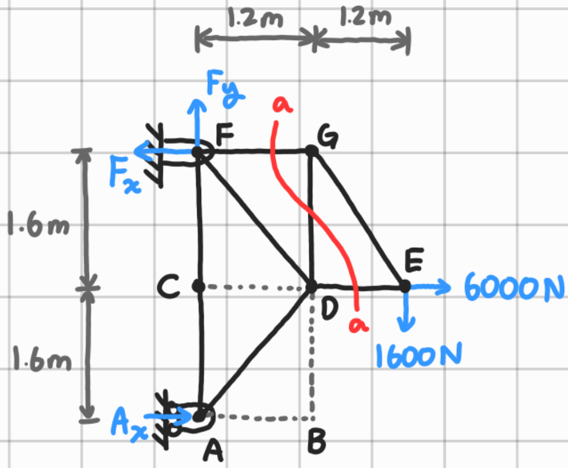
CV1011 PYP 2017/18 Sem 1

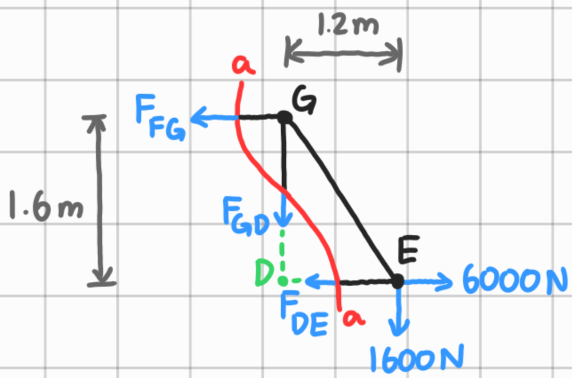
by Renardi Muliawan

1. (a)



- This is a truss as all loads are applied at the joints.
- Members AB, BD & CD are zero-force members, so we can ignore them.
- We are going to use the method of sections to solve the question, slicing the truss along a-a.

⇓



$$+\uparrow \Sigma F_y = -1600 - F_{GD} = 0$$

$$F_{GD} = -1600\text{ N (C)}$$

$$\circlearrowleft + \Sigma M_D = F_{FG}(1.6) - 1600(1.2) = 0$$

$$F_{FG} = +1200\text{ N (T)}$$

$$\circlearrowleft + \Sigma M_G = 6000(1.6) - 1600(1.2) - F_{DE}(1.6) = 0$$

$$F_{DE} = +4800\text{ N (T)}$$

$$\boxed{\begin{aligned} \therefore\ F_{GD} &= -1600\text{ N (C)} \\ F_{FG} &= +1200\text{ N (T)} \\ F_{DE} &= +4800\text{ N (T)} \end{aligned}}\ //$$

OTHER METHOD: Another way to solve the question is to use the tensile-positive method of joints.

For the whole truss:

$$\circlearrowleft + \Sigma M_F = A_x(3.2) + 6000(1.6) - 1600(2.4) = 0$$

$$A_x = -1800\text{ N}\ *$$

$$\overset{+}{\rightarrow} \Sigma F_x = A_x + 6000 - F_x = 0$$

$$(-1800) + 6000 - F_x = 0$$

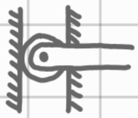
$$F_x = 4200\text{ N}$$

$$+\uparrow \Sigma F_y = F_y - 1600 = 0$$

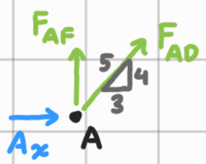
$$F_y = 1600\text{ N}$$

* As A_x is negative, A_x is supposed to be directed into the wall ($\leftarrow$).
However, a roller only provides a reaction <u>away</u> from the surface/wall.
Hence, this truss is <u>not</u> even in equilibrium.

Nevertheless, for the sake of showing the calculation, let's assume the roller is confined in a smooth slot instead, hence able to provide the reaction force needed.

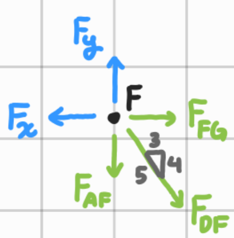
i.e. 

Joint A:



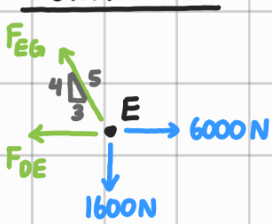
$$\overset{+}{\rightarrow}\Sigma F_x = A_x + \frac{3}{5}F_{AD} = 0$$
$$F_{AD} = -\frac{5}{3}A_x = -\frac{5}{3}(-1800) = +3000\text{ N (T)}$$
$$+\uparrow\Sigma F_y = F_{AF} + \frac{4}{5}F_{AD} = 0$$
$$F_{AF} = -\frac{4}{5}F_{AD} = -\frac{4}{5}(3000) = -2400\text{ N (C)}$$

Joint F:



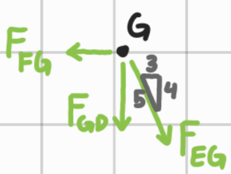
$$+\uparrow\Sigma F_y = F_y - F_{AF} - \frac{4}{5}F_{DF} = 0$$
$$F_{DF} = \frac{5}{4}(F_y - F_{AF}) = \frac{5}{4}[1600 - (-2400)] = +5000\text{ N (T)}$$
$$\overset{+}{\rightarrow}\Sigma F_x = F_{FG} + \frac{3}{5}F_{DF} - F_x = 0$$
$$F_{FG} = F_x - \frac{3}{5}F_{DF} = 4200 - \frac{3}{5}(5000) = +1200\text{ N (T)} \checkmark$$

Joint E:



$$+\uparrow\Sigma F_y = \frac{4}{5}F_{EG} - 1600 = 0$$
$$F_{EG} = \frac{5}{4}(1600) = +2000\text{ N (T)}$$
$$\overset{+}{\rightarrow}\Sigma F_x = 6000 - \frac{3}{5}F_{EG} - F_{DE} = 0$$
$$F_{DE} = 6000 - \frac{3}{5}(2000) = +4800\text{ N (T)} \checkmark$$

Joint G:

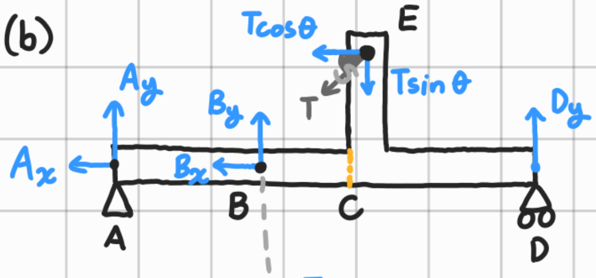


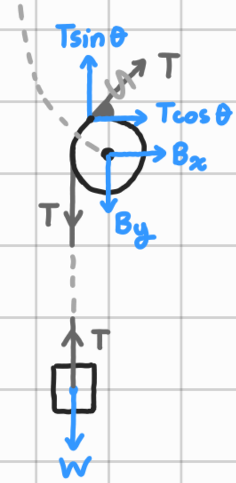
$$+\uparrow\Sigma F_y = -\frac{4}{5}F_{EG} - F_{GD} = 0$$
$$F_{GD} = -\frac{4}{5}(2000) = -1600\text{ N (C)} \checkmark$$

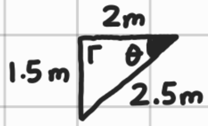
$$\boxed{\begin{aligned} \therefore\ F_{GD} &= -1600\text{ N (C)} \\ F_{FG} &= +1200\text{ N (T)} \\ F_{DE} &= +4800\text{ N (T)} \end{aligned}}$$

//

(b)







$$\sin\theta = \frac{1.5}{2.5} = \frac{3}{5}$$

$$\cos\theta = \frac{2}{2.5} = \frac{4}{5}$$

- For frames, it is a good practice to separate the components.
- Pay attention to the action-reaction pairs when separating the components.

(e.g. pay attention to the way B_x, B_y & T were drawn on the beam and the pulley. They must be in opposite directions.)

Weight: $+\uparrow\Sigma F_y = T - W = 0 \Rightarrow T = W = 27\text{ kN}$

Pulley:

$$+\uparrow\Sigma F_y = T\sin\theta - T - B_y = 0$$

$$B_y = T\sin\theta - T = \frac{3}{5}(27) - 27 = -10.8\text{ kN}$$

$$\overset{+}{\rightarrow}\Sigma F_x = B_x + T\cos\theta = 0$$

$$B_x = -T\cos\theta = -\frac{4}{5}(27) = -21.6\text{ kN}$$

Beam:

$$\circlearrowleft + \Sigma M_A = B_y(2) + T\cos\theta(1.5) - T\sin\theta(4) + D_y(6) = 0$$

$$D_y = \frac{4T\sin\theta - 1.5T\cos\theta - 2B_y}{6}$$

$$= \frac{4(27)\left(\frac{3}{5}\right) - 1.5(27)\left(\frac{4}{5}\right) - 2(-10.8)}{6} = 9\text{ kN}$$

$$+\uparrow\Sigma F_y = A_y + B_y + D_y - T\sin\theta = 0$$

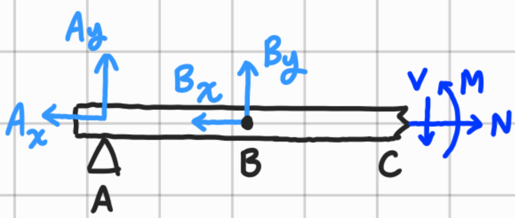
$$A_y = -B_y - D_y + T\sin\theta$$

$$= -(-10.8) - 9 + 27\left(\frac{3}{5}\right) = 18\text{ kN}$$

$$\overset{+}{\rightarrow}\Sigma F_x = -A_x - B_x - T\cos\theta = 0$$

$$A_x = -B_x - T\cos\theta = -(-21.6) - \frac{4}{5}(27) = 0\text{ kN}$$

Internal forces at C:



$$\xrightarrow{+} \Sigma F_x = N - A_x - B_x = 0$$

$$N = B_x + A_x = -21.6 + 0 = -21.6 \text{ kN}$$

$$+\uparrow \Sigma F_y = A_y + B_y - V = 0$$

$$V = A_y + B_y = 18 + (-10.8) = 7.2 \text{ kN}$$

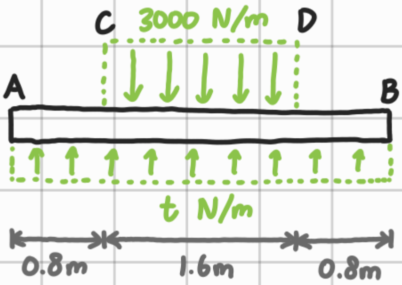
$$\circlearrowleft + \Sigma M_C = M - B_y(2) - A_y(4) = 0$$

$$M = 2B_y + 4A_y = 2(-10.8) + 4(18) = 50.4 \text{ kN}\cdot\text{m}$$

$$\boxed{\begin{aligned} \therefore\ N &= -21.6 \text{ kN (compression)} \\ V &= 7.2 \text{ kN} \\ M &= 50.4 \text{ kNm} \end{aligned}}$$

//

② (a)



For the whole beam,

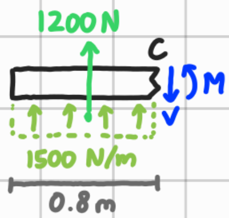
$$+\uparrow \Sigma F_y = 3.2t - 3000(1.6) = 0$$

$$t = 1500$$

We are going to use the sketching method, as finding the functions $V(x)$ and $M(x)$ would take ages.

Finding V & M at critical points:

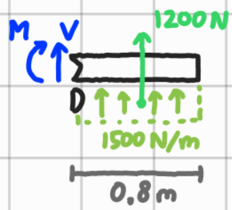
Point C →



$$+\uparrow \Sigma F_y = 1200 - V = 0 \Rightarrow V = 1200 \text{ N}$$

$$\circlearrowleft + \Sigma M_C = M - 1200(0.4) = 0 \Rightarrow M = 480 \text{ N·m}$$

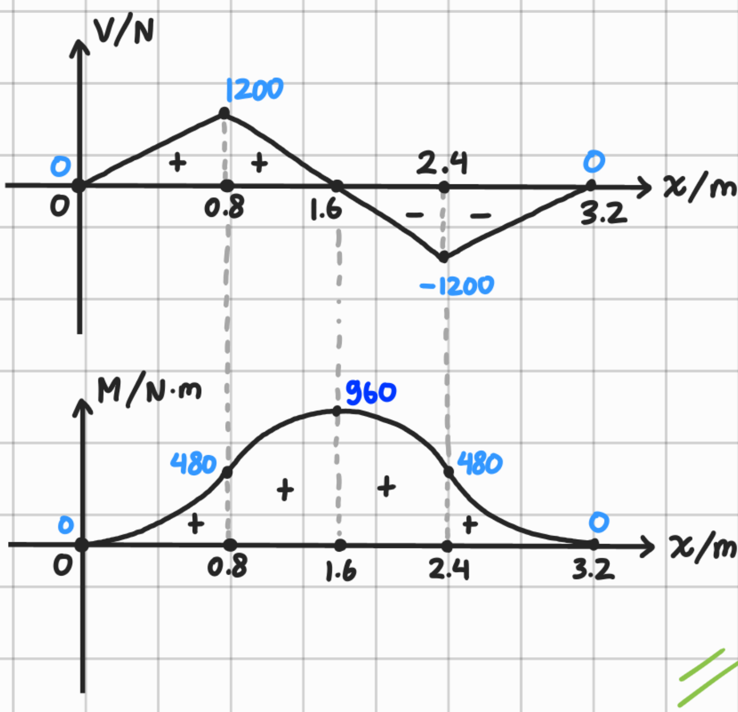
Point D →



$$+\uparrow \Sigma F_y = 1200 + V = 0 \Rightarrow V = -1200 \text{ N}$$

$$\circlearrowleft + \Sigma M_D = -M + 1200(0.4) = 0 \Rightarrow M = 480 \text{ N·m}$$

Sketching the shear & bending moment diagram...

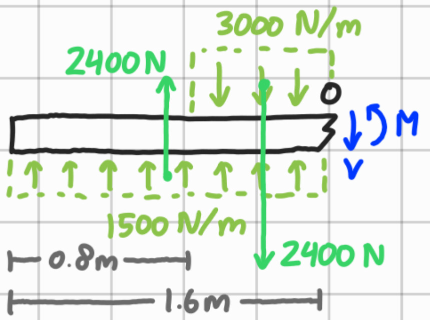


Explanation:

- $V(x) = \int w(x)\,dx$, so if $w(x)$ is constant then $V(x)$ is linear with $\frac{dV}{dx} = w(x)$.
- $M(x) = \int V(x)\,dx$, so if $V(x)$ is linear then $M(x)$ is quadratic with $\frac{dM}{dx} = V(x)$.

//

- As $V(x) = 0$ at $x = 1.6$ m, (in this case) max. $M(x)$ is also at $x = 1.6$ m. Finding max. $M(x)$ ...



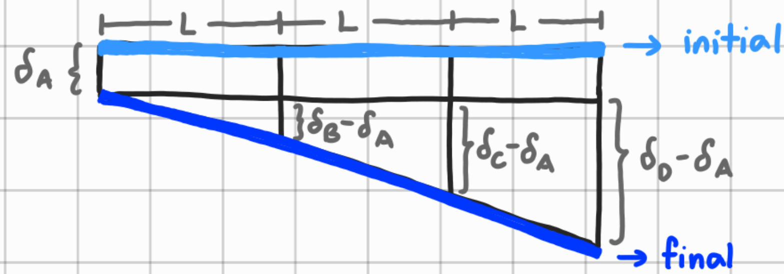
$$+\uparrow \sum F_y = 2400 - 2400 - V = 0$$

$$V = 0 \quad \text{(as expected)}$$

$$\circlearrowleft + \sum M_o = M + 2400(0.4) - 2400(0.8) = 0$$

$$M = 960 \text{ N}\cdot\text{m}$$

(b) (i)



Similar triangles:

$$\frac{\delta_B - \delta_A}{L} = \frac{\delta_C - \delta_A}{2L} = \frac{\delta_D - \delta_A}{3L} \quad \times 3L$$

$$3(\delta_B - \delta_A) = \frac{3}{2}(\delta_C - \delta_A) = \delta_D - \delta_A \qquad \delta = \frac{NL}{AE}$$

$$3\left[\frac{F_B l}{AE} - \frac{F_A l}{AE}\right] = \frac{3}{2}\left[\frac{F_C l}{AE} - \frac{F_A l}{AE}\right] = \frac{F_D l}{AE} - \frac{F_A l}{AE}$$

$\div \frac{l}{AE}$, as all the wires had the same l, A, E.

$$3(F_B - F_A) = \frac{3}{2}(F_C - F_A) = F_D - F_A$$

$$3F_B - 3F_A = \frac{3}{2}F_C - \frac{3}{2}F_A = F_D - F_A$$

$$3F_B - 3F_A = F_D - F_A$$

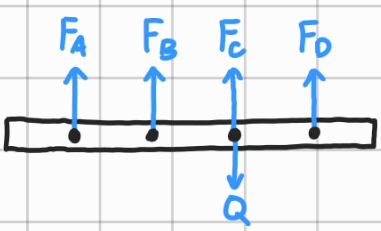
$$2F_A - 3F_B + F_D = 0 \quad \cdots (1)$$

$$3F_B - 3F_A = \frac{3}{2}F_C - \frac{3}{2}F_A$$

$$\frac{3}{2}F_A - 3F_B + \frac{3}{2}F_C = 0$$

$$F_A - 2F_B + F_C = 0 \quad \cdots (2)$$

Beam FBD:



$$\circlearrowleft + \sum M_c = F_D \cdot L - F_B \cdot L - F_A \cdot 2L = 0$$

$$2F_A + F_B - F_D = 0 \quad \cdots \text{(3)}$$

$$+\uparrow \sum F_y = F_A + F_B + F_C + F_D - Q = 0$$

$$F_A + F_B + F_C + F_D = Q \quad \cdots \text{(4)}$$

Solving all the four equations ...

(4) − (2): $3F_B + F_D = Q \Rightarrow 6F_B + 2F_D = 2Q \cdots$ (5)

(3) − (1): $4F_B - 2F_D = 0 \cdots$ (6)

(5) + (6): $10 F_B = 2Q$

$$F_B = Q/5 \xrightarrow{(6)} 4Q/5 - 2F_D = 0$$

$$F_D = 2Q/5$$

(1):

$$2F_A - 3Q/5 + 2Q/5 = 0$$

$$F_A = Q/10$$

(2):

$$Q/10 - 2Q/5 + F_C = 0$$

$$F_C = 3Q/10$$

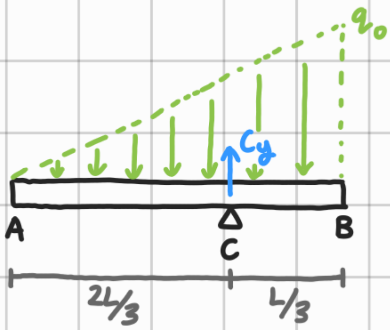
$$\boxed{\therefore\ F_A = Q/10,\ F_B = Q/5,\ F_C = 3Q/10,\ F_D = 2Q/5}$$

(b) (ii) Despite the wires elongating, the distribution of force would still be the same as part (i).

This is because the 4 wires are identical, hence it expands at the <u>same rate</u>, so the compatibility requirements remain the same. (You can imagine this by replacing all δ_i with $\frac{N_i \ell}{AE} + \ell \alpha \Delta T$)

Moreover, none of the wires are restricted to expand, hence there are <u>no</u> additional axial forces.

3. (a) (i)

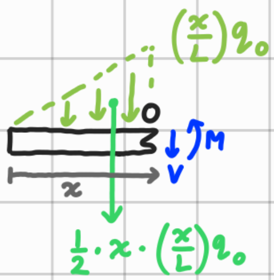


For the whole beam:

$$+\uparrow \Sigma F_y = C_y - \frac{1}{2} \cdot L \cdot q_o = 0$$

$$C_y = \frac{1}{2} L q_o$$

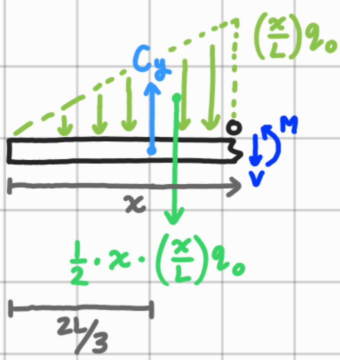
Section AC:



$$+\uparrow \Sigma F_y = -V - \frac{1}{2} x \left(\frac{x}{L}\right) q_o = 0 \Rightarrow V = -\frac{1}{2}\left(\frac{q_o}{L}\right) x^2$$

$$\circlearrowleft + \Sigma M_o = M + \left[\frac{1}{2} x \left(\frac{x}{L}\right) q_o\right]\left(\frac{x}{3}\right) \Rightarrow M = -\frac{1}{6}\left(\frac{q_o}{L}\right) x^3$$

Section CB:



$$+\uparrow \Sigma F_y = -V - \frac{1}{2} x \left(\frac{x}{L}\right) q_o + C_y = 0$$

$$-V - \frac{1}{2} x \left(\frac{x}{L}\right) q_o + \frac{1}{2} L q_o = 0$$

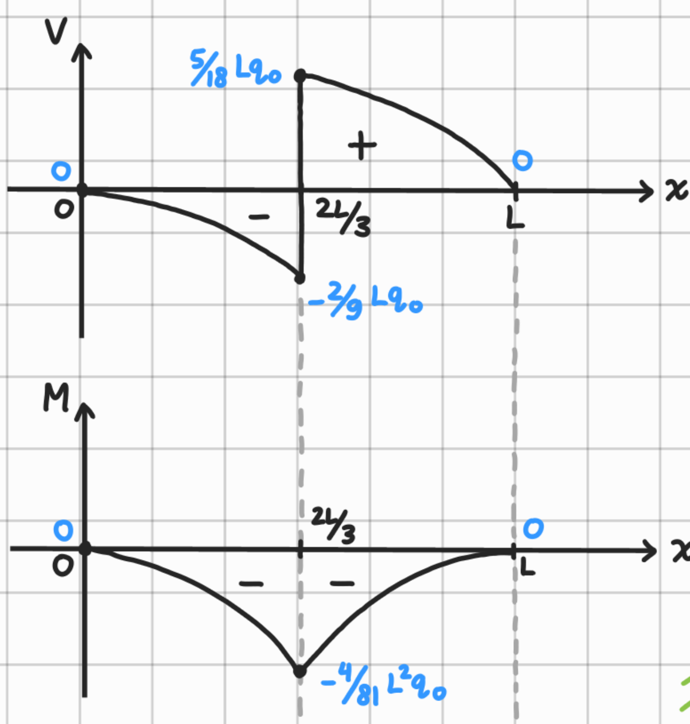
$$V = -\frac{1}{2}\left(\frac{q_o}{L}\right) x^2 + \frac{1}{2} L q_o$$

$$\circlearrowleft + \Sigma M_o = M + \left[\frac{1}{2} x \left(\frac{x}{L}\right) q_o\right]\left(\frac{x}{3}\right) - C_y\left(x - \frac{2L}{3}\right) = 0$$

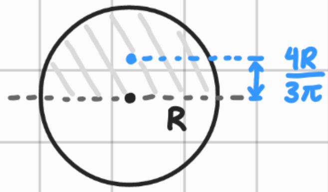
$$M + \frac{1}{6}\left(\frac{q_o}{L}\right) x^3 - \frac{1}{2} L q_o x + \frac{1}{3} L^2 q_o = 0$$

$$M = -\frac{1}{6}\left(\frac{q_o}{L}\right) x^3 + \frac{1}{2} L q_o x - \frac{1}{3} L^2 q_o$$

Shear & Bending Moment Diagram:



(a) (ii)



$I = \frac{\pi}{4}R^4$

- Largest bending stress is at the top/bottom of the beam, so $y_{max} = R$.

$$\sigma_a = \frac{M_{max}\,y_{max}}{I} = \frac{\left(\frac{4}{81}L^2 q_o\right)(R)}{\frac{\pi}{4}R^4} = \frac{16L^2 q_o}{81\pi R^3}$$

$$\Rightarrow q_o = \frac{81\pi R^3 \sigma_a}{16L^2}$$

- Largest shear stress happens at the centroid of the cross-sectional area, so $Q_{max} = A y' = \left(\frac{1}{2}\pi R^2\right)\left(\frac{4R}{3\pi}\right) = \frac{2}{3}R^3$

$$\tau_a = \frac{V_{max} \cdot Q_{max}}{I \cdot t} = \frac{\left(\frac{5}{18}Lq_o\right)\left(\frac{2}{3}R^3\right)}{\left(\frac{\pi}{4}R^4\right)(2R)} = \frac{10Lq_o}{27\pi R^2}$$

$$\Rightarrow q_o = \frac{27\pi R^2 \tau_a}{10L}$$

$$\boxed{\therefore \text{Largest permissible value of } q_o = \min\left\{\frac{81\pi R^3\sigma_a}{16L^2},\ \frac{27\pi R^2 \tau_a}{10L}\right\}}$$ //

(b)

$n = \frac{c_1}{c_2}$

As both shafts have the same cross-sectional area,

$$A_h = A_s$$

$$\pi(c_2^2 - c_1^2) = \pi r^2$$

$$r^2 = (c_2^2 - c_1^2)$$

$$r^2 = c_2^2\left[1 - \left(\frac{c_1}{c_2}\right)^2\right] \Rightarrow r = c_2\sqrt{1-n^2}$$

Hollow $\rightarrow$ $J_h = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2} c_2^4 \left[1 - \left(\frac{c_1}{c_2}\right)^4\right] = \frac{\pi}{2} c_2^4 (1 - n^4)$

Solid $\rightarrow$ $J_s = \frac{\pi}{2} r^4 = \frac{\pi}{2}\left(c_2\sqrt{1-n^2}\right)^4 = \frac{\pi}{2} c_2^4 (1-n^2)^2$

(i) $(\tau_{max})_h = (\tau_{max})_s$

$$\frac{T_h \cdot c_2}{J_h} = \frac{T_s \cdot r}{J_s}$$

$$T_s = \frac{J_s c_2}{J_h r} \cdot T_h$$

$$T_s = \frac{\cancel{\frac{\pi}{2}}\, \cancel{c_2^4} (1-n^2)^2 \cdot \cancel{c_2}}{\cancel{\frac{\pi}{2}}\, \cancel{c_2^4} (1-n^4) \cdot \cancel{c_2} \sqrt{1-n^2}} \cdot T_h$$

$$T_s = \frac{(1-n^2)^{\cancel{2}} \cdot T_h}{(1+n^2)\cancel{(1-n^2)}\sqrt{1-n^2}}$$

$$T_s = \boxed{\frac{(1-n^2)\, T_h}{(1+n^2)\sqrt{1-n^2}}}$$

//

(ii) $\phi_h = \phi_s$

$$\frac{T_h \cdot \cancel{L}}{J_h \cdot \cancel{G}} = \frac{T_s \cdot \cancel{L}}{J_s \cdot \cancel{G}}$$

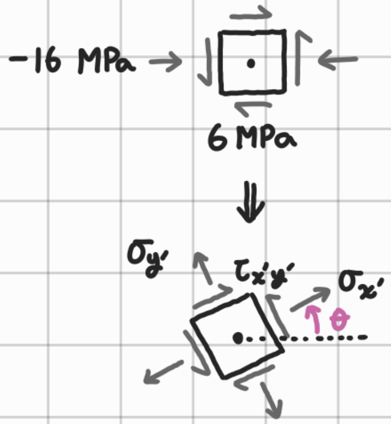
$$T_s = \frac{J_s}{J_h} \cdot T_h = \frac{\cancel{\frac{\pi}{2}}\, \cancel{c_2^4} (1-n^2)^2}{\cancel{\frac{\pi}{2}}\, \cancel{c_2^4} (1-n^4)} \cdot T_h$$

$$T_s = \frac{(1-n^2)^{\cancel{2}} \cdot T_h}{(1+n^2)\cancel{(1-n^2)}}$$

$$\boxed{T_s = \frac{(1-n^2)}{(1+n^2)} T_h}$$

//

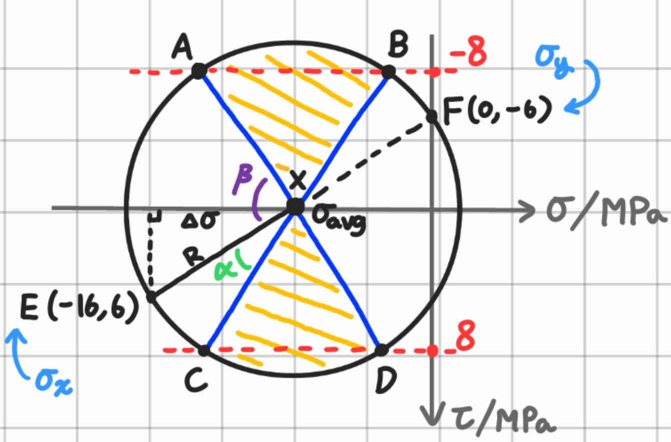
4. (a)



$\sigma_x = -16\text{MPa}$ $\quad \tau_{xy} = 6 \text{ MPa}$

$\sigma_y = 0$ $\quad \tau_{yx} = -6 \text{ MPa}$

- Notice that θ is measured from the original σ_x, counter-clockwise.



Radius of circle

$$R = \sqrt{\Delta\sigma^2 + \tau^2}$$

$$= \sqrt{\left(\frac{-16-0}{2}\right)^2 + (6)^2} = 10 \text{ MPa}$$

$$\sigma_{avg} = \frac{-16+0}{2} = -8 \text{ MPa} \Rightarrow X(-8,0)$$

- So, eqn. of circle is $(\sigma+8)^2 + \tau^2 = 100 \rightarrow (x-a)^2 + (y-b)^2 = r^2$

- For $\tau = \pm 8$ MPa : $(\sigma+8)^2 + (\pm 8)^2 = 100 \rightarrow$ To find coordinates of A, B, C, D

$$(\sigma+8)^2 = 36$$

$$\sigma + 8 = \pm 6$$

$$\sigma = -8 \pm 6 = \begin{cases} -2 \\ -14 \end{cases} \quad \left.\begin{matrix} A(-14,-8) & B(-2,-8) \\ C(-14,8) & D(-2,8) \end{matrix}\right\}$$

- $\vec{XE} = \begin{pmatrix} -16-(-8) \\ 6-0 \end{pmatrix} = \begin{pmatrix} -8 \\ 6 \end{pmatrix}$ $\quad |\vec{XE}| = R = 10$

$\vec{XC} = \begin{pmatrix} -14-(-8) \\ 8-0 \end{pmatrix} = \begin{pmatrix} -6 \\ 8 \end{pmatrix}$ $\quad |\vec{XC}| = R = 10$

$\vec{XA} = \begin{pmatrix} -14-(-8) \\ -8-0 \end{pmatrix} = \begin{pmatrix} -6 \\ -8 \end{pmatrix}$ $\quad |\vec{XA}| = R = 10$

- $\alpha = \cos^{-1}\left(\frac{\vec{XE}\cdot\vec{XC}}{|\vec{XE}||\vec{XC}|}\right) = \cos^{-1}\left(\frac{(-8)(-6)+(6)(8)}{(10)(10)}\right) = 16.26°$

$\beta = \cos^{-1}\left(\frac{\vec{XE}\cdot\vec{XA}}{|\vec{XE}||\vec{XA}|}\right) = \cos^{-1}\left(\frac{(-8)(-6)+(6)(-8)}{(10)(10)}\right) = 90°$

- As $\tau_{x'y'}$ cannot exceed $\pm 8\,MPa$, we cannot go into the yellow-shaded sector of the Mohr's circle above. This means, our permitted range of angles ON THE MOHR CIRCLE is :

$$\underbrace{-\beta \le 2\theta \le \alpha}_{\text{LHS of the circle}} \quad \text{or} \quad \underbrace{(180^\circ - \beta) \le 2\theta \le (180^\circ + \alpha)}_{\text{RHS of the circle}}$$

→ Keep in mind these angles are measured CCW from σ_x (line XE)

Substituting α and β,

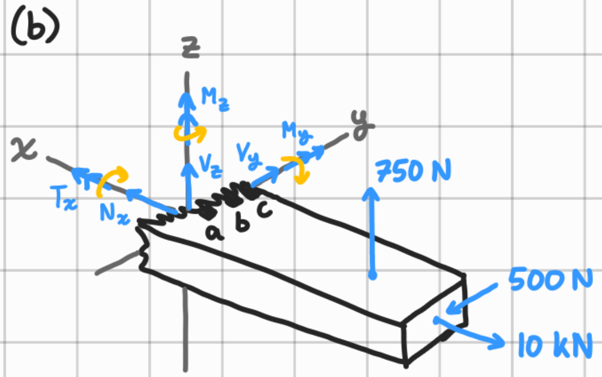
$$-90^\circ \le 2\theta \le 16.26^\circ \quad \text{or} \quad 180^\circ - (90^\circ) \le 2\theta \le 180^\circ + 16.26^\circ$$

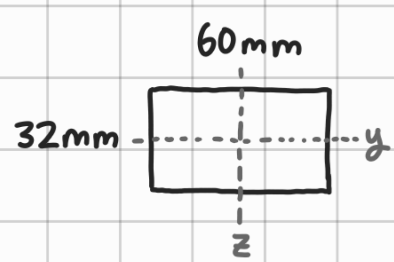
$$-90^\circ \le 2\theta \le 16.26^\circ \quad \text{or} \quad 90^\circ \le 2\theta \le 196.26^\circ$$

Dividing by 2 to get the physical angle θ of the stress element,

$$\boxed{\therefore \quad -45^\circ \le \theta \le 8.13^\circ \quad \text{or} \quad 45^\circ \le \theta \le 98.13^\circ}$$ //

(b)





$I_z = \frac{1}{12}(32)(60)^3 \times 10^{-12} = 5.76 \times 10^{-7}\ m^4$

$I_y = \frac{1}{12}(60)(32)^3 \times 10^{-12} = 1.638 \times 10^{-7}\ m^4$

$A = (60 \times 32) \times 10^{-6} = 1.92 \times 10^{-3}\ m^2$

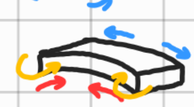
$\Sigma F_x = 0 \Rightarrow N_x = 10\ kN$

$\Sigma F_y = 0 \Rightarrow V_y = 500\ N$

$\Sigma F_z = 0 \Rightarrow V_z = -750\ N$

$\Sigma M_x = 0 \Rightarrow T_x = 0$

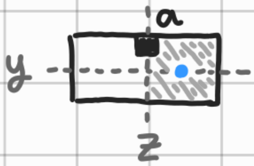
$\Sigma M_y = M_y - 750\,(180 \times 10^{-3}) = 0 \Rightarrow M_y = +135\ N\cdot m$ →  *

$\Sigma M_z = M_z - 500\,(220 \times 10^{-3}) = 0 \Rightarrow M_z = +110\ N\cdot m$ →  **

* $\sigma_y = -\dfrac{M_y z}{I_y}$ as points above y-axis (+z) experiences compression ($-\sigma$). A negative sign is needed in the formula to make the LHS have the same sign as the RHS.

** $\sigma_z = \dfrac{M_z y}{I_z}$ as points to the right of z-axis (+y) experiences tension ($+\sigma$).

Point a

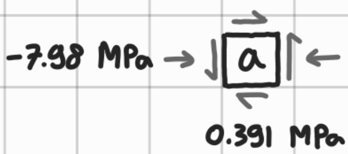


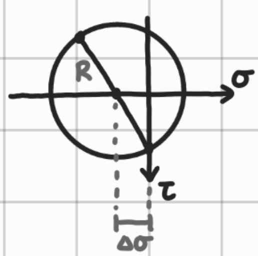
$$(Q_y)_a = Ay' = (32 \times 30)(15) \times 10^{-9} = 1.44 \times 10^{-5}\ \text{m}^3$$

$$(Q_z)_a = 0 \Rightarrow (\tau_z)_a = 0$$

$$(\tau_y)_a = \frac{V_y \cdot (Q_y)_a}{I_z \cdot t_z} = \frac{(500)(1.44 \times 10^{-5})}{(5.76 \times 10^{-7})(32 \times 10^{-3})} = 0.391\ \text{MPa}$$

$$\sigma_a = -\frac{M_y z}{I_y} + \frac{M_z y}{I_z} + \frac{N}{A} = -\frac{(135)(16 \times 10^{-3})}{1.638 \times 10^{-7}} - 0 + \frac{10\,000}{1.92 \times 10^{-3}} = -7.98\ \text{MPa (C)}$$





$$R = \sqrt{\Delta\sigma^2 + \tau^2} = \sqrt{\left(\frac{7.98}{2}\right)^2 + (0.391)^2} = 4.01\ \text{MPa}$$

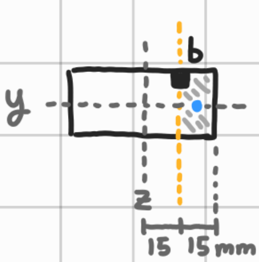
$$(\sigma_{max})_a = \sigma_{avg} \pm R$$

$$\frac{-7.98}{2} + 4.01 = 0.02\ \text{MPa (T)}$$

$$\frac{-7.98}{2} - 4.01 = -8.00\ \text{MPa (C)}$$

$$(\tau_{max})_a = R = 4.01\ \text{MPa}$$

Point b

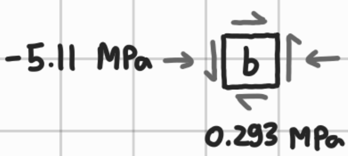


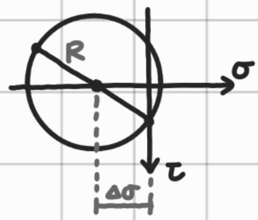
$$(Q_y)_b = Ay' = (32 \times 15)\left(15 + \frac{15}{2}\right) \times 10^{-9} = 1.08 \times 10^{-5}\ \text{m}^3$$

$$(Q_z)_b = 0 \Rightarrow (\tau_z)_b = 0$$

$$(\tau_y)_b = \frac{V_y \cdot (Q_y)_b}{I_z \cdot t_z} = \frac{(500)(1.08 \times 10^{-5})}{(5.76 \times 10^{-7})(32 \times 10^{-3})} = 0.293 \text{ MPa}$$

$$\sigma_b = -\frac{M_y z}{I_y} + \frac{M_z y}{I_z} + \frac{N}{A} = -\frac{(135)(16 \times 10^{-3})}{1.638 \times 10^{-7}} + \frac{(110)(15 \times 10^{-3})}{5.76 \times 10^{-7}} + \frac{10\,000}{1.92 \times 10^{-3}} = -5.11 \text{ MPa (C)}$$





$$R = \sqrt{\Delta\sigma^2 + \tau^2}$$

$$= \sqrt{\left(\frac{5.11}{2}\right)^2 + (0.293)^2} = 2.57 \text{ MPa}$$

$$(\sigma_{max})_b = \sigma_{avg} \pm R \;\begin{cases} \dfrac{-5.11}{2} + 2.57 = 0.015 \text{ MPa (T)} \\[2ex] \dfrac{-5.11}{2} - 2.57 = -5.125 \text{ MPa (C)} \end{cases}$$

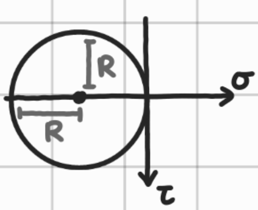
$$(\tau_{max})_b = R = 2.57 \text{ MPa}$$

Point c

$$(Q_y)_c = 0 \Rightarrow (\tau_y)_c = 0$$

$$(Q_z)_c = 0 \Rightarrow (\tau_z)_c = 0$$

$$\sigma_b = -\frac{M_y z}{I_y} + \frac{M_z y}{I_z} + \frac{N}{A} = -\frac{(135)(16 \times 10^{-3})}{1.638 \times 10^{-7}} + \frac{(110)(30 \times 10^{-3})}{5.76 \times 10^{-7}} + \frac{10\,000}{1.92 \times 10^{-3}} = -2.25 \text{ MPa (C)}$$



$$(\sigma_{max})_c = \begin{cases} 0 \text{ MPa (T)} \\ -2.25 \text{ MPa (C)} \end{cases}$$

$$(\tau_{max})_c = R = \frac{2.25}{2} = 1.13 \text{ MPa}$$