CV1011 - Mechanics of Materials
2015/2016 Semester 1

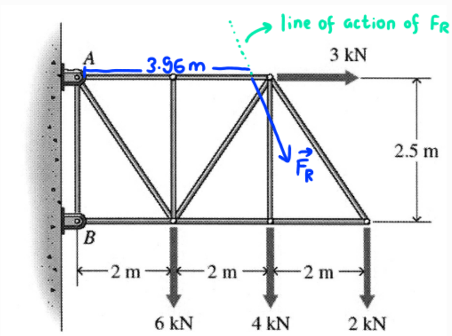
By Daniel Rahmatcipta

For comments / queries, please contact:

- 90553427 (WhatsApp) - danielrahmatcipta (Line)

$\boxed{1}$ A/ a. $\vec{F_R} = F_x \hat{x} + F_y \hat{y} = (3\hat{x} - 12\hat{y})$ kN

$\Rightarrow F_R = \sqrt{F_x^2 + F_y^2} \approx 12.369$ kN $\Rightarrow \theta = \tan^{-1}\left(\frac{-12}{3}\right) \approx -75.96°$



b. $\circlearrowright\!+\ \sum M_B = 6(2) + 4(4) + 2(6) + 3(2.5) = 47.5$ kNm

c. $\sum M_B = 47.5 = 12x \iff x \approx 3.96$ m

B/ a. $A = \int_0^a y\,dx = \frac{h}{3a^2} x^3 \Big|_0^a = \frac{ah}{3}$

b. $x_{cm} = \frac{x\int_A dA}{\int_A dA} = \frac{1}{A}\int_0^a \frac{h}{a^2} x^3\,dx = \frac{1}{A}\frac{h}{4a^2} x^4 \Big|_0^a = \frac{3}{ah}\frac{a^2 h}{4} = \frac{3}{4}a$

c. $y = \frac{h}{a^2}x^2 \iff x = \frac{a}{\sqrt{h}} y^{\frac{1}{2}}$ for $x, y \geq 0$

$y_{cm} = \frac{y\int_A dA}{\int_A dA} = \frac{1}{A}\int_0^a\int_0^y y\,dy\,dx = \frac{1}{2A}\int_0^a y^2\,dx = \frac{1}{2A}\int_0^a \frac{h^2}{a^4} x^4\,dx = \frac{3}{2\cancel{ah}}\frac{h^{\cancel{2}}}{5\cancel{a^4}}\cancel{a^5} = 0.3h$

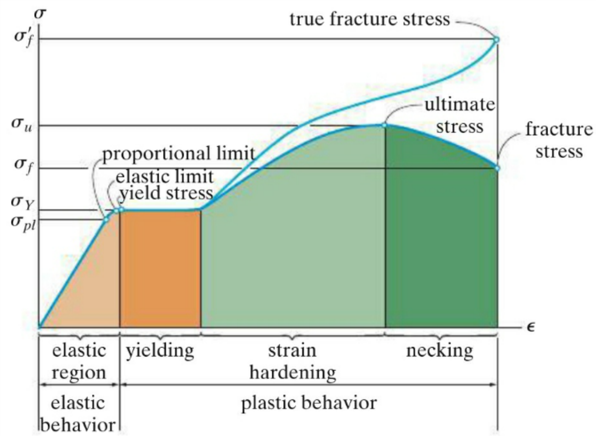
$\Rightarrow$ centroid is at $(x_{cm}, y_{cm}) = \left(\frac{3}{4}a, \frac{3}{10}h\right)$

d. $I_y = x^2\int_A dA = \int_0^a\int_0^y x^2\,dy\,dx = \int_0^a x^2 y\,dx = \frac{h}{a^2}\int_0^a x^4\,dx = \frac{h}{a^2}\frac{1}{5}a^5 = \frac{1}{5}ha^3$

Nb. using $I_y = \int_{I_y} dI = \int_0^h \frac{1}{3} x_i^3\,dy = \frac{1}{3}\int_0^h \left(a - \frac{a}{\sqrt{h}} y^{\frac{1}{2}}\right)^3 dy$

is more complicated mathematically albeit more comprehensive conceptually

2 A/

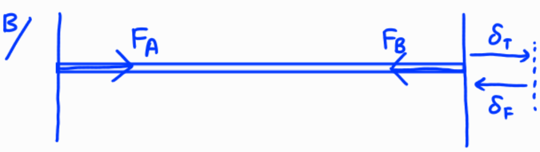


Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

a. Engineering / conventional $\sigma - \varepsilon$ diagram uses the original cross-sectional area and length before any load is applied

b. True $\sigma - \varepsilon$ diagram uses both data at an instant the load is measured

B/

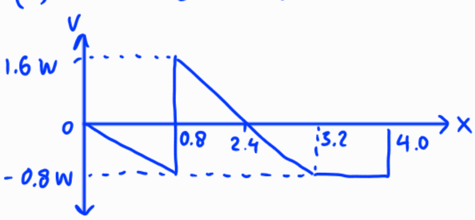


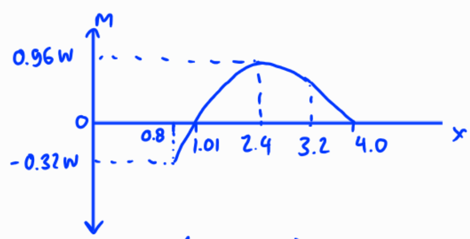
$\Sigma F_x = 0 \Leftrightarrow F_A = F_B$

(i) $0 = \delta_T - \delta_F \Leftrightarrow \alpha \Delta T L = \sigma \frac{L}{E} \Leftrightarrow \sigma = 119$ MPa

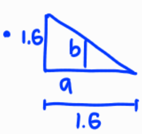
(ii) $P = F_A = F_B = \sigma A = 892.5$ kN

3 A/ (i) a. $\cancel{3.2} B = \cancel{3.2} w \, 2.4 \Leftrightarrow B = \frac{12}{5} w$ b. $\cancel{3.2} D = 0.8 \cdot \cancel{3.2} w \Leftrightarrow D = \frac{4}{5} w$





c.



$\frac{b}{1.6 - a} = 1 \Rightarrow 0.32 = \frac{1}{2} a (1.6 + b)$

$0.64 = a(3.2 - a) \rightarrow 3.2a - a^2 - 0.64 = 0$

$a = 2.986$ m (reject) $\vee$ $a = 0.214$ m

$\Rightarrow M = 0$ at $x = 0.8 + 0.214 \approx 1.01$ m

(ii) a. $\bar{y} = \frac{(0.07 \times 0.02 \times 0.06) + (0.03 \times 0.06 \times 0.02)}{(0.02 \times 0.06) + (0.02 \times 0.06)} = 0.05$ m

b. $I = \left(\frac{1}{12} 0.06 \times 0.02^3\right) + \left(\frac{1}{12} 0.02 \times 0.06^3\right) + \left(0.02^2 \times 0.06 \times 0.02\right) + \left(0.02^2 \times 0.02 \times 0.06\right)$

$I = 1.36 \times 10^{-6}$ m^4

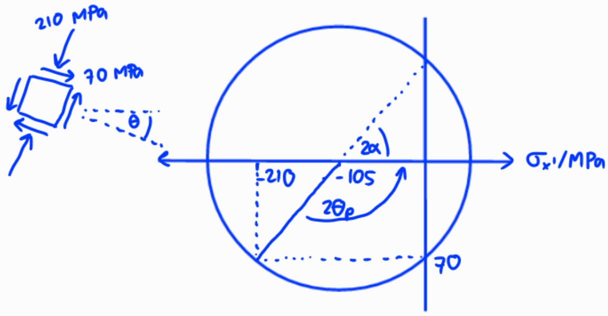
$\sigma_a = 160 \times 10^6 = \frac{0.96 w_1 \times 0.05}{I} \Leftrightarrow w_1 \approx 4.533$ kNm^{-1}

c. $\tau_a = 80 \times 10^6 = \frac{1.6 w_2 \, 0.025 \times 0.05 \cancel{\times 0.02}}{I \, \cancel{0.02}} \Leftrightarrow w_2 \approx 54.4$ kNm^{-1}

B/ a. $\phi = \frac{0.015}{0.3} = \frac{600 \cdot 0.3 \cdot 0.5}{\frac{\pi}{2} r_1^4 \, 77 \times 10^9} \Leftrightarrow r_1 \approx 11.045$ mm

b. $\tau_a = 80 \times 10^6 = \frac{Tr}{J} = \frac{600 \cdot 0.3 r_2}{\frac{\pi}{2} r_2^4} = \frac{360}{\pi r_2^3} \Leftrightarrow r_2 \approx 11.27$ mm

4 A/



Notice :

① $\theta = 0°$ indicates principal plane i.e. $\tau_{xy'} = 0$

② Plane b-b needs to rotate θ angle CCW to become principal plane

(i) a. $2\alpha = 180° - 2\theta_p \Leftrightarrow \tan 2\alpha = -\tan 2\theta_p \Leftrightarrow |\alpha| = |\theta_p|$

b. Magnitude $\theta_p = \alpha = \frac{1}{2}\tan^{-1}\left(\frac{140}{210}\right) \approx 16.8°$ //

Nb. solving without Mohr's circle gives the same answer : $\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{140}{210}$

(ii) $\sigma_{min} = -105 - \sqrt{(-105)^2 + 70^2} \approx -231.19$ MPa //

B/ $r = 0.4$ m

a. 25 kN : $\Rightarrow \sigma = -\frac{P}{A} = -\frac{25\times10^3}{\pi\, 0.4^2} = -49735.9$ Pa

b. 6 kN : $\Rightarrow \tau_T = \frac{6\times10^3 \times 2.5 \times 0.4}{\pi/2 \times 0.4^4} = 298415$ Pa

$\Rightarrow \tau_V = \frac{QV}{It} = \frac{4\cdot\cancel{0.4}}{\cancel{3\pi}}\,\frac{\cancel{\pi\, 0.4^2}}{\cancel{2}}\,\frac{\cancel{6}\times10^3}{\pi/4\ 0.4^4\cdot 0.8} = 15915$ Pa

$\Rightarrow \sigma_B = \frac{6\times10^3\times 2.5\times0.4}{\pi/4\ 0.4^4} = 298415$ Pa

c. H : $\Rightarrow \sigma = -\frac{P}{A} + \sigma_B = 248679$ Pa

$\Rightarrow \tau = \tau_T = 298415$ Pa

$\tau_{xy'} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = 323282$ Pa //

$\sigma_{1,2} = \frac{\sigma}{2} \pm \tau_{xy'} \begin{cases} 447622 \text{ Pa} \\ -198943 \text{ Pa} \end{cases}$ //

d. K : $\Rightarrow \sigma = -\frac{P}{A} = -49735.9$ Pa

$\Rightarrow \tau = \tau_T + \tau_V = 314330$ Pa

$\tau_{xy'} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = 315312$ Pa //

$\sigma_{1,2} = \frac{\sigma}{2} \pm \tau_{xy'} \begin{cases} 290444 \text{ Pa} \\ -340180 \text{ Pa} \end{cases}$ //

DanilRahmatipta