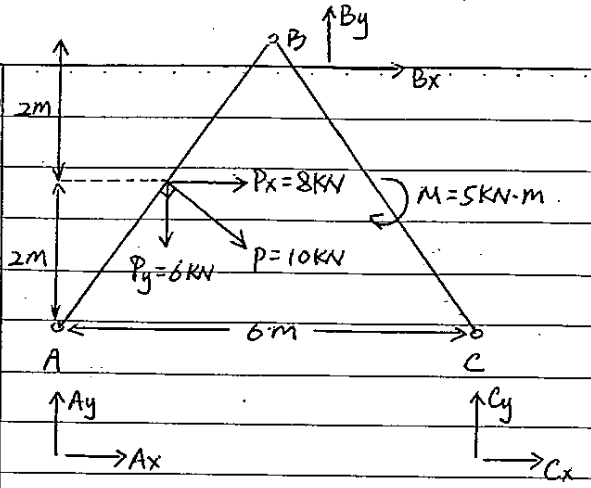
NANYANG TECHNOLOGICAL UNIVERSITY

CV1011 SEM 1 EXAM 2012-2013.

Date No.

Q1.(a)



(1) Consider the whole frame

$\circlearrowleft + M_A = 0$

$\Rightarrow 6C_y - 5 - 10\times 2.5 = 0 \Rightarrow C_y = 5\,KN$ // (↑)

$\circlearrowleft + M_C = 0$

$\Rightarrow -6A_y - 5 - 8\times 2 + 4.5\times 6 = 0 \Rightarrow A_y = 1\,KN$ // (↑)

(2) Consider member BC

$\circlearrowleft + M_B = 0$

$\Rightarrow -5 + 3\times 5 + 4C_x = 0 \Rightarrow C_x = -2.5\,KN$ // (←)

$\Sigma F_y = 0 \Rightarrow B_y = 5\,KN$ // (↓)

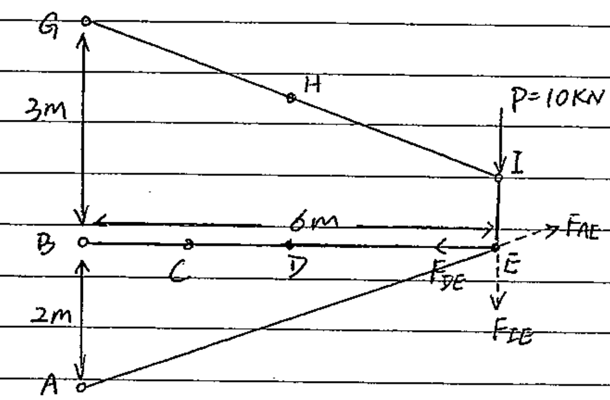
$\Sigma F_x = 0 \Rightarrow B_x = 2.5\,KN$ // (→)

(3) Consider the whole frame again

$\Sigma F_x = 0 \Rightarrow A_x + 8 - 2.5 = 0$

$\Rightarrow A_x = -5.5\,KN$ // (←)

Q1.(b) Zero-force Members: FI, EF, HD, CH, AC.



(1) Consider joint I.

$\Sigma F_x = 0 \Rightarrow F_{HI} = 0\,KN$ //

$\Sigma F_y = 0 \Rightarrow F_{EI} = 10\,KN\,(C)$ //

(2) Consider joint H.

$F_{GH} = F_{HI} = 0\,KN$ //.

(3) Consider joint E.

$\Sigma F_y = 0 \Rightarrow F_{AE}\times\frac{1}{\sqrt{10}} = 10$

$\Rightarrow F_{AE} = 31.6\,KN\,(C)$ //

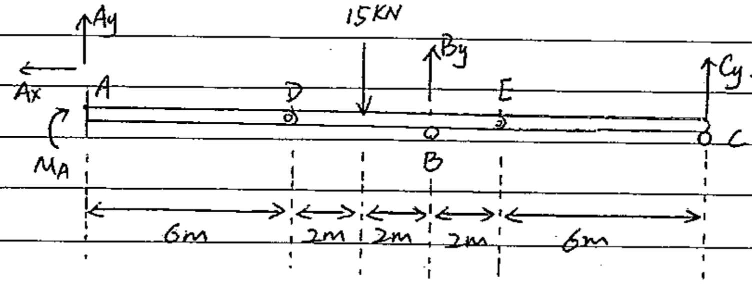
$\Sigma F_x = 0 \Rightarrow F_{DE} = F_{AE}\times\frac{1}{\sqrt{10}}\times 3$

$= 30\,KN\,(T)$ //

(4) Consider joint D: $\Sigma F_x = 0 \Rightarrow F_{CD} = F_{DE} = 30\,KN\,(T)$ //

(5) Consider joint C: $\Sigma F_x = 0 \Rightarrow F_{BC} = F_{CD} = 30\,KN\,(T)$ //

Q2(a).



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(1) Consider member EC

↺+ $M_E=0 \Rightarrow 6C_y=0 \Rightarrow C_y=0$ //

$\Sigma F_y=0 \Rightarrow E_y=0$ //

$\Sigma F_x=0 \Rightarrow E_x=0$ //

(2) Consider member DE.

↺+ $M_D=0 \Rightarrow 4B_y-15\times 2=0 \Rightarrow B_y=7.5\,kN$ //

(3) Consider the whole beam.

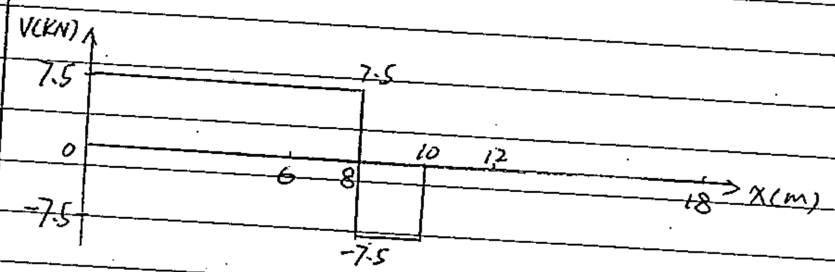
$\Sigma F_x=0 \Rightarrow A_x=0$ //

$\Sigma F_y=0 \Rightarrow A_y+7.5-15=0 \Rightarrow A_y=7.5\,kN$ //

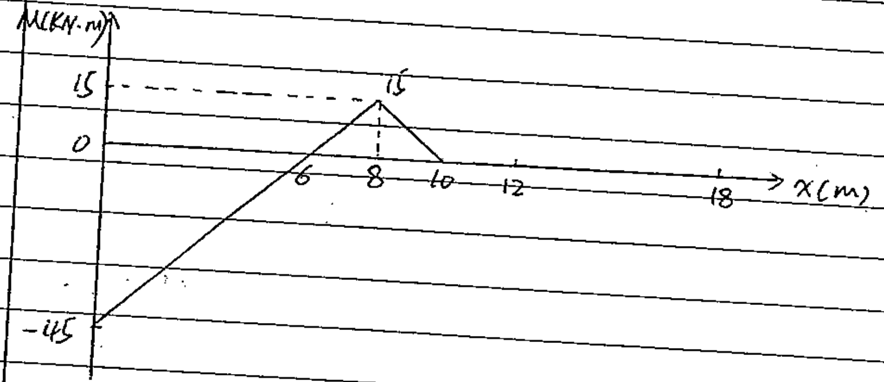
↺+ $M_A=0 \Rightarrow (6+4)\times 7.5-15\times 8+M_A=0$

$\Rightarrow M_A=45.0\,kN\cdot m$ (anti-clockwise)

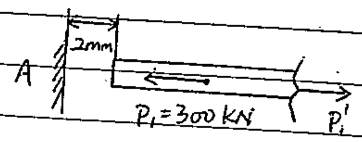
Shear force diagram.



Bending moment diagram.



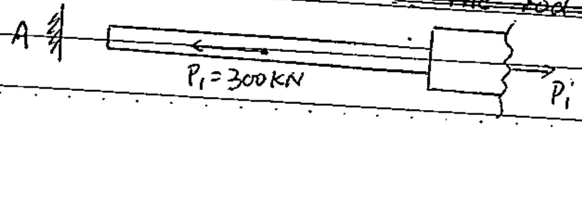
Q2.(b)(i) First check whether the rod system will touch wall at A after subjected to three axial forces.



Axial force exerted in centre

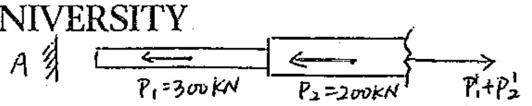
$\delta_1 = \frac{P_1' L}{(EA)_{st}} = \frac{300\,kN \times 300\,mm \times \frac{1}{2}}{200\,GPa \times 200\,mm^2} = 1.125\,mm$ // $= \cancel{2.25\,mm > 2\,mm}$

~~∴ The rod system will touch wall at A.~~



$\delta_2 = \frac{P_1' L}{(EA)_{br}} = \frac{300\,kN \times 100\,mm}{100\,GPa \times 400\,mm^2} = 0.75\,mm$ //

Why δ_2 use P_1?



$\delta_3 = \frac{(P_1' + P_2')L}{(EA)_{br}} = \frac{500KN \times 100mm}{100GPa \times 400mm^2} = 1.25mm$ //

$\because \delta_1 + \delta_2 + \delta_3 = 1.125 + 0.75 + 1.25 = 3.125mm > 2mm.$

$\therefore$ The rod system will touch wall at B.

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(2) $\Sigma F_x = 0 \Rightarrow F_A + F_B - 300 - 200 - 100 = 0 \Rightarrow F_A + F_B - 600 = 0$

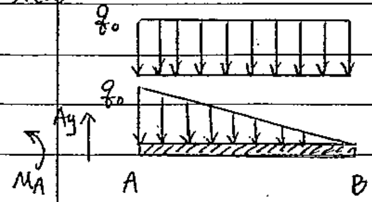
Compatibility:

$$\delta_{B/A} = 2mm = -\frac{F_A \times 150}{E_{st}A_{st}} - \frac{(F_A - 300)\times 150}{E_{st}A_{st}} - \frac{(F_A - 300)\times 100}{E_{br}A_{br}} - \frac{(F_A - 500)\times 100}{E_{br}A_{br}} - \frac{(F_A - 500)\times 60}{E_{cu}A_{cu}} - \frac{(F_A - 600)\times 60}{E_{cu}A_{cu}}$$

$\Rightarrow F_A = 153 KN$ // (towards right)

$\Rightarrow F_B = 600 - F_A = 600 - 153 = 447 KN$ // (towards right).

Q3.(a)



(1) $\Sigma F_y = 0 \Rightarrow A_y = q_0 L + \frac{q_0 L}{2} = \frac{3}{2} q_0 L$ //

$\circlearrowleft + M_A = 0 \Rightarrow M_A - q_0 \cdot L \cdot \frac{L}{2} - q_0 \cdot L \cdot \frac{1}{2} \cdot \frac{1}{3} L = 0 \Rightarrow M_A = \frac{2}{3} q_0 L^2$ //

$\therefore V_{max} = \frac{3}{2} q_0 L$ // $= 15 q_0$ KN·m

$M_{max} = \frac{2}{3} q_0 L^2$ // $= 66.67 q_0$ KN·m

(2) From the cross-section of the beam.

$I = 2[\frac{1}{12}(250)(25)^3 + (250)(25)(312.5)^2] + \frac{1}{12}(15)(600)^3 = 1.491 \times 10^9 mm^4$ //

(3) $\sigma_{allow} = 90 MPa$

$\sigma_{allow} = My/I \Rightarrow 90 = 66.67 q_0 \times 10^6 \times 325 / 1.491 \times 10^9 \Rightarrow q_0 = 6.193 KN$

(4) $\tau_{allow} = 50 MPa.$

$\tau_{allow} = VQ/It$

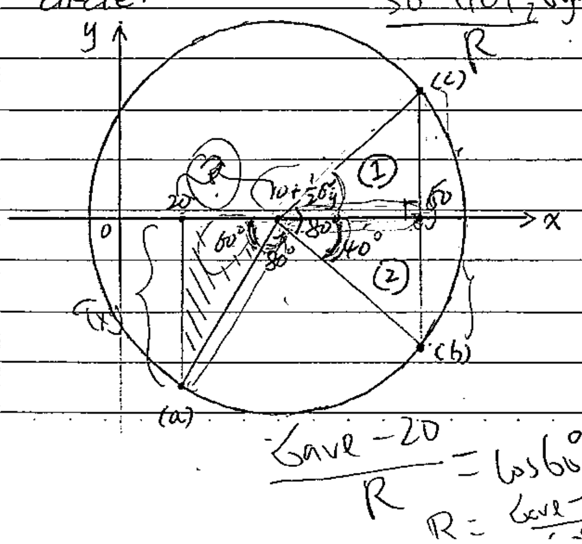
$Q = 312.5 \times 250 \times 25 + 300 \times \frac{1}{2} \times 15 \times 300 = 2.628 \times 10^6 mm^3$

$\Rightarrow 50 = (15 q_0 \times 10^6 / 1.491 \times 10^9 \times 2.628 \times 10^6 / 15) \Rightarrow q_0 = 0.0284 KN = 28.4 N$ //

Choose the smaller q_0, thus $q_0 = 28.4 N$.

Q3.(b) Using Mohr's circle, cases (a), (b), (c) can be determined as points on the circle.

$\frac{50 - (10 + \frac{1}{2}\sigma_y)}{R} = \cos 40^\circ \Rightarrow R = \frac{50 - (10 + \frac{1}{2}\sigma_y)}{\cos 40}$



(1) center of the circle: $(10 + \frac{1}{2}\sigma_y, 0)$

(2) consider the shaded triangle.

How to know $\angle 60^\circ$

$2(10 + \frac{1}{2}\sigma_y - 20) = R \Rightarrow \sigma_y - 20 = R.$

$\Rightarrow (\sigma_y - 20)^2 = R^2 = (10 - \frac{1}{2}\sigma_y)^2 + \tau_{xy}^2$ ①

$\tau_{xy} = \sqrt{3}(10 + \frac{1}{2}\sigma_y - 20)$ ②

$\sin 60 = \frac{\tau_{xy}}{R}$

~~From ① & ② $\Rightarrow (\sigma_y - 20)^2 = (10 - \frac{1}{2}\sigma_y)^2 + 3(\frac{1}{2}\sigma_y - 10)^2$~~

~~$\Rightarrow \sigma_y =$~~

(3) consider triangle ① $\Rightarrow \frac{R}{50 - (10 + \frac{1}{2}\sigma_y)} = \cos^{-1} 40^\circ$

$\frac{\sigma_{ave} - 20}{R} = \cos 60^\circ \Rightarrow \frac{\sigma_y - 20}{40 - \frac{1}{2}\sigma_y} = \cos^{-1} 40^\circ \Rightarrow \sigma_y = 43.7$ MPa //

$R = \frac{\sigma_{ave} - 20}{\cos 60}$

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$\therefore \tau_{xy} = \sqrt{3}\,(10 + \frac{1}{2}\sigma_y - 20) = \sqrt{3}\,(10 + \frac{1}{2} \times \overset{43.7}{\cancel{36.6}} - 20) = \overset{20.5}{\cancel{\cancel{14.4}}}$ MPa // (Below x axis, should be negative)

(4) consider triangle ~~①~~ ②. τ_{xy} at (b) $= (40 - \frac{1}{2} \times \underset{43.7}{\cancel{36.6}}) \times \underset{\tan}{\cancel{\sin}} 40° = \underset{15.2}{\cancel{7.4}}$ MPa // (should be negative as well)

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$\therefore \tau_{xy}$ at (c) $= 15.2$ MPa // (positive).

Q4. (a) (1) Magnitude of wind force : $2.0\,kPa \times 2 \times 0.75 = 3\,KN$ //

Direction of the wind force : acting perpendicular to the sign and on the centroid of the sign

(2) At point A : $V = 3000\,N$, $M = 3000 \times (3.2 + 0.75 \times \frac{1}{2}) = 10725\,N \cdot m$

$T = 3000 \times 1 = 3000\,N \cdot m$.

Shear force V : $\tau_A = \frac{VQ}{It} = 0$

Bending moment M : $\sigma_A = \frac{My}{I} = \frac{10725 \times 50 \times 10^{-3}}{\frac{1}{4}\pi(50^4 - 40^4) \times (10^{-3})^4} = \cancel{68.3}\ 185$ MPa

Torsional moment T : $\tau_A = Tc/J$

$= 3000 \times 50 \times 10^{-3} / \frac{1}{2}\pi(50^4 - 40^4) \times (10^{-3})^4$

$= 25.9$ MPa

$\Rightarrow$ Maximum In-Plane Shear Stress :

$\tau_{max} = \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2} = \sqrt{(\frac{185}{2})^2 + 25.9^2} = 96.1$ MPa //

$\Rightarrow \sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2} = 185 \div 2 \pm 96.1 = 188.6$ MPa / -3.6 MPa //

(3) At point B ~~在~~ : $V = 3000\,N$, $M = 10725\,N \cdot m$ $T = 3000\,N \cdot m$.

Shear force V : $\tau_B = \frac{VQ}{It} = \frac{3000 \times [\pi \times (50 \times 10^{-3})^2 \times \frac{4 \times (50 \times 10^{-3})}{3\pi} - \pi \times (40 \times 10^{-3})^3 \times \frac{4}{3\pi}]}{\frac{1}{4}\pi(50^4 - 40^4) \times (10^{-3})^4 \times (100 - 80) \times 10^{-3}}$

$= 4.21$ MPa

Bending Moment M : $\sigma_B = \frac{My}{I} = 0$.

Torsional moment T : $\tau_B = 25.9$ MPa (same as τ_A)

$\Rightarrow \tau_{max} = \sqrt{(25.9 + 4.21)^2} = 30.11$ MPa //

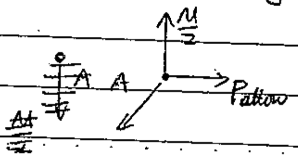
$\Rightarrow \sigma_{1,2} = \pm 30.11$ MPa //

(4) At point C : if at point B. shear force $V \Rightarrow \tau_B$ and torsional moment $T \Rightarrow \tau_B$ are in the same direction, then at point C, ~~there~~ they should in the opposite direction, but in same magnitude

$\therefore \Rightarrow \tau_{max} = 25.9 - 4.21 = 21.69$ MPa //

$\Rightarrow \sigma_{1,2} = \pm 21.69$ MPa //

Q4. (b) Assume the weight of the pipe is M. consider joint A (or B)



$P_{allow} \frac{M}{14} \times 10 = \frac{5}{7} M$.

~~$P_{allow} = P_{cr} / F.S = \frac{5}{7} M \times \frac{1}{2} = \frac{5}{14} M$~~

$\Rightarrow P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \times 200 \times 10^9 \times \frac{1}{4}\pi \times [(35 \times 10^{-3})^4 - (28.5 \times 10^{-3})^4]}{2.6^2}$

$= 193\,KN \Rightarrow F.S = P_{cr} / P_{allow} = 2 \Rightarrow 193 = \frac{10}{7} M \Rightarrow M = 135.1\,KN$