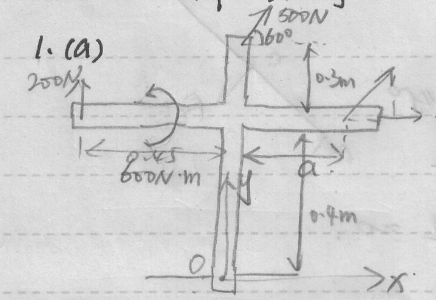
CV1011 April / May 2016.

1. (a)



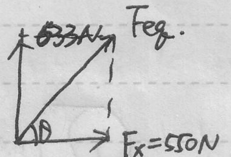
$F_{eqx} = \Sigma F_x = 300 + 500 \cdot \cos 60°$

$= 550N$

$F_{eqy} = \Sigma F_y = 200 + 500 \cdot \sin 60°$

$\approx 633N$

$F_{eq} = \sqrt{F_{eqx}^2 + F_{eq}^2} = \sqrt{550^2 + 633^2} \approx 839N$



$\tan\theta = \frac{633}{550} = 1.15$

$\theta = \tan^{-1}(1.15) \approx 49.0°$

Suppose moment anticlockwise is positive $M \circlearrowleft +$.

$\circlearrowleft + \Sigma M_{eq} = 600 - 200 \times 0.45 - 500 \cdot \cos 60° \times 0.7 - 300 \times 0.4 = 215 Nm$

Assume the location of equivalent force is located at $(a, 0.4)$

$215 = 633 \times a - 550 \times 0.4$

$\Rightarrow a = 0.6872\ m \quad (> 0.45m))$

which is actually outside the frame

Assume the location is at $(0, b)$

$215 = -550 \times b$

$\Rightarrow b = -0.3909 m \quad (< 0)$

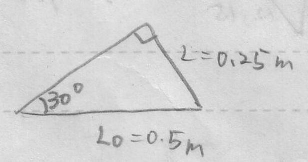
which is below the original point O)

(b) additional condition: $L = 25\ cm$ (in exam)

$\because$ the lever in the position 45°

$F = k(L - L_0)$

$\therefore L_{AD} = \sqrt{2} \cdot L$



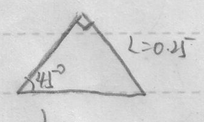
$= 500(0.354 - 0.5)$

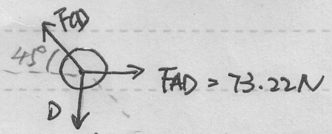
$= \sqrt{2} \times 0.25$

$= -73.22 N$

$\approx 0.354\ m$

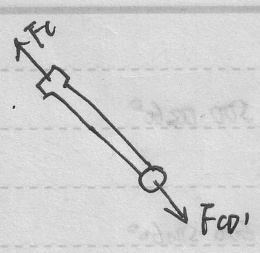
negative sign means the force is compressive.





$F_{CD} = \sqrt{2} F_{AD}$

$= 103.55 N$

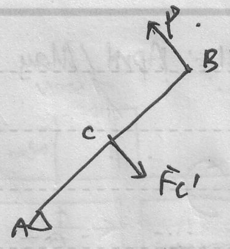


$F_C = F_{CD}' = F_{CD} = 103.55\,N$

$F_C' = F_C = 103.55\,N$

$\because$ the lever in the position 45°

$l_{CD} = l_{AC} = 0.25\,m = L$

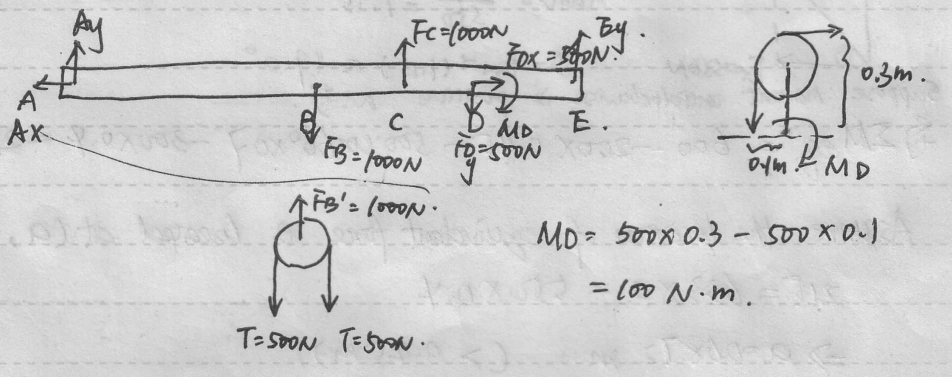


$\overset{+}{\circlearrowleft} \Sigma M_A = P \cdot 2L - F_C' \times L = 0.$

$\therefore P = \frac{1}{2} F_C'$

$P = 51.8\,N$ #

2. (a)



$M_D = 500 \times 0.3 - 500 \times 0.1$

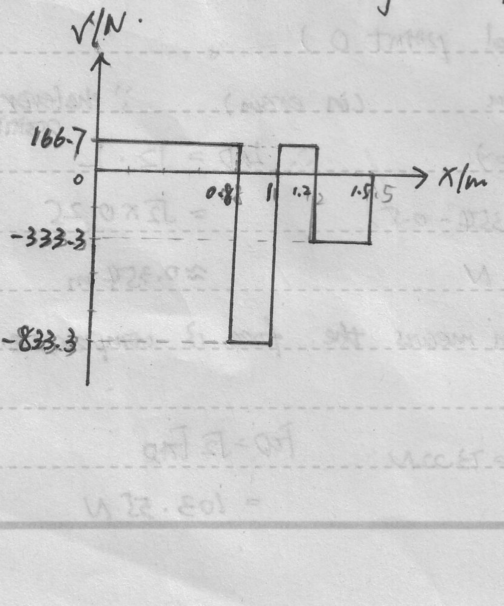
$= 100\ N \cdot m.$

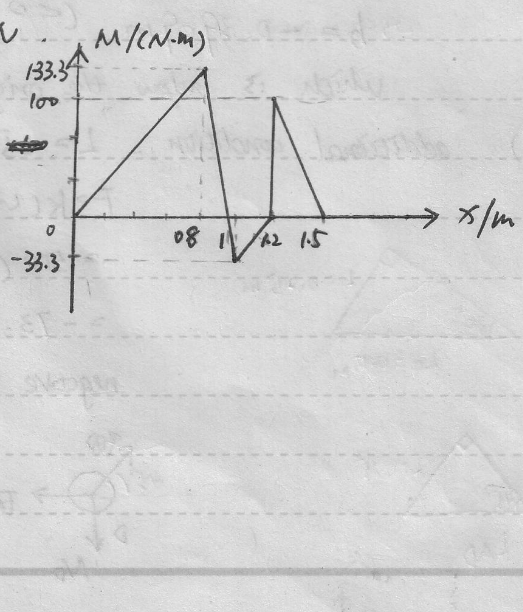
$\overset{+}{\circlearrowleft} \Sigma M_A = 1000 \times 0.8 - 1000 \times 1 + 100 + (-E_y) \times 1.5 = 0.$

$E_y = 333.3\,N$

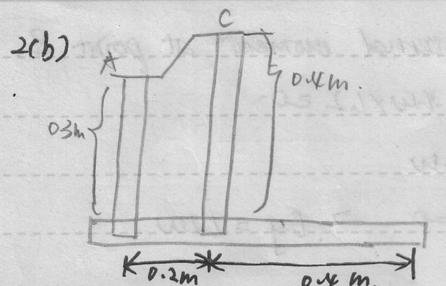
$+\uparrow \Sigma F_y = A_y + 1000 - 1000 - 500 - 333.3 = 0$

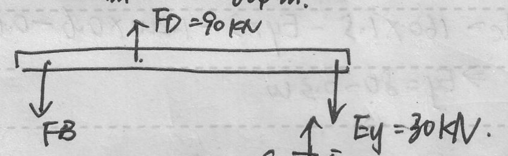
$A_y = 166.7\,N.$

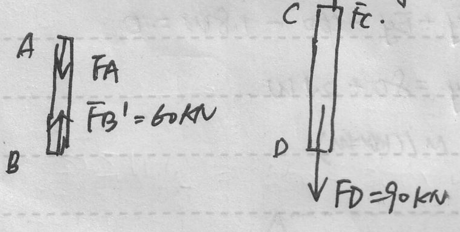


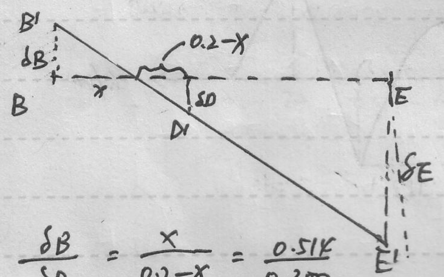


2(b)









$$\frac{\delta B}{\delta D} = \frac{x}{0.2-x} = \frac{0.514}{0.300}$$

$\rightarrow x = 0.1263$ mm

$\overset{+}{\curvearrowleft} \Sigma M_B = 30 \times 0.6 - F_D \times 0.2 = 0.$

$\Rightarrow F_D = 90$ kN.

$+\uparrow \Sigma F_y = 90 - F_B - 30 = 0$

$F_B = 60$ kN.

$A_{AB} = 500\ mm^2 \qquad A_{CD} = 600\ mm^2$

~~$E_{al} = 700$ GPa~~

$E_{al} = 70$ GPa $\qquad E_{st} = 200$ GPa.

$$\delta_{AB} = \frac{P_{AB} L}{E_{al} \cdot A_{AB}} = \frac{60 \times 10^3 \times 0.3}{70 \times 10^9 \times 500 \times 10^{-6}}$$

$= 5.14 \times 10^{-4}$ m

$= 0.514$ mm $(\uparrow)$

$$\delta_{CD} = \frac{P_{CD} L}{E_{st} A_{CD}} = \frac{90 \times 10^3 \times 0.4}{200 \times 10^9 \times 600 \times 10^{-6}}$$

$= 0.300$ mm $(\downarrow)$

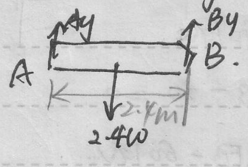
$$\frac{\delta B}{x} = \frac{\delta E}{0.6 - x}$$

$$\delta E = \frac{0.514 \times (0.6 - 0.1263)}{0.1263}$$

$\delta E = 1.928$ mm $(\downarrow)$ #

3.(a) B is a hinge, therefore there is no internal moment at point B.

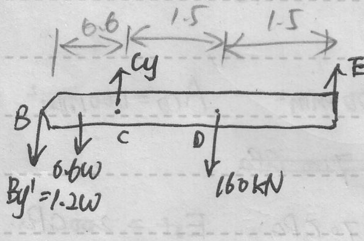
(i)



$+\circlearrowleft \Sigma M_B = A_y \times 2.4 - 2.4W \times 1.2 = 0$

$\Rightarrow A_y = 1.2W$

$+\uparrow \Sigma F_y = A_y + B_y - 2.4W = 0 \Rightarrow B_y = 1.2W.$

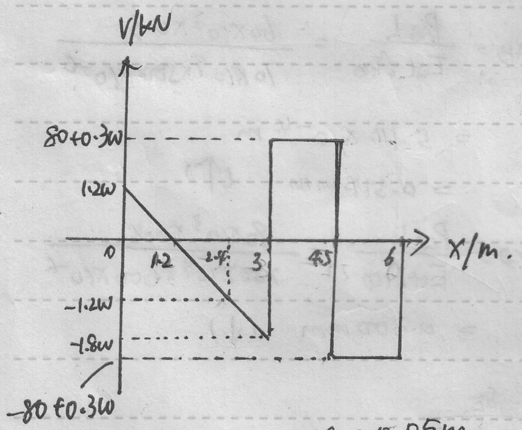


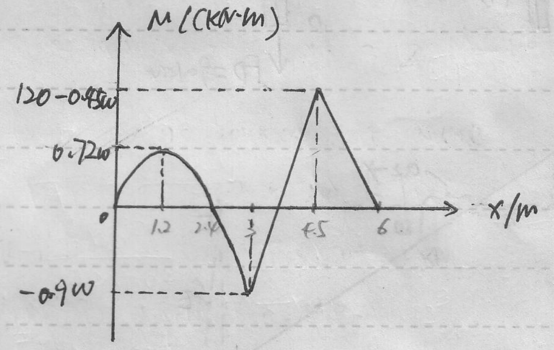
$+\circlearrowleft \Sigma M_C = 160 \times 1.5 - E_y \times 3 - 1.2W \times 0.6 - 0.6W \times 0.3 = 0$

$\Rightarrow E_y = 80 - 0.3W$

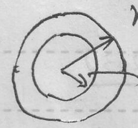
$+\uparrow \Sigma F_y = C_y + E_y - 160 - 1.8W = 0$

$\Rightarrow C_y = 80 + 2.1W.$





(ii)



① $80 + 0.3W \geq 1.8W$

$W \leq 53.33$ kN/m

$Q = \frac{1}{2}\pi r_2^2 \times \frac{4r_2}{3\pi} - \frac{1}{2}\pi r_1^2 \cdot \frac{4r_1}{3\pi}$

$= \frac{2}{3}(0.05^3 - 0.033^3)$

$= 5.937 \times 10^{-5}\ m^3$

$\tau = \frac{VQ}{It} = \frac{(80+0.3W) \times 5.937 \times 10^{-5}}{3.977 \times 10^{-6} \times (0.1 - 0.066)} \leq 60 \times 10^3$ kN/m²

$80 + 0.3W \leq 136.65$

$W \leq -144.5$ kN/m (X)

$I = \frac{1}{4}\pi(r_2^4 - r_1^4)$

$= \frac{\pi}{4}(0.05^4 - 0.033^4)$

$= 3.977 \times 10^{-6}\ m^4$

② $80 + 0.3W < 1.8W$　　$W > 53.33$

$\tau = \frac{VQ}{It} = \frac{1.8W \times 5.937 \times 10^{-5}}{3.977 \times 10^{-6} \times (0.034)} \leq 60 \times 10^3$ kN/m²

$1.8W \times 439.07 \leq 60 \times 10^3$

$W \leq 75.91$ kN/m. #

③ $120 - 0.45W \geq 0.9W$ $\quad W \leq 88.89$.

$\sigma = \frac{My}{I} = \frac{(120-0.45w)\times 0.05}{3.977\times10^{-6}} \leq 120\times10^{3}\ kN/m^2$

$w \geq 245.5$. (×)

④ $w > 88.89\ kN/m$ $\quad \sigma = \frac{My}{I} = \frac{0.9w \times 0.05}{3.977\times10^{-6}} \leq 120\times10^{3}\ kN/m^2$

$w \leq 10.6$ (×)

Therefore, the largest permissible value of w is 75.91 kN/m.

(b)
$$\begin{cases} \phi_{st} = \phi_{al} \\ T = T_{st} + T_{al} \\ \tau_{st} \leq 300\ MPa. \end{cases}$$

$\tau_{max_{st}} = \frac{T\rho}{J} = \frac{T_{st}\times 0.04}{2.749\times10^{-6}} \leq 300\times10^{-6}$.

$T_{st} = 20.6\ kN\cdot m$ ※

$J_{st} = \frac{\pi}{2}(0.04^4 - 0.03^4)$
$= 2.749\times10^{-6}\ m^4$

$J_{al} = \frac{\pi}{2}(0.03^4)$
$= 1.272\times10^{-6}\ m^4$

$\frac{\phi_{st}}{\phi_{al}} = \frac{\frac{T_{st}L}{G_{st}J_{st}}}{\frac{T_{al}L}{G_{al}J_{al}}} = \frac{T_{st}}{T_{al}} \cdot \frac{G_{al}J_{al}}{G_{st}J_{st}} = 1$

$\Rightarrow \frac{T_{al}}{T_{st}} = \frac{G_{al}}{G_{st}} \cdot \frac{J_{al}}{J_{st}}$

$= \frac{27\times10^{9}\times1.272\times10^{-6}}{77.2\times10^{9}\times2.749\times10^{-6}}$.

$= 0.1618$

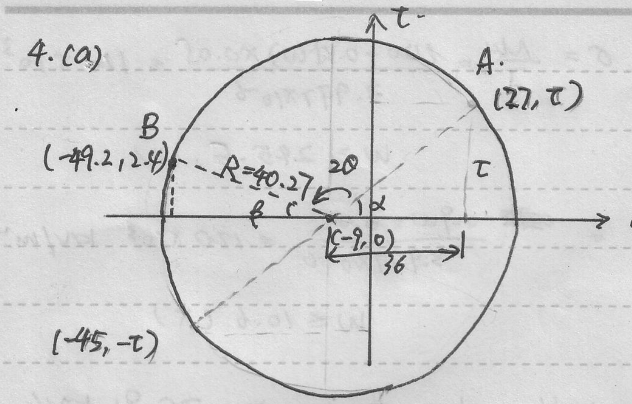
$T_{al} = 20.6\times0.1618 \approx 3.33\ kN\cdot m$ ※.

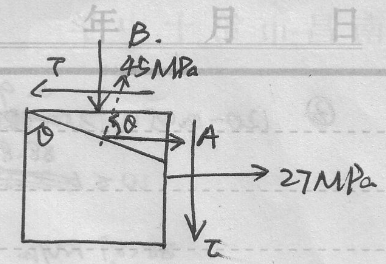
$T = T_{st} + T_{al}$
$= 20.6 + 3.33$
$= 23.93\ kN\cdot m$

All the best for your exam 😊　　ZHANG YULU
u1520564G

授课日期　年　月　日

4. (a)





$$\sigma_{ave} = \frac{27+(-45)}{2} = -9$$

$$R = \sqrt{[-49.2-(-9)]^2 + 2.4^2}$$

$$R = 40.27$$

$$R = \sqrt{36^2 + \tau^2} = 40.27$$

$$\Rightarrow \tau = 18.05 \text{ MPa} \ \#$$

$$\tan\beta = \frac{2.4}{49.2} \Rightarrow \beta = 2.8^\circ$$

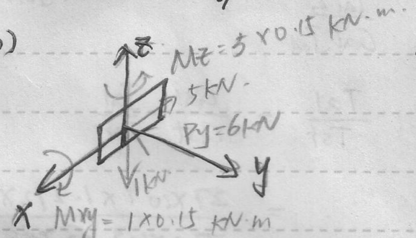
$$\tan\alpha = \frac{\tau}{36} = \frac{18.05}{36} \approx 0.5014$$

$$\tan^{-1}(0.5014) = 26.6^\circ$$

$$\theta = \frac{180 - 2.8 - 26.6}{2}$$

$$\Rightarrow \theta = 75.3^\circ \ \#$$

(b)

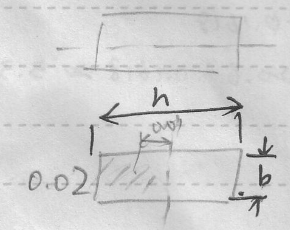


$$\sigma_x = \frac{P_y}{A} + \frac{M_x \cdot y}{I_x}$$

$$= -\frac{6\times10^3}{0.02\times0.04} + \frac{0.15\times10^3\times0.01}{\frac{1}{12}\times0.04\times0.02^3}$$

$$= -7.5 + 56.26 \text{ MPa}$$

$$= 48.75 \text{ MPa}.$$



$$\tau_{xy} = \frac{VQ}{It} = \frac{5\times10^3\times4\times10^{-6}}{1.067\times10^{-7}\times0.02} = 9.372 \text{ MPa}.$$

$$Q = \underbrace{0.01}_{y} \times \underbrace{0.02\times0.02}_{A} = 4\times10^{-6} \text{ m}^3$$

$$I = \frac{1}{12}\times\underbrace{0.02}_{b}\times\underbrace{0.04^3}_{h} = 1.067\times10^{-7} \text{ m}^4$$

maximum In plane shear stress:

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} = 26.12 \text{ MPa} \ \#$$

$$\sigma_{1,2} = \frac{\sigma_x + 0}{2} \pm \sqrt{\left(\frac{\sigma_x - 0}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = 24.375 + \sqrt{(24.375)^2 + 9.372^2}$$

maximum tensile stress, $\sigma_1 \approx 50.49$ MPa #

Maximum compressive stress, $\sigma_2 = 24.375 - 26.115 = -1.74$ MPa #.