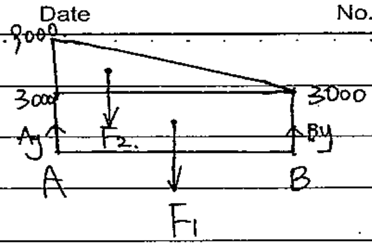
NANYANG TECHNOLOGICAL UNIVERSITY

12|13 S2

CV 1011 Solution

Date No.



1\. (a) $F_1 = 3000 \times 12 = 36000N$

$F_2 = (9000-3000) \times 12/2 = 36000N$

$\Sigma M_A = 0 \Rightarrow 12B_y - 36000 \times 12/2 - 36000 \times 12/3 = 0$

$\Rightarrow B_y = 30000\ N \uparrow.$

$\Sigma F_y = 0 \Rightarrow A_y + B_y = F_1 + F_2 \quad A_y + 30000 = 36000 \times 2$

$\Rightarrow A_y = 42000\ N \uparrow.$

$F = F_1 + F_2 = 72000N$

$\Sigma M_A = 0 \Rightarrow 12 \times B_y - F \cdot x = 0$

$\Rightarrow x = \frac{12 \times 30000}{72000} = 5m.$

the equivalent concentrated load is 72000N, act 5m from A.

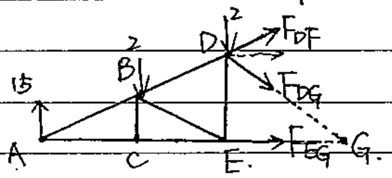
(b) $\Sigma M_A = 0 \quad L_y \cdot 60 - 2\times10 - 2\times20 - 12\times30 - 12\times40 - 12\times50 = 0.$

$\Rightarrow L_y = 25kN \uparrow$

$\Sigma F_y = 0 \quad A_y + L_y = 2\times5 + 10\times3$

$\Rightarrow A_y = 15\ kN \uparrow.$

Use the method of section cut DF, DG, EG.



$\Sigma M_G = 0$

$2\times10 + 2\times20 - 15\times30 - F_{DF} \cdot \frac{16/3}{\sqrt{(\frac{16}{3})^2 + 10^2}} \cdot 10 - F_{DF} \cdot \frac{10}{\sqrt{(\frac{16}{3})^2+10^2}} \cdot \frac{32}{3} = 0$

$\Rightarrow F_{DF} = -27.625\ kN\ (C)$

$\Sigma F_y = 0 \quad F_{DF} \cdot \frac{16/3}{\sqrt{(16/3)^2 + 100}} + 15 - 4 - F_{DG} \cdot \frac{32/3}{\sqrt{(32/3)^2 + 100}} = 0$

$\Rightarrow F_{DG} = -2.74\ kN\ (C)$

$\Sigma F_x = 0 \quad F_{DG} \cdot \frac{10}{\sqrt{(32/3)^2 + 100}} + F_{EG} = 0$

$\Rightarrow F_{EG} = 1.875\ kN\ (T)$

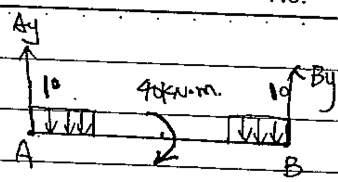
2. (a) $M = 20 \times 2 = 40\ \text{kN.m}$

$\Sigma M_A = 0 \Rightarrow -10\times2\times1 - 40 - 10\times2\times7 + B_y \cdot 8 = 0$

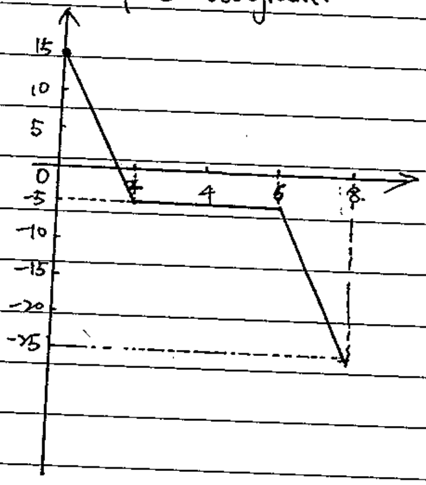
$\Rightarrow B_y = 25\ \text{kN}$

$\Sigma F_y = 0 \Rightarrow A_y + B_y = 10\times2\times2$

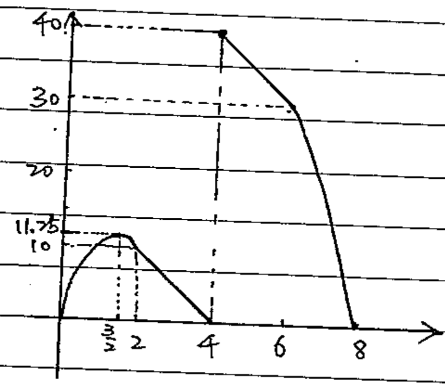
$\Rightarrow A_y = 15\ \text{kN}$



Shear force diagram:



Bending Moment diagram:



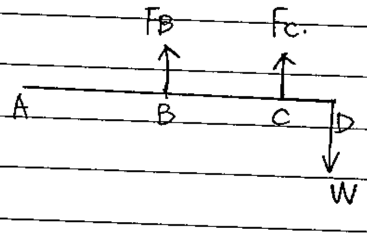
$V=0$ at $x = \frac{3}{2}$ m. where $M = 11.25\ \text{kN.m}$.

(b) $\Sigma M_A = 0 \quad F_B \cdot 2a + F_C \cdot 4a - W \cdot 5a = 0$

$2\delta_B = \delta_C$

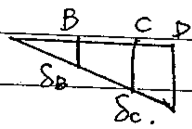
and $\delta_B = \dfrac{F_B \cdot L}{A_B \cdot E} + \alpha \cdot \Delta T \cdot L$

$= \dfrac{F_B \cdot L}{\frac{\pi}{4}(12\times10^{-3})^2 \cdot 140\times10^9} + 12\times10^{-6}\times 60 L.$



$\delta_C = \dfrac{F_C \cdot L}{A_C \cdot E} + \alpha \cdot \Delta T \cdot L$

$= \dfrac{F_C \cdot L}{\frac{\pi}{4}(20\times10^{-3})^2 \times 140\times10^9} + 12\times10^{-6}\times 60 L.$



$\Rightarrow \dfrac{2F_B \cancel{L}}{\frac{\pi}{4}(12\times10^{-3})^2 \cdot 140\times10^9} + \cancel{2}\times 12\times10^{-6}\times 60 \cancel{L} = \dfrac{F_C \cdot \cancel{L}}{\frac{\pi}{4}(20\times10^{-3})^2\times140\times10^9} + \cancel{12\times10^{-6}\times 60 L}$

$1.263\times10^{-7} F_B + 7.2\times10^{-4} = 2.274\times10^{-8} \cdot F_C.$

(Cont).

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$F_C = 5.554 F_B + 31662.3$

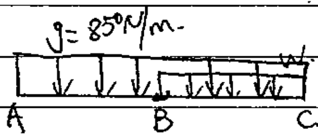
$\Rightarrow F_B \cdot 2\cancel{x} + (5.554 F_B + 31662.3) 4\cancel{x} = 5\cancel{x} W$

$24.216 F_B = 5W - 126649.2$

$\Rightarrow F_B = 0.2065 W - 5230$

$\Rightarrow F_C = 1.147 W + 2614.88$

3.(a) $\Sigma M_A = 0 \Rightarrow B_y \cdot l - 850 \times 2l \cdot l - Wl \cdot \frac{3}{2} l = 0$

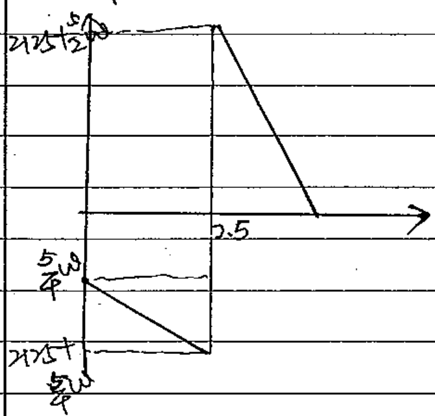


$\Rightarrow B_y = 4250 + \frac{15}{4} W \uparrow$

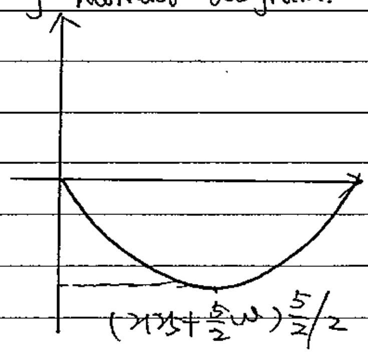
$\Sigma F_y = 0 \Rightarrow A_y + B_y = 850 \times 2 \times 2.5 + \frac{5}{2} W$

$\Rightarrow A_y = \frac{5}{4} W \downarrow$

shear force diagram.



Bending Moment diagram.



failure by Bending stress: $\sigma = \frac{M \cdot y}{I} \Rightarrow 7.5 \times 10^6 = \frac{(2125 + \frac{5}{2} W) \frac{5}{4} \cdot \frac{300 \times 10^{-3}}{2}}{\frac{\pi}{64} (300 \times 10^{-3})^4}$

$\Rightarrow W = 5511.73$ N·m

failure by shear stress: $\tau = \frac{VQ}{It} \Rightarrow 0.8 \times 10^6 = \frac{(2125 + \frac{5}{2} W) \frac{\pi}{8} (300 \times 10^{-3})^2 \cdot \frac{2 \times 300 \times 10^{-3}}{3\pi}}{\frac{\pi}{64} (300 \times 10^{-3})^4 \cdot 300 \times 10^{-3}}$

$\Rightarrow W = 16114.60$ N·m.

$\Rightarrow W_{max} = 5511.73$ N·m

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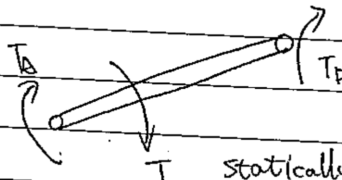
3. (b) $T_A + T_D = T$

$$\frac{T_A \cdot L_{AB}}{G \cdot J_{AB}} - \frac{T_D \cdot L_{CD}}{G \cdot J_{CD}} = \frac{1.5}{180} \cdot \pi$$



statically Indeterminate.

$$\Rightarrow \frac{T_A \cdot 600 \times 10^{-3}}{77 \times 10^9 \times \frac{\pi}{2}\left(\frac{30}{2} \times 10^{-3}\right)^4} - \frac{T_D \cdot 900 \times 10^{-3}}{77 \times 10^9 \times \frac{\pi}{2}\left(\frac{36}{2} \times 10^{-3}\right)^4} = \frac{1.5}{180} \pi$$

$$9.8 \times 10^{-5} T_A - 7.09 \times 10^{-5} T_D = \frac{\pi}{120}$$

$$T_A = 267.14 + 0.723 T_D$$

$$267.14 + 0.723 T_D + T_D = 300 \Rightarrow T_D = 19.07 \, N \cdot m$$

$$\Rightarrow T_A = 280.93 \, N \cdot m$$

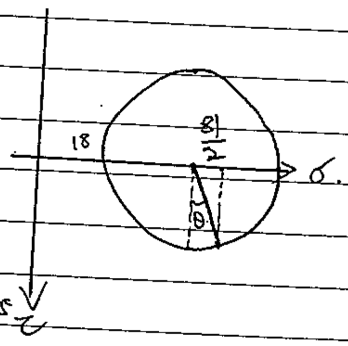
$$\tau_{AB} = \frac{T_A \cdot \rho}{J} = \frac{280.93 \times 15 \times 10^{-3}}{\frac{\pi}{2}(15 \times 10^{-3})^4} = 53 \, MPa.$$

$$\tau_{CD} = \frac{T_D \cdot \rho}{J} = \frac{19.07 \times 18 \times 10^{-3}}{\frac{\pi}{2}(18 \times 10^{-3})^4} = 2.08 \, MPa.$$

4. (a) $\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{49+32}{2} = \frac{81}{2} \, MPa$

$\sigma_{min} = \sigma_{avg} - R = 18 \, MPa.$

$\sigma_{max} = \sigma_{avg} + R = \sigma_{avg} + \sigma_{avg} - 18 = 63 \, MPa$



$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{81}{2} - 18$$

$\Rightarrow \tau_{xy} = 20.833 \, MPa.$: Max in-plane shear stress

$$\tan\theta = \frac{49 - \frac{81}{2}}{20.833} \Rightarrow \theta = 22.2^\circ$$

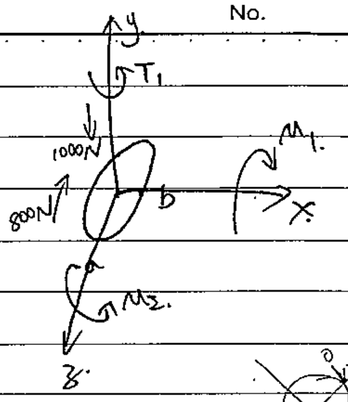
$\varphi = \frac{\theta}{2} = -11.1^\circ$ (negative as it is clockwise)

4.(b) $P = 1000\,N \qquad V = 800\,N$

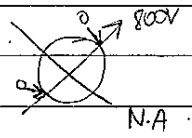
$T_1 = 90\times10^{-3}\times 800 = 72\,N\cdot m.$

$M_1 = 75\times10^{-3}\times 800 = 60\,N\cdot m.$

$M_2 = 45\times10^{-3}\times 1000 = 45\,N\cdot m.$



$$\sigma_1 = \frac{P}{A} = \frac{1000}{\frac{\pi}{4}\left((42\times10^{-3})^2-(35\times10^{-3})^2\right)} = 2.36\,MPa$$ (compress).



At a: σ of $M_2 = 0$. τ of $V = 0$.

$$\sigma_a = \frac{M_1\cdot y}{I} = \frac{60\times 42\times10^{-3}/2}{\frac{\pi}{4}\left((21\times10^{-3})^4-(\frac{35}{2}\times10^{-3})^4\right)} = 7.97\,MPa \text{ (Tensile)}$$

$$\tau_1 = \frac{T_1\cdot \rho}{J} = \frac{72\times 42\times10^{-3}/2}{\frac{\pi}{2}\left((21\times10^{-3})^4-(\frac{35}{2}\times10^{-3})^4\right)} = 9.56\,MPa$$

Max: $$\tau = \sqrt{\left(\frac{\sigma}{2}\right)^2+\tau_1^2} = \sqrt{\left(\frac{7.97-2.36}{2}\right)^2+9.56^2} = 9.96\,MPa.$$

principle stress: $$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \tau_{max} = \frac{-7.97+2.36}{2} \pm 9.96 = -2.805 \pm 9.96\,MPa$$

Max tensile stress: 12.765 MPa. Max compressive stress: 7.875 MPa.

At b: $$\sigma_b = \frac{M_2\cdot y}{I} = \frac{45\times 21\times10^{-3}}{\frac{\pi}{4}\left((21\times10^{-3})^4-(\frac{35}{2}\times10^{-3})^4\right)} = 5.98\,MPa \text{ (compressive)}$$

shear stress of shear force:

$$\tau_1 = \frac{VQ}{It} = \frac{800\times\frac{\pi}{2}\left((21\times10^{-3})^2-(\frac{35}{2}\times10^{-3})^2\right)\cdot\left(\frac{4\times(21\times10^{-3}-\frac{35}{2}\times10^{-3})}{3\pi}\right)}{\frac{\pi}{4}\left((21\times10^{-3})^4-(\frac{35}{2}\times10^{-3})^4\right)\cdot 42\times10^{-3}}$$

$$= 0.037864\,MPa.$$

$$\tau_2 = \frac{T_1\cdot \rho}{J} = \frac{72\times 42\times10^{-3}/2}{\frac{\pi}{2}\left((21\times10^{-3})^4-(\frac{35}{2}\times10^{-3})^4\right)} = 9.56\,MPa.$$

$$\tau_{total} = 9.56-0.037864 = 9.52\,MPa$$

Max: $$\tau = \sqrt{\left(\frac{\sigma}{2}\right)^2+\tau_{total}^2} = \sqrt{\left(\frac{2.36+5.98}{2}\right)^2+9.52^2} = 10.39\,MPa.$$

principle stress: $$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \tau_{max} = \frac{2.36+5.98}{2} \pm 10.39 = 4.17 \pm 10.39\,MPa.$$

Max tensile stress: 6.22 MPa Max compressive stress: 14.56 MPa.