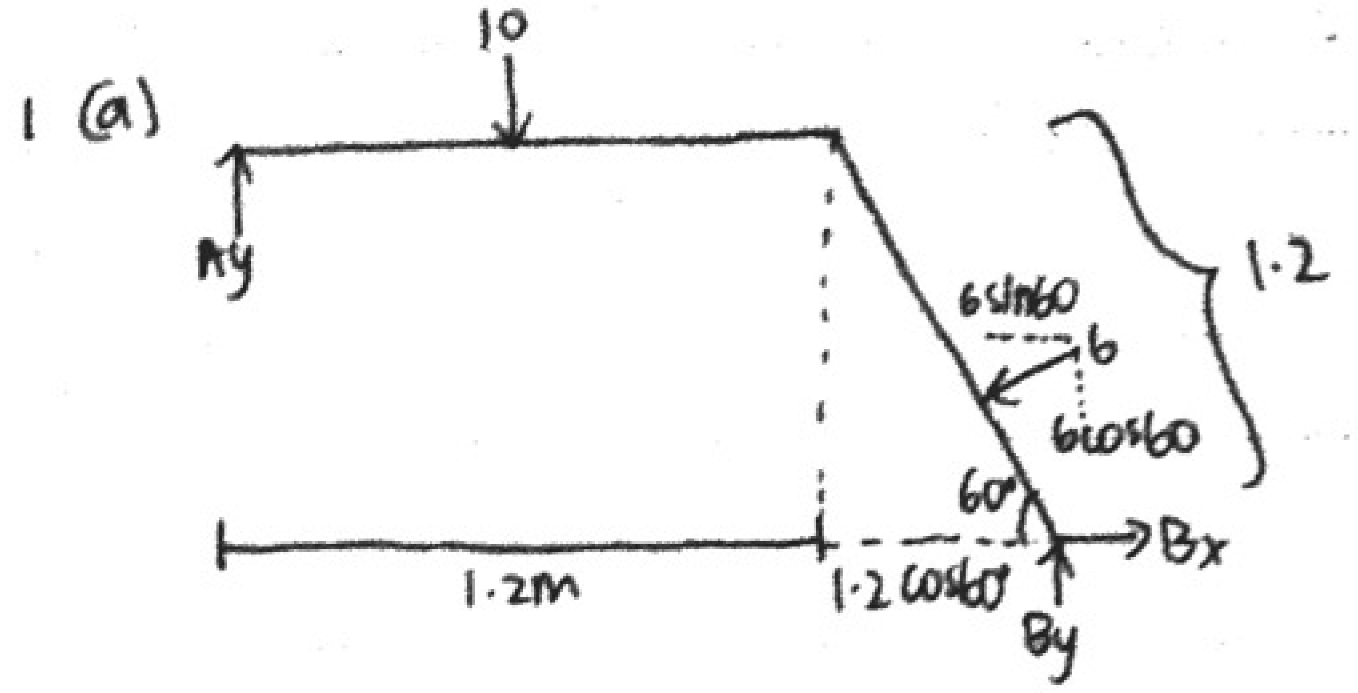
CV2011 Mechanics of Material Nov/Dec 2016 Solutions

1 (a)



$\Sigma M_B = 0$

$-A_y \times (1.2 + 1.2\cos 60°) + 10(0.6 + 1.2\cos 60°) + 6(0.4) = 0$

$A_y = \underline{8\ \text{KN}}$

$\Sigma F_y = 0$

$B_y + 8 - 10 - 6\cos 60° = 0$

$\underline{B_y = 5\ \text{KN}}$

$\Sigma F_x = 0$

$B_x - 6\sin 60° = 0$

$\underline{B_x = 5.196\ \text{KN}}$

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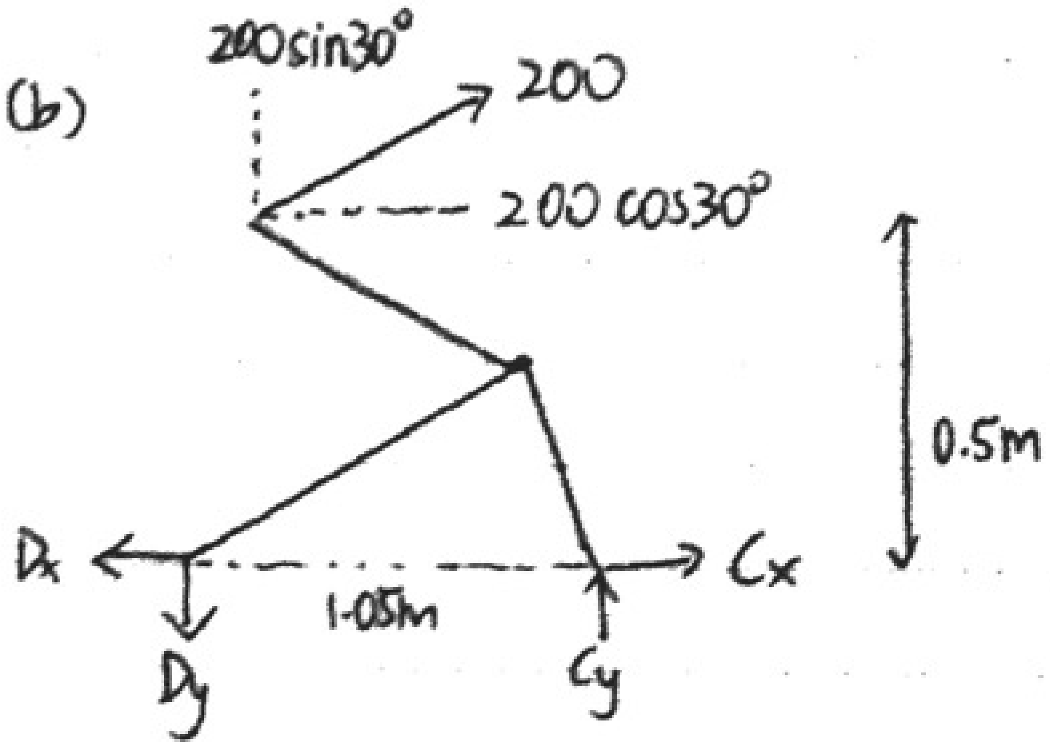
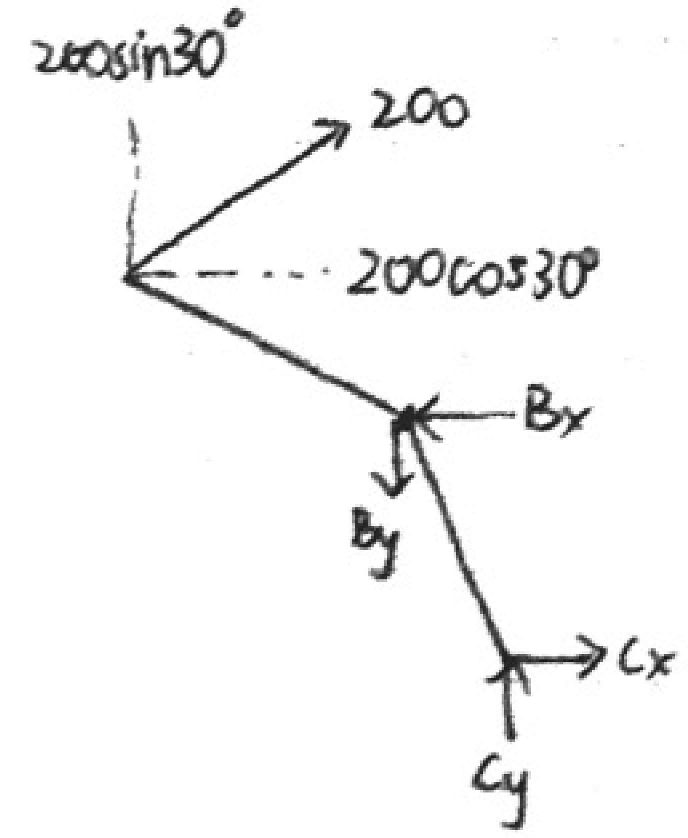
1 (b)



$\Sigma M_D = 0$

$200\sin 30° \times 0.15 - 200\cos 30° \times 0.5 + C_y(1.05) = 0$

$\underline{C_y = 68.19\ \text{N}}$



$\Sigma M_B = 0$

$-200\cos 30° \times 0.25 - 200\sin 30° \times 0.75 + 68.19 \times 0.15 + C_x \times 0.25 = 0$

$\underline{C_x = 432.3\ \text{N}}$

$\Sigma F_x = 0$

$B_x = 432.289 + 200\cos 30°$

$= 605.49\text{N}$

$\Sigma F_y = 0$

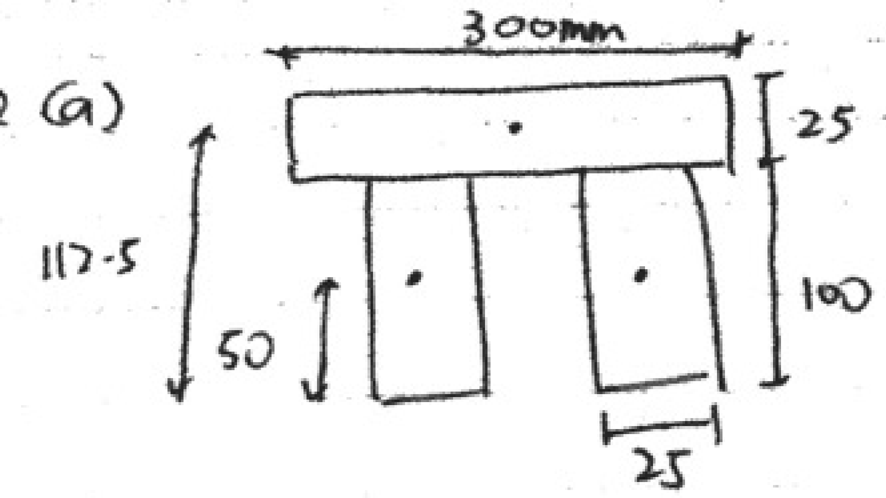
$B_y = 68.19 + 200\sin 30°$

$= 168.19\text{N}$

$F_{BD} = \sqrt{605.49^2 + 168.19^2}$

$= 628.4\text{N}$

2 (a)



centroid $\bar{y} = \frac{\Sigma \bar{y} A}{A}$

$$= \frac{112.5 \times 300 \times 25 + 2 \times 25 \times 100 \times 50}{300 \times 25 + 2 \times 25 \times 100}$$

$$= \underline{87.5 \text{ mm}}$$

Moment of Inertia about the centroid axis

$$= \Sigma(I + Ad^2)$$

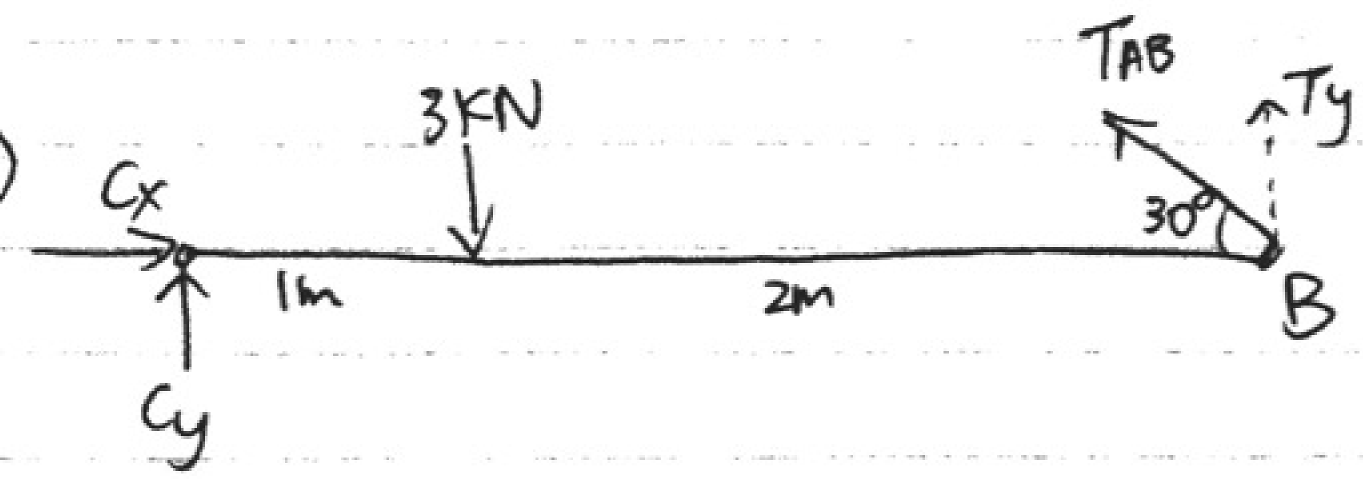
$$= \frac{1}{12}(300)(25)^3 + 300 \times 25 \times (112.5 - 87.5)^2$$

$$+ \left[\frac{1}{12}(25)(100)^3 + 25 \times 100 \times (87.5 - 50)^2\right] \times 2$$

$$= 5078125 \quad + \quad 11197916$$

$$= \underline{16276041} \text{ mm}^4$$

2 (b)



$\Sigma M_B = 0$

$-C_y \times 3 + 3 \times 2 = 0$

$C_y = 2$ KN

$\Sigma F_y = 0 \Rightarrow T_y = 1$ KN

$T_{AB} = \frac{1}{\sin 30^\circ} = 2$ KN

Area of steel wire $= \pi(5^2) = 25\pi$ mm^2

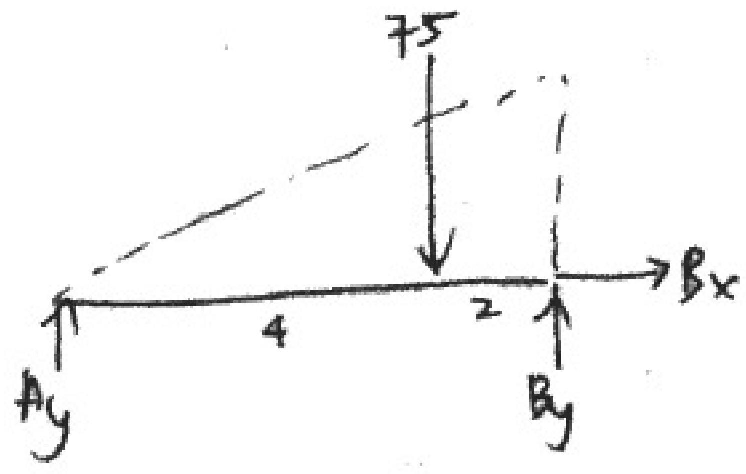
$$\delta = \frac{PL}{AE} = \frac{2000\text{N} \times 3.46 \times 10^3 \text{ mm}}{25\pi \text{ mm}^2 \times 200 \times 10^3 \frac{\text{N}}{\text{mm}^2}}$$

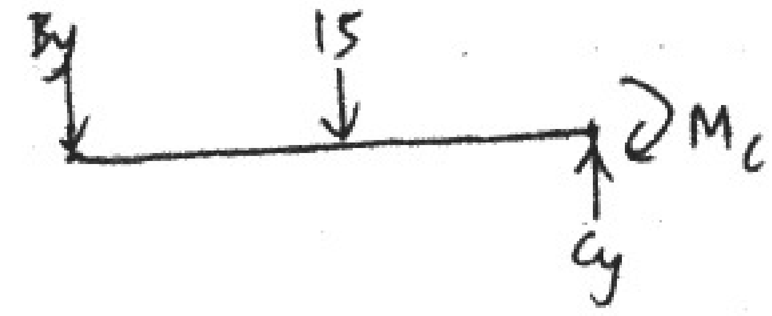
$$= \underline{0.44 \text{ mm}}$$

To check whether the steel wire fails, $\tau_{max} = \frac{2000}{25\pi} = 25.46$ MPa < 250

Do not fail.

3 (a)





$\Sigma M_A = 0$, $B_y \times 6 = 75 \times 4$

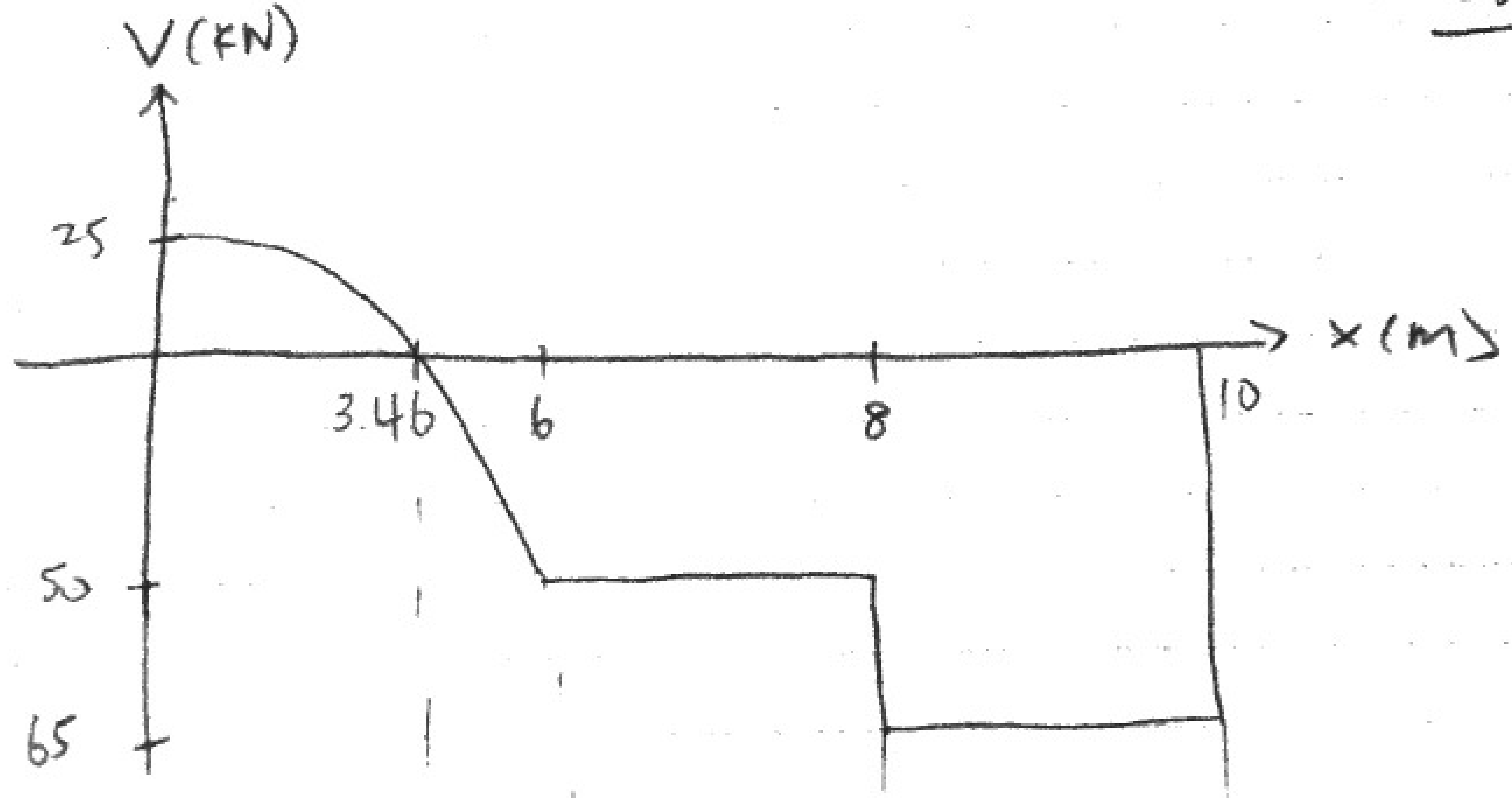
$\underline{B_y = 50 KN}$, $\underline{A_y = 25 KN}$

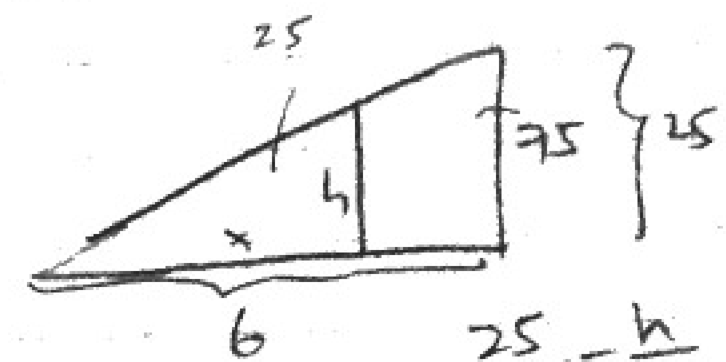
$B_x = 0$.

$C_y = 50 + 15 = \underline{65 KN}$

$M_c = 15 \times 2 + 50 \times 4$

$= \underline{230 KNm}$



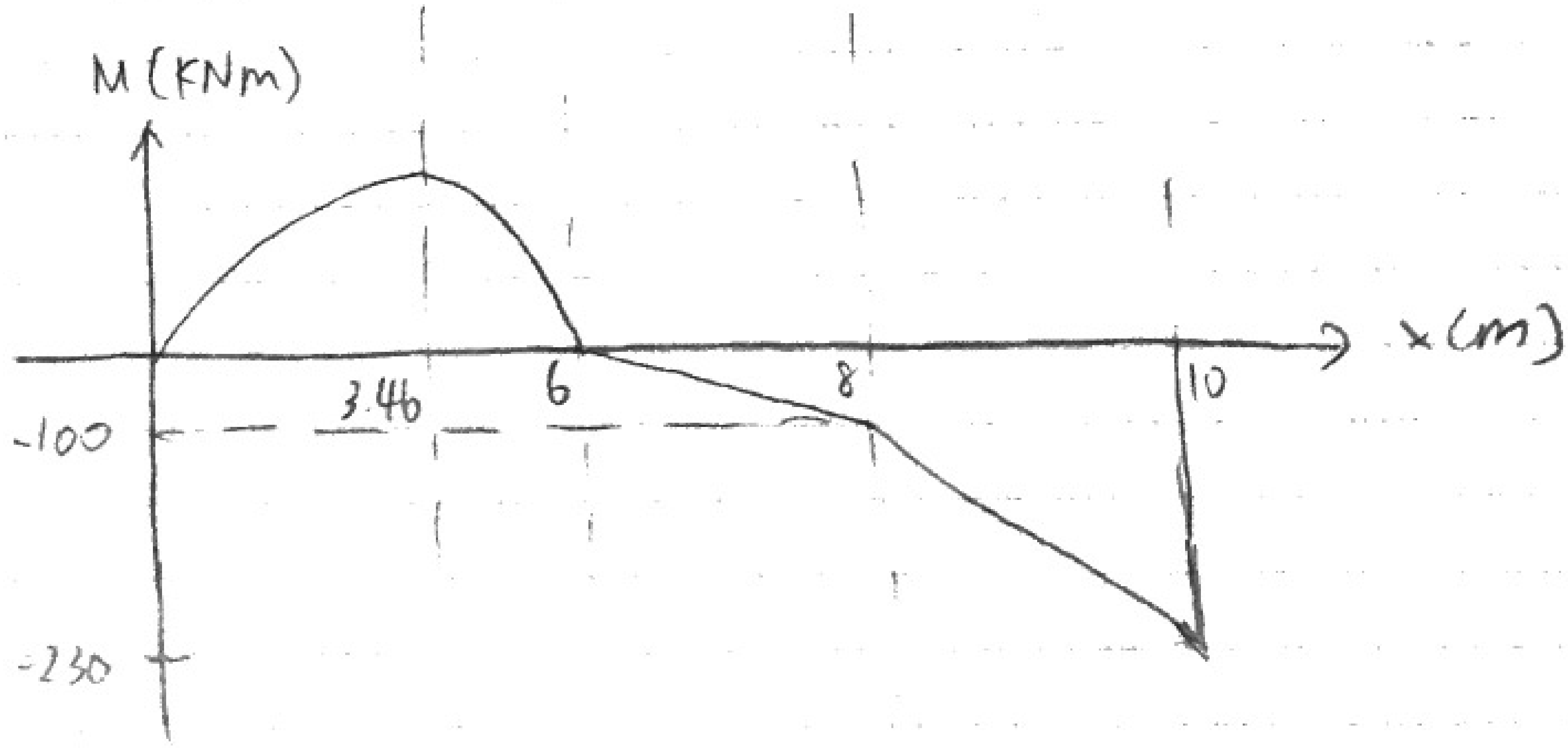


$\frac{25}{6} = \frac{h}{x}$ $h = \frac{25x}{6}$

$\frac{1}{2}h \cdot x = 25$

$\frac{1}{2}\left(\frac{25x^2}{6}\right) = 25$

$x = \sqrt{12} = 3.46$



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3 (b)

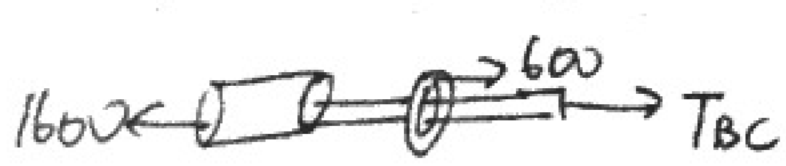
$\rightarrow$ = anticlockwise
$\leftarrow$ = clockwise

$J = \frac{\pi}{2} r^4 \text{ mm}^4$

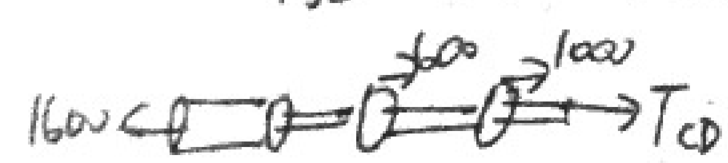
$T_{AB} = +1600 Nm$

$\tau = \frac{Tc}{J} = \frac{1600r}{\frac{\pi}{2}r^4} = \frac{3200}{\pi r^3} = 120 MPa \Rightarrow r = 8.488 mm$

(This is the max r for shear force as T is the highest).



$T_{BC} = +1000 Nm$



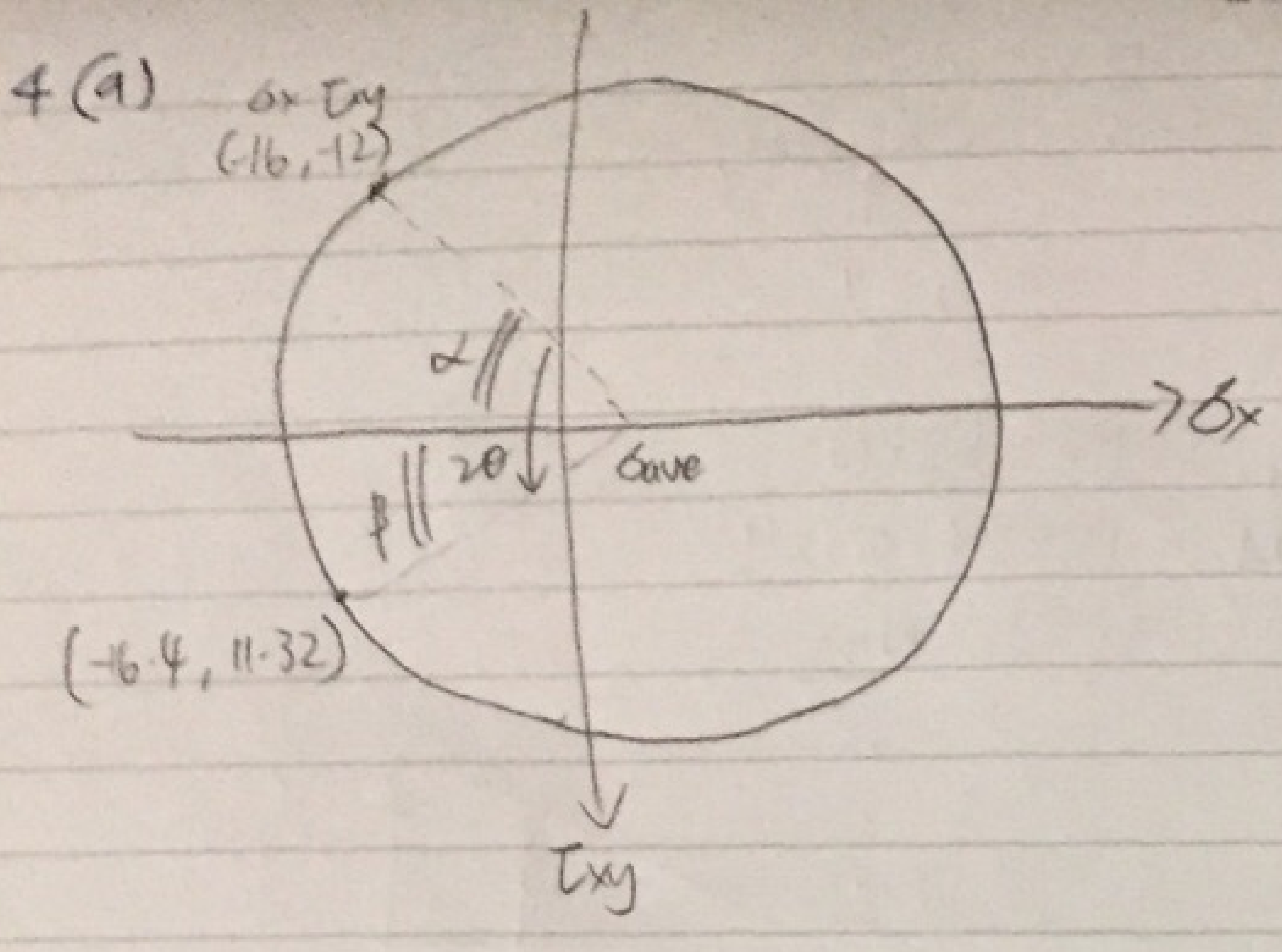
$T_{CD} = 0$.

$$\phi = \frac{1600 \times 10^3 Nmm \times 400mm}{J(77 \times 10^3)} + \frac{1000 \times 10^3 \times 600}{J(77 \times 10^3)} = \frac{3}{180}\pi$$

$\frac{16103.8}{J} = 0.05259$, $J = 307561 = \frac{\pi}{2}r^4$

$r = 21.03 mm$, $\boxed{\text{Max } d = 42.06 mm}$

4 (a)



$(-16-\sigma_{ave})^2+12^2 = R^2 = (-16.4-\sigma_{ave})^2+11.32^2$

Using GC,

$\sigma_{ave} = 3.622 = \dfrac{\sigma_x+\sigma_y}{2}$

$\sigma_y = \underline{\underline{23.244\ MPa}}$

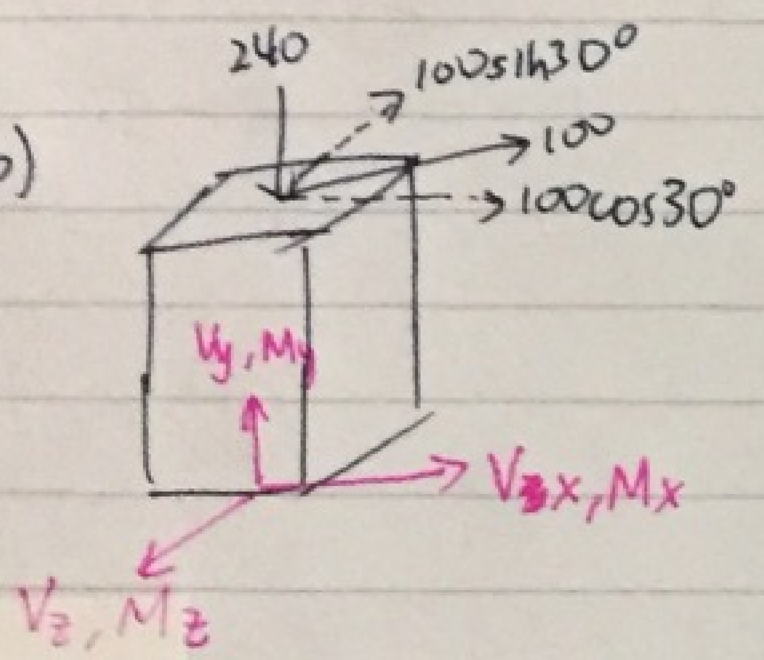
$R = 23$

$\alpha = \sin^{-1}\dfrac{12}{23} = 31.44°$

$\beta = \sin^{-1}\dfrac{11.32}{23} = 29.48°$

$\theta = \dfrac{31.44+29.48}{2} = \underline{\underline{30.47°}}$

4(b)



$\Sigma F_x = 0$, $V_x = -100\cos 30° = -86.6\ kN$ (comp)

$\Sigma M_x = 0$, $M_x - 100\sin 30°(375) = 0$

$M_x = 18750\ kNmm$

$\Sigma F_y = 0$, $V_y = 240\ kN$ (comp)

$M_y = 0 \Rightarrow$ Torsion $= 0$.

$\Sigma F_z = 0$, $V_z = 100\sin 30° = 50\ kN$ (tension)

$\Sigma M_z = 0$, $M_z - 100\cos 30°(375) = 0$

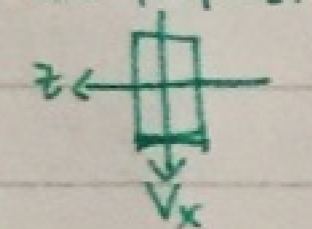
$M_z = 32475\ kNmm$.

Area $= 100\times150 = 15000\ mm^2$

$I_x = \frac{1}{12}(150)(100^3) = 125\times10^5\ mm^4$

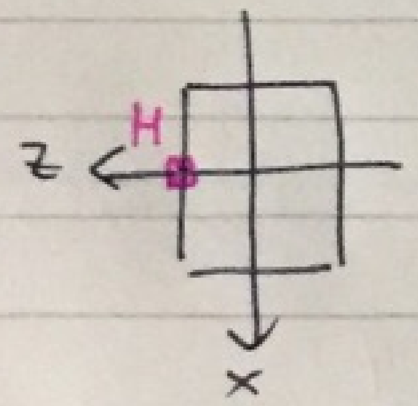
$I_z = \frac{1}{12}(100)(150^3) = 28125000\ mm^4$

Note: For shear force, the moment of inertia is about the neutral axis (ie axis perpendicular to shear).



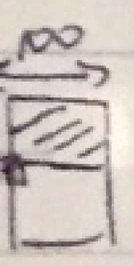
ie for V_x, take moment of inertia about the z axis.

For point H,



$\sigma_H = \dfrac{P}{A} + \dfrac{M_x c}{I_x} = -\dfrac{240000}{15000} - \dfrac{18750\times10^3\times50}{125\times10^5}$

$= -59\ MPa$



$Q = \frac{75}{2}(75\times100) = 281250\ mm^3$

$\tau = \dfrac{V_x Q}{I_z t} = \dfrac{86.6\times10^3\times281250}{28125000\times100} = 8.66\ MPa$

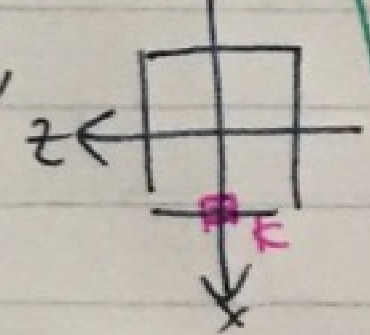
$\tau_{max} = \sqrt{\left(\frac{59}{2}\right)^2 + 8.66^2} = 30.74$

$\sigma_{1,2} = -59+30.74$ or $59-30.74$

$= -89.7$ or $-28.25\ MPa$

(max comp) (max tensile)

For point K,



$\sigma_K = -\dfrac{P}{A} + \dfrac{M_z c}{I_z} = -16 - \dfrac{32475\times75}{28125000}$

$= -102.6\ MPa$

$Q = \frac{50}{2}(50\times150) = 187500\ mm^3$

$\tau = \dfrac{V_z Q}{I_x t} = \dfrac{50\times10^3\times187500}{125\times10^5\times150} = 5\ MPa$

$\tau_{max} = \sqrt{\left(\frac{102.62}{2}\right)^2 + 5^2} = 51.54\ MPa$

$\sigma_{1,2} = \underline{-51.05}$ and $\underline{-154.15\ MPa}$

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