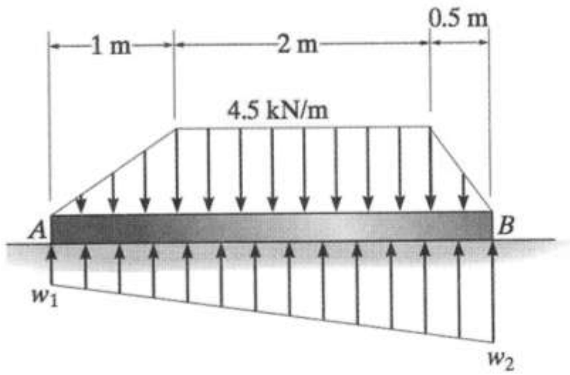
NTU 16/17 Semester 2 CV2011 Final Exam Solution

1.(a)



The resultant force on top of beam = The resultant force at below of beam

$$\frac{1}{2}(4.5)(2+3.5) = \frac{1}{2}(3.5)(w_1 + w_2)$$

$$\mathbf{w_2 + w_1 = 7.0714 ------(1)}$$

At point A,

The moment due to the force on top of beam = The moment due to the force below of beam

Devide the force distribution on top into 3 parts, the force distribution below into 2 parts, then

$$F_1\left(\frac{2}{3}\right) + F_2(2) + F_3\left(3\frac{1}{6}\right) = F_4(1.75) + F_5\left(\frac{7}{3}\right)$$

where

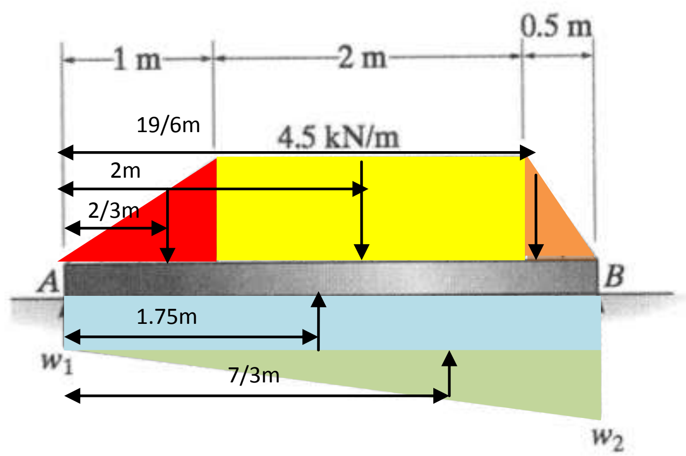
$$F_1 = \frac{1}{2}(4.5)(1) = 2.25\text{kN}$$

$$F_2 = (4.5)(2) = 9\text{kN}$$

$$F_3 = \frac{1}{2}(4.5)(0.5) = 1.125\text{kN}$$

$$F_4 = (w_1)(3.5) = 3.5w_1\text{kN}$$

$$F_5 = \frac{1}{2}(w_2 - w_1)(3.5) = 1.75(w_2 - w_1)\text{kN}$$



==》 $\mathbf{2w_2 + w_1 = 11.2960 ------(2)}$

By solving equation (1) and (2) ,

w_2 =4.22kN/m, w_1 =2.85 kN/m

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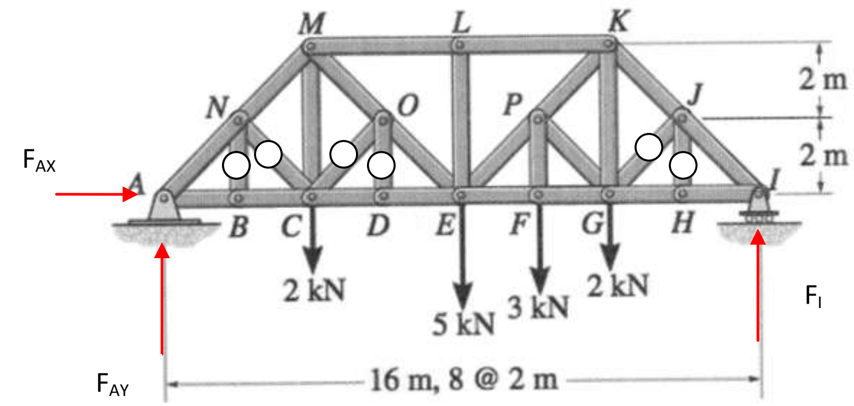
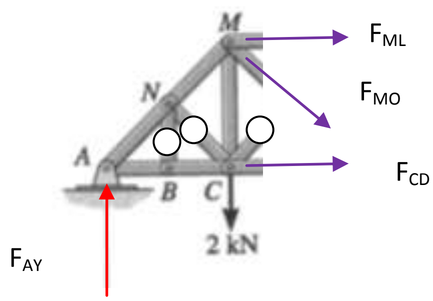
1.(b) First, remove all the obvious zero force members, identify support reactions



Support Reaction:

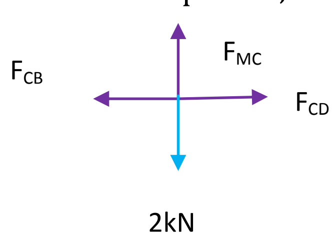
Take moment about point A,
We get F_I =6.375 kN
ΣF_y =0, we get F_{Ay}=5.625kN
ΣF_x =0, we get F_{Ax} = 0



Take moment about point M,
-5.625(4)+ F_{CD}(4)=0

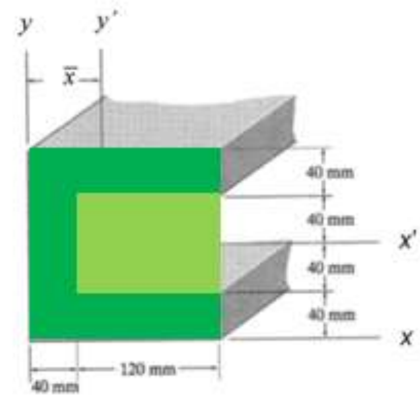
F_{CD}=5.625kN (Tension)

Consider point C,



ΣF_y =0, we get **F_{MC}=2kN (Tension)**

2(a)



Moment of big square- moment of small rectangle= moment of U shape

(160)(160)(80)-(120)(80)(100)=x(160*160-120*80)
X=68mm

Moment of inertia about y'-axis
= Moment of intertia of big square about y'-axis – Moment of intertia of small rectangle about y'-axis
$=[\frac{1}{12}(160)(160)^3 + (160)^2(80-68)^2] -$
$[\frac{1}{12}(80)(120)^3 + (80)(120)(100-68)^2]$
=36.9 x 10^8 (mm^4)

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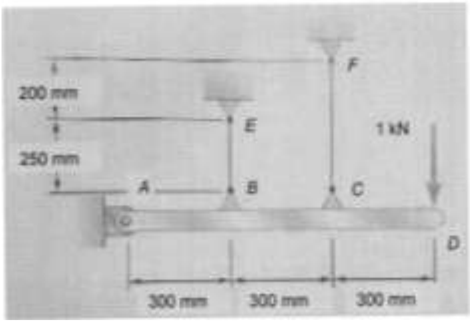
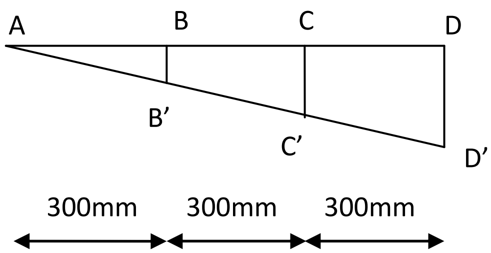
2(b)



Take moment about point A,

$F_{BE}(300)+F_{FC}(600)-1(900)=0$

$F_{BE} + 2F_{FC}=3$-----(1)



Due to property of similar triangle,

$$\frac{\delta_{BE}}{\delta_{CF}} = \frac{300}{300+300}$$

$$\frac{\frac{P_1L_1}{E_1A_1}}{\frac{P_2L_2}{E_2A_2}} = \frac{1}{2}$$

Take note that $E_1=E_2$, $A_1=A_2$

$2F_{BE}(250)=F_{FC}(450)$

$F_{BE} - 0.9F_{FC} = 0$-----(2)

Solve equation (1) and (2),

We will get $\mathbf{F_{BE}=\frac{27}{29}}$ **kN**, $\mathbf{F_{FC}=\frac{30}{29}}$ **kN**

$$\delta_B = \frac{P_1L_1}{E_1A_1}$$

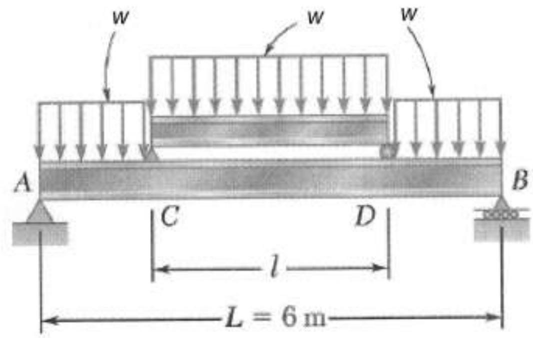
$$\delta_B = \frac{\frac{27}{29} \times 250}{200G \times \pi \times (\frac{0.0015}{2})^2}$$

= 0.6585mm

Due to properties of similar triangle,

$\mathbf{\delta_D = 3\delta_B = 1.976mm}$

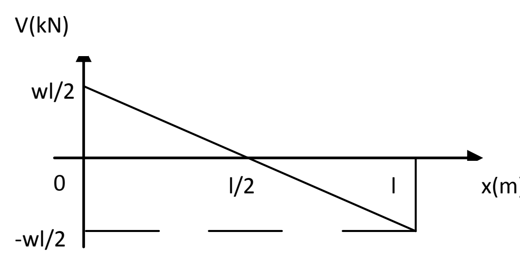
3(a)



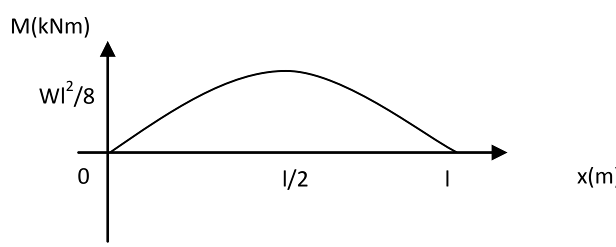
FBD of Bar CD.

(By symmetry)

Shear Diagram of Bar CD



Bending Moment Diagram of Bar CD



Remarks: After drawing the shear diagram, you should be able to tell how does the shape of Bending Moment Diagram look like, it is a very basic pattern.

Q: How we know $wl^2/8$ is the largest bending moment?

A: Find the area from 0 to l/2 in Shear Diagram. Since this area is above x-axis, it means increment of Bending Moment

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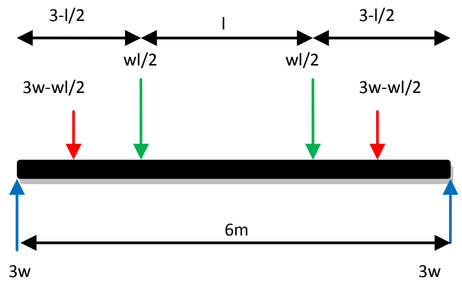
Continue 3 (a)

FBD of Bar AB



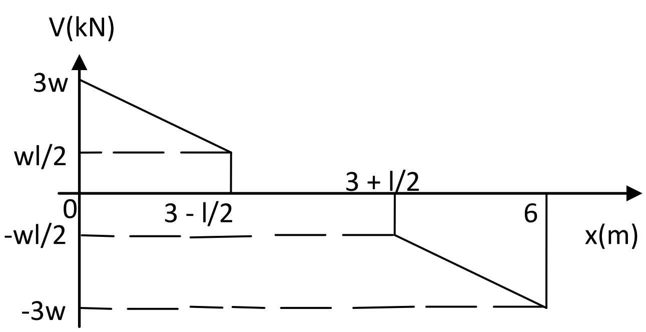
Too fast? Don't worry, follow the steps:

1. Determine the length accordingly, assume **symmetry**, else will be too troublesome=.=

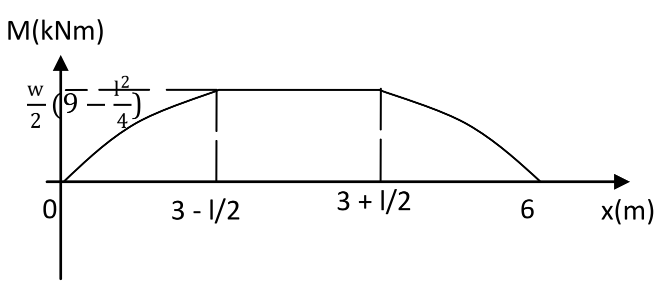
Though I think this should be stated at the beginning of question rather than part (b)...

2. Copy the green arrow from Bar CD based on Newton Third Law.
3. Find the value of red arrow (Distributed loading) by length*loading per length
4. Use $\Sigma F_y = 0$, to find support reaction (blue arrow)

Shear Diagram of Bar AB



Bending Moment Diagram of Bar AB



Q: How to get $\frac{w}{2}(9 - \frac{l^2}{4})$?

A: Area from Shear Diagram.

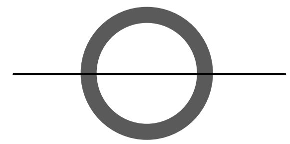
**Try to master the "lazy" method to draw shear and bending moment diagram. It helps a lot in exam since you need to write significantly less.

3(b)

When l = 3m,

Shear Max of Bar CD = 1.5w

Shear Max of Bar AB =3w ←Choose this (larger)



Cross section properties:

$$Q = \frac{1}{2}\pi(50^2)\left(\frac{4(50)}{3\pi}\right) - \frac{1}{2}\pi(30^2)\left(\frac{4(30)}{3\pi}\right)$$

Q = 65333.33 (mm³)

$$I = \frac{1}{4}\pi(50^4 - 30^4)$$

I = 4.2725 x 10⁶ (mm⁴)

t = 100 - 60

t = 40(mm)

max allow transverse shear stress **τ = 80 MPa**

Use formula

$$\tau = \frac{VQ}{It}$$

we will get w_{max} = 69.76kN/m

When l = 3m,

B. Moment Max of Bar CD = 9w/8

B. Moment Max of Bar AB =27w/8←Choose this (larger)

Given max allow bending stress **σ = 140Mpa**

Use Formula

$$\sigma = \frac{My}{I}$$

where **y = 50mm**

we will get w_{max}= 3.54kN/m

Compare the two w_{max} we get, it is obvious that structure will fail when w > 3.54kN/m.

Therefore, largest permessible **w = 3.54kN/m.**

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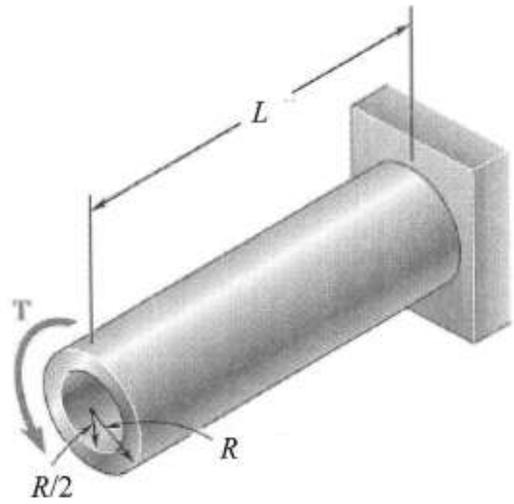
3(b)



Let φ_1 be the angle of thrist of first case,
φ_2 be the angle of thrist of second case

Given $A_1 = A_2$

$\pi[(2R)^2 - R^2] = \pi {r_2}^2$

$r_2 = \sqrt{3}R$

Using Formula

$$\varphi = \frac{TL}{JG}$$

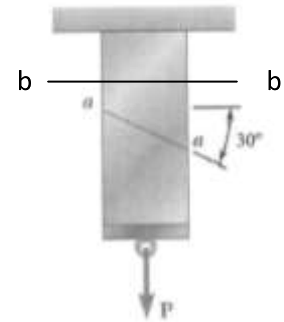
Take note that T, L, G are same for both cases,
Therefore,

$$\frac{\varphi_1}{\varphi_2} = \frac{J_2}{J_1}$$

$$\frac{3^o}{\varphi_2} = \frac{\frac{\pi}{2}(\sqrt{3}R)^4}{\frac{\pi}{2}[(2R)^4 - R^4]}$$

Simplify, we get **$\varphi_2 = 5^o$**

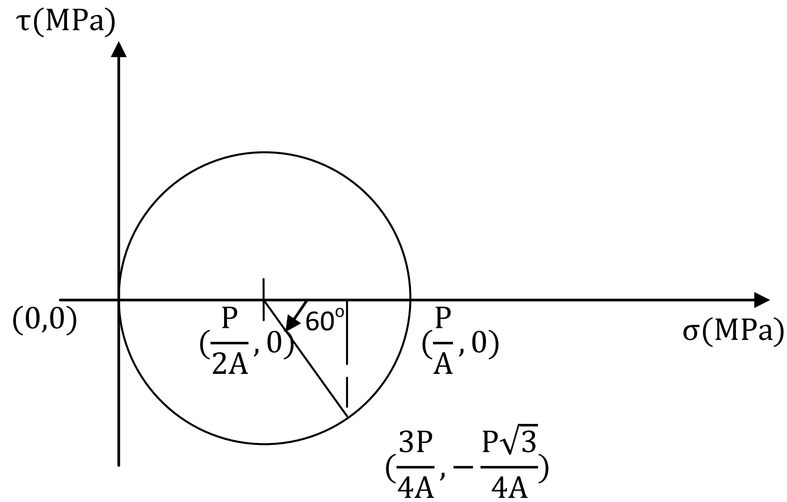
4(a)



Given
At b-b surface, horizontal, based on force equilirbrium

$\sigma_x = 0, \sigma_y = \frac{P}{A}, \quad \tau = 0$ Take note A = 3600mm²

$\sigma_{ave} = \frac{\frac{P}{A}+0}{2} = \frac{P}{2A}$



Q: How to find coordinate correspond to a-a surface?
A: Use the trigo-ratio in the right triangle.

Cross section area = 3600 (mm²)

Let $\frac{3P}{4A} = 800$, we will get P_{max} = 3840kN

Let $\frac{P\sqrt{3}}{4A} = 600$, we will get P_{max} = 4988kN

Therefore, the largest load that can applied is **3840kN.**

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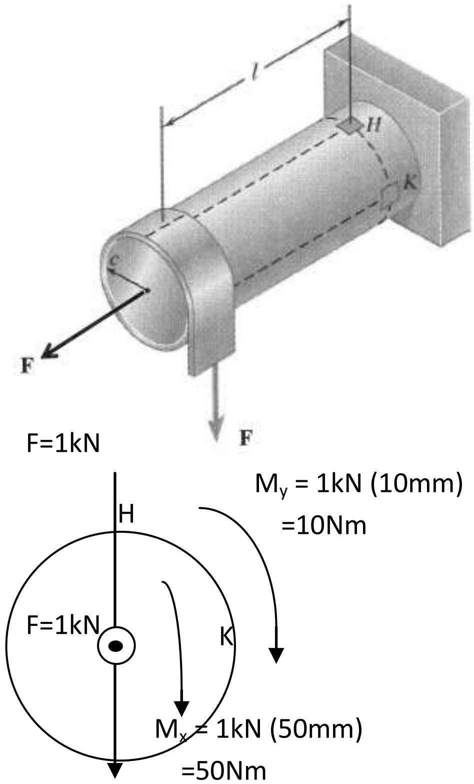
4(b)



Cross section propersites:

$A = \pi(0.01)^2$

A = 3.141 x 10^{-4} m^2

$Q = \frac{2}{3}(0.01)^3$

Q = 6.667 x 10^{-7} m^3

$I = \frac{\pi}{4}(0.01)^4$

I = 7.85 x 10^{-9} m^4

J = 2I = 1.57 x 10^{-8} m^4

y = ρ = 0.01m

t = 0.02 m

At point H,

Normal stress $\sigma = \frac{F}{A} + \frac{M_x y}{I}$

Normal stress σ = 66.85MPa

Shear stress

$$\tau = \frac{T\rho}{J}$$

τ = 6.37 MPa

At point K,

Normal stress $\sigma = \frac{F}{A}$

Normal stress σ = 3.18MPa

Shear stress

$$\tau = \frac{T\rho}{J} + \frac{VQ}{It}$$

τ = 10.61 MPa

Use these formulas to find the answer required,

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$\sigma_{1,2} = \sigma_{ave} \pm \tau_{max}$$

At point H,

$\sigma_x = 66.85$Mpa, $\sigma_y = 0$, $\tau_{x,y} = 6.37$ MPa

$\boldsymbol{\tau_{max} = 34.04}$**MPa**

$\boldsymbol{\sigma_1 = 67.45}$**MPa (Tension)**

$\boldsymbol{\sigma_2 = -0.615}$**MPa (Compression)**

At point K,

$\sigma_x = 3.18$Mpa, $\sigma_y = 0$, $\tau_{x,y} = 10.61$ MPa

$\boldsymbol{\tau_{max} = 10.73}$**MPa**

$\boldsymbol{\sigma_1 = 12.32}$**MPa (Tension)**

$\boldsymbol{\sigma_2 = -9.14}$**MPa (Compression)**

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