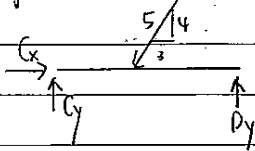


Yes, U can!

11-12 Sem 2 CV1011 Mechanics of Material

1 (a) Segment CD:  $P=20\text{kN}$



$$\sum F_x = 0$$

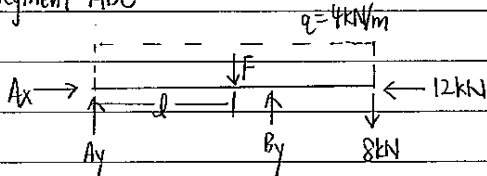
$$C_x - 20\left(\frac{3}{5}\right) = 0$$

$$C_x = 12\text{ kN}$$

Due to symmetry,  $C_y = D_y = \frac{1}{2} \times 20 \times \left(\frac{4}{5}\right)$

$$C_y = D_y = 8\text{ kN}$$

Segment ABC:



$$F = 4 \times 6 = 24\text{ kN}$$

$$l = \frac{1}{2} \times 6 = 3\text{ m}$$

$$\sum F_x = 0$$

$$A_x = 12\text{ kN}$$

$$\sum M_A = 0$$

$$-24 \times 3 + B_y \cdot 4 - 8 \times 6 = 0$$

$$B_y = 30\text{ kN}$$

$$\sum F_y = 0$$

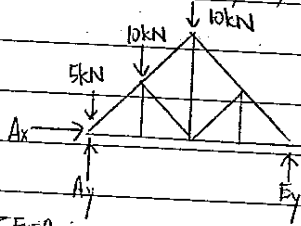
$$A_y - 24 + 30 - 8 = 0$$

$$A_y = 2\text{ kN}$$



Yes, U can!

1. (b) Zero force member: BH, DJ, CJ



$$\sum F_x = 0$$

$$A_x = 0$$

$$\sum M_A = 0$$

$$-10 \times 3 - 10 \times 6 + E_y \cdot 12 = 0$$

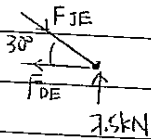
$$E_y = 7.5 \text{ kN}$$

$$\sum F_y = 0$$

$$A_y + 7.5 - 5 - 10 - 10 = 0$$

$$A_y = 17.5 \text{ kN}$$

Joint E:



$$\sum F_y = 0$$

$$F_{JE} \sin 30^\circ - 7.5 = 0$$

$$F_{JE} = 15 \text{ kN (C)}$$

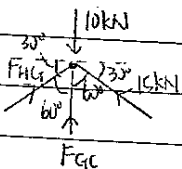
$$\therefore F_{GJ} = 15 \text{ kN (C)}$$

$$\sum F_x = 0$$

$$F_{DE} - 15 \cos 30^\circ = 0$$

$$F_{DE} = 13 \text{ kN (T)}$$

Joint G:



$$\sum F_x = 0$$

$$F_{HG} \cos 30^\circ - 15 \cos 30^\circ = 0$$

$$F_{HG} = 15 \text{ kN (C)}$$

$$\sum F_y = 0$$

$$15 \times \sin 30^\circ + 15 \times \sin 30^\circ - 10 + F_{GC} = 0$$

$$F_{GC} = -5$$

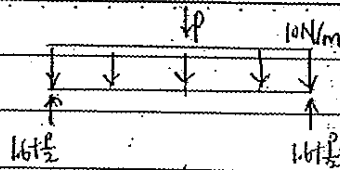
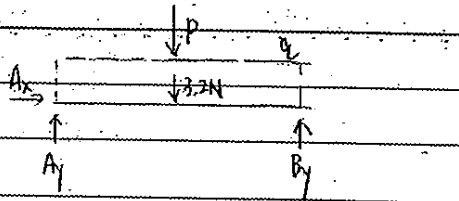
$$F_{GC} = 5 \text{ kN (T)}$$

Date

No.

3

(a)



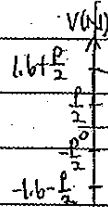
$\sum F_x = 0$

$A_x = 0$

Due to symmetry

$A_y = B_y = \frac{3.2P}{2} = 1.6 \frac{P}{2}$

$Q = \frac{3.2}{0.32} = 10 \text{ N/m}$



For  $0 \leq x < 0.16 \text{m}$ ,

$\Delta V = \int -w dx = -10 \times 0.16$

$\Delta V = -1.6 \text{ N}$

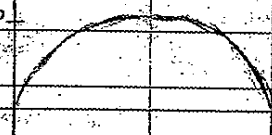
For  $0.16 < x \leq 0.32 \text{m}$ ,

$\Delta V = \int -w dx = -10 \times 0.16$

$\Delta V = -1.6 \text{ N}$

$M(\text{Nm})$

$0.128 + 0.16P$



For  $0 \leq x < 0.16 \text{m}$ ,

$\Delta M = \int V dx = \frac{1}{2} \times (1.6P) \times 0.16$

$\Delta M = 0.128 + 0.16P$

For  $0.16 < x \leq 0.32 \text{m}$ ,

$\Delta M = \int V dx = \frac{1}{2} \times (-1.6P) \times 0.16$

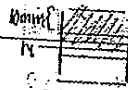
$\Delta M = -0.128 - 0.16P$

$V_{\text{max}} = 1.6 \frac{P}{2}$

$M_{\text{max}} = 0.128 + 0.16P$

Date

No.



Neutral axis (measured from top)

$$\bar{y} = 15 \text{ mm}$$

$$I = \left[ \frac{1}{12} \times (30) \times (10)^3 + (10)(30)(15-5)^2 \right] + \left[ \frac{1}{12} (30)(10)^3 + (10)(30)(25-15)^2 \right]$$

$$I = 67500 \text{ mm}^4$$

As maximum bending stress occurs the farthest from neutral axis

$$c = \bar{y} = 15 \text{ mm}$$

allowable bending stress  $\sigma_b = 8 = \frac{M_{\max} c}{I}$

$$8 \times 10^6 = \frac{P \times 67500 \times 10^{-12} \times \frac{1}{15 \times 10^{-3}}}{1}$$

$$P = 224.2 \text{ (N)}$$

∴ Maximum transverse shear stress occurs at neutral axis

$$t = 30 \text{ mm}$$

$$Q = \bar{y} A' \bar{y}' = (15 \times 30) \times (15 - 7.5)$$

$$Q = 3375 \text{ (mm}^3\text{)}$$

$$\tau = \frac{V_{\max} Q}{I t}$$

$$0.3 \times 10^6 = \frac{(1.67 \times 10^4) (3375 \times 10^{-9})}{67500 \times 10^{-4} \times 30 \times 10^{-3}}$$

$$P = 356.8 \text{ (N)}$$

∴ maximum permissible load  $P = 224.2 \text{ (N)}$

$$2) b) L_{AB} = L$$

$$D_{AB} = d_1$$

$$T = T_0$$

$$J_{AB} = \frac{1}{2} \pi d_1^4$$

$$i) \phi = \frac{TL}{GJ}$$

$$= \frac{T_0 \times L}{G \times \frac{1}{2} \pi d_1^4}$$

$$= \frac{2 T_0 L}{G \pi d_1^4}$$

ii) When  $T_0$  are removed, the sleeve will stop twisting when stress on the sleeve is equal to the stress on the flange.

$$J_1 = \frac{1}{2} \pi d_2^4, J_2 = \frac{1}{2} \pi (d_3^4 - d_2^4)$$

$$\tau_1 = \tau_2$$

$$\frac{T_1 d_2}{\frac{1}{2} \pi d_2^4} = \frac{T_2 d_2}{\frac{1}{2} \pi (d_3^4 - d_2^4)}$$

$$T_1 (d_3^4 - d_2^4) = T_2 d_2^4$$

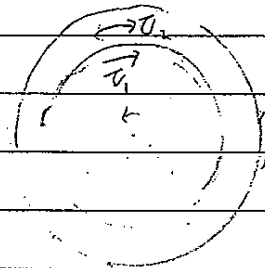
$$T_1 = \frac{T_2 d_2^4}{d_3^4 - d_2^4}$$

$$\therefore T_1 + T_2 = T_0$$

$$\frac{d_2^4}{d_3^4 - d_2^4} T_2 + T_2 = T_0$$

$$\frac{d_2^4}{d_3^4 - d_2^4} T_2 = T_0 - T_2$$

$$T_2 = \frac{d_3^4 - d_2^4}{d_3^4} T_0$$



$\therefore$  Angle of twist of the sleeve =  $\frac{T_2 L}{G J_2}$

$$= \frac{T_2 L}{G \frac{1}{2} \pi (d_3^4 - d_2^4)}$$

$$= \frac{2 T_2 L}{G \pi (d_3^4 - d_2^4)}$$

$$= \frac{2 T_0 L}{G \pi d_3^4}$$

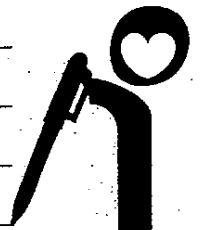
$$= \frac{2 T_0 L}{G \pi d_3^4}$$

Maximum shear stress =  $\tau = \frac{T_1 d_2}{J_{AB}}$

$$= \frac{T_1 \times d_2}{\frac{1}{2} \pi d_1^4}$$

$$= \frac{d_2^4}{d_1^4} \times \frac{d_3^4 - d_2^4}{d_3^4} T_0 \frac{d_1}{\frac{1}{2} \pi d_1^4}$$

$$= \frac{T_0 d_2}{\pi d_1^3 d_3^4} 2 T_0 d_2^4$$

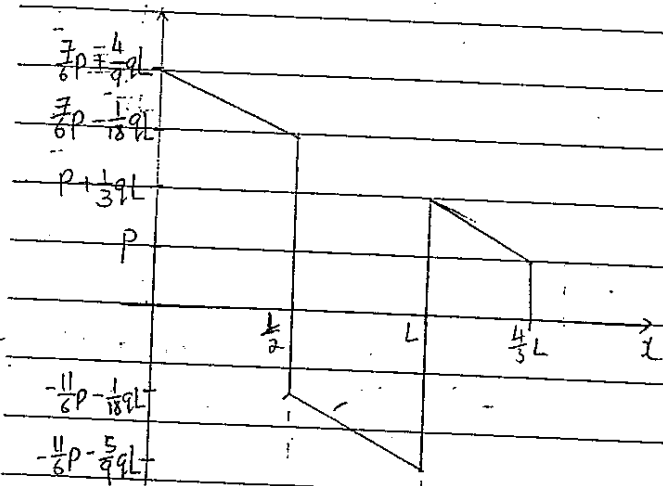


$$2) a) \sum M_A = -3P \times \frac{L}{2} - P \times \frac{4}{3}L + F_B \times L - q \times \frac{4}{3}L \times \frac{2}{3}L = 0$$

$$\therefore F_B = \frac{17}{6}P + \frac{8}{9}qL$$

$$\sum F_y = F_A + \frac{8}{9}qL + \frac{17}{6}P - 4P - q \times \frac{4}{3}L = 0$$

$$F_A = \frac{7}{6}P + \frac{4}{9}qL$$



For  $0 \leq x \leq \frac{L}{2}$

$$\sum F_y = \frac{7}{6}P + \frac{4}{9}qL - qL - V = 0$$

$$\therefore V = -qL + \frac{7}{6}P + \frac{4}{9}qL$$

$$\sum M = -(\frac{7}{6}P + \frac{4}{9}qL)x + qL \cdot \frac{x}{2} + M = 0$$

$$\therefore M = -\frac{q}{2}x^2 + (\frac{7}{6}P + \frac{4}{9}qL)x$$

For  $\frac{L}{2} \leq x \leq L$

$$\sum F_y = \frac{7}{6}P + \frac{4}{9}qL - qL - 3P - V = 0$$

$$V = -qL + \frac{7}{6}P + \frac{4}{9}qL - 3P$$

$$\sum M = -(\frac{7}{6}P + \frac{4}{9}qL)x + qL \cdot \frac{x}{2} + 3P(x - \frac{L}{2}) + M = 0$$

$$\therefore M = -\frac{q}{2}x^2 + (\frac{7}{6}P + \frac{4}{9}qL)x - 3P(x - \frac{L}{2})$$

For  $L \leq x \leq \frac{4}{3}L$

$$\sum F_y = \frac{7}{6}P + \frac{4}{9}qL - qL - 3P + \frac{17}{6}P + \frac{8}{9}qL - V = 0$$

$$V = P + \frac{1}{3}qL - qL$$

$$\sum M = -(\frac{7}{6}P + \frac{4}{9}qL)x + qL \cdot \frac{x}{2} + 3P(x - \frac{L}{2}) -$$

$$(\frac{17}{6}P + \frac{8}{9}qL)(x - L) + M = 0$$

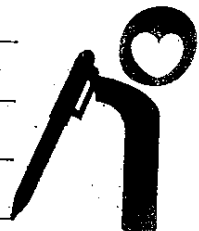
$$M = -\frac{q}{2}x^2 + (\frac{7}{6}P + \frac{4}{9}qL)x - 3P(x - \frac{L}{2}) + (\frac{17}{6}P + \frac{8}{9}qL)(x - L)$$

\*  $\therefore$  For maximum shear force,

when  $x = L$ ,  $V = -\frac{11}{6}P - \frac{5}{9}qL$ ,

- maximum bending moment,

when  $x = \frac{L}{2}$ ,  $M = \frac{7}{12}PL + \frac{7}{12}qL^2$ ,



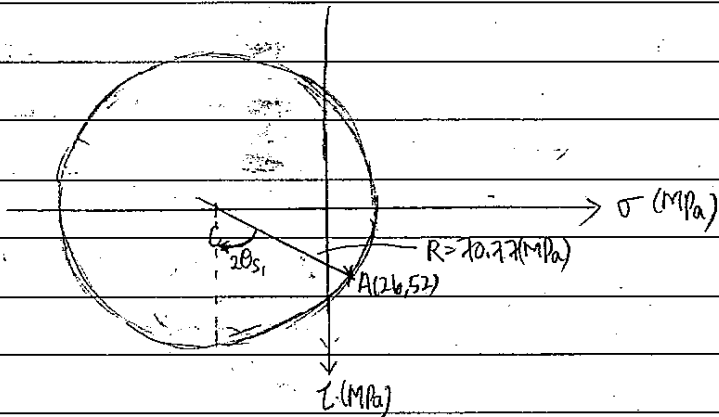
3b) (i)  $\sigma_x = 26 \text{ MPa}$ ,  $\sigma_y = -70 \text{ MPa}$ ,  $\tau_{xy} = 52 \text{ MPa}$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{26 - 70}{2} = -22 \text{ MPa}$$

$$A(26, 52)$$

$$C(-22, 0)$$

$$R = \sqrt{(26 - (-22))^2 + (52)^2} = 70.77 \text{ MPa}$$



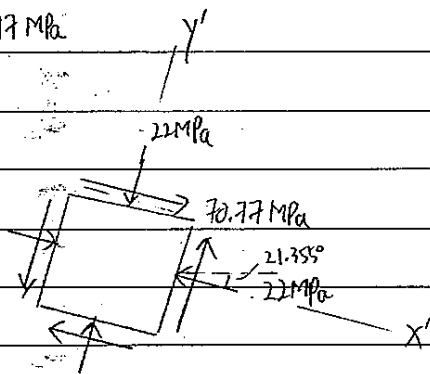
$$\tan 2\theta_{s1} = \frac{26 - (-22)}{52}$$

$$2\theta_{s1} = 42.71^\circ$$

$$\theta_{s1} = 21.355^\circ$$

$$\tau_{max} = R = 70.77 \text{ MPa}$$

$$\sigma_{ave} = -22 \text{ MPa}$$



pg 11 -

Date

No.

internal - opposite in direction

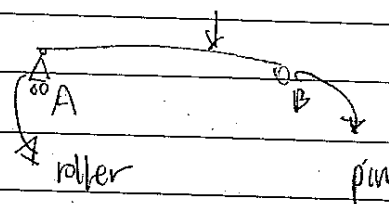
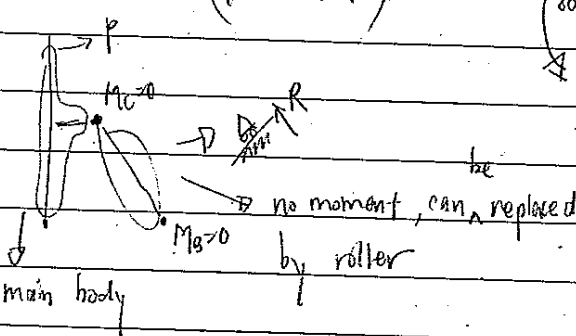
At A, external force (E.R.)

At B, D, W, W<sub>2</sub>  $\perp$  External Impaired force

pg 12  
at a point \*

pg 17

(10-02 am)



pg 18 read -

pg 19 read - (internal force)

sketchon 27



(3b) (i)  $\sigma_x = 26 \text{ MPa}$ ,  $\sigma_y = -70 \text{ MPa}$ ,  $\tau_{xy} = 70.77 \text{ MPa}$

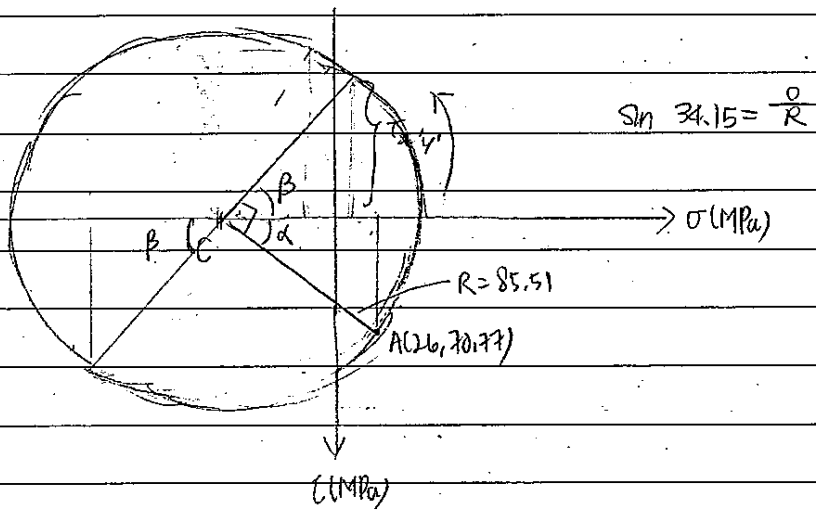
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = \frac{26 - 70}{2}$$

$$\sigma_{\text{ave}} = -22 \text{ MPa}$$

$$A(26, 70.77)$$

$$C(-22, 0)$$

$$R = \sqrt{[26 - (-22)]^2 + (70.77)^2} = 85.51 \text{ MPa}$$



$$\sin 34.15 = \frac{\tau}{R}$$

$$\tan \alpha = \frac{70.77}{92.26}$$

$$\alpha = 55.85^\circ$$

$$\beta = 90^\circ - 55.85^\circ = 34.15^\circ$$

$$\sigma_{x'} = -22 + 85.51 \times \cos 34.15^\circ$$

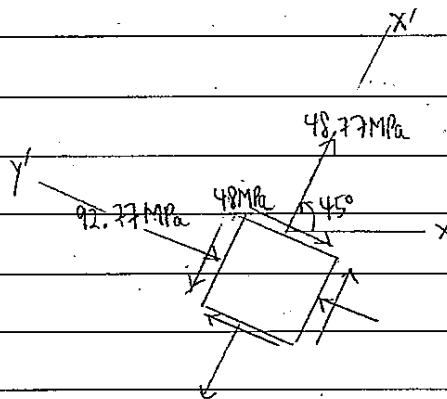
$$\sigma_{x'} = 48.77 \text{ MPa}$$

$$\tau_{x'y'} = -85.51 \times \sin 34.15^\circ$$

$$\tau_{x'y'} = -48 \text{ MPa}$$

$$\sigma_{y'} = -85.51 \times \cos 34.15^\circ - 22$$

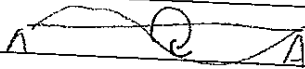
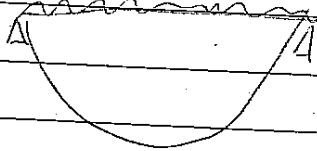
$$\sigma_{y'} = -92.77 \text{ MPa}$$



bending moment depends on deflection profile.

Q2

deflection profile



Q3

$$75.833 \text{ kN} = F_D$$

$$A_y = 50 \text{ kN}$$

$$A_x = 20.833 \text{ kN}$$

Q4

AC - need support from CB (dependent structure)

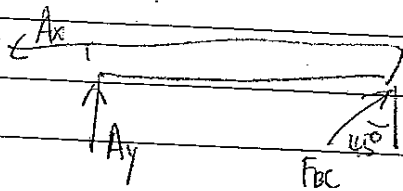
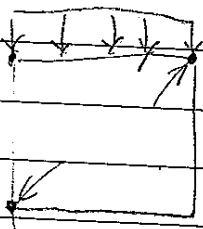
CB - can sustain itself

remove distributed on AC, internal force in AC = 0

" " " CB, CB will still have internal force to sustain AC.

Q5

pin-pin connect = roller



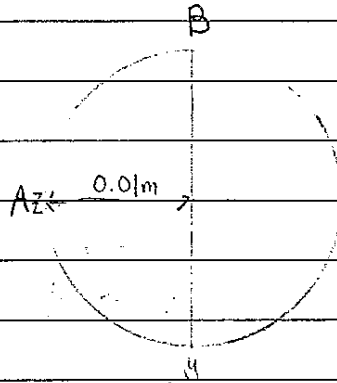
4) a)  $V = 1000 \text{ kN}$

$M_y = 1000 \times 120 \text{ mm}$   
 $= 120 \text{ Nm}$

$M_x = 1000 \times 120 \text{ mm}$   
 $= 120 \text{ Nm}$

$M_z = 0 \text{ Nm}$

$N = 0 \text{ N}$



$I_y = \frac{1}{4} \pi (0.01)^4$   
 $= 7.85 \times 10^{-9} \text{ m}^4 = I_z$

$J = \frac{1}{2} \pi (0.01)^4$   
 $= 1.57 \times 10^{-8} \text{ m}^4$

i) - Tensile and Compressive stress :

$0 + \frac{120 \times 0.01}{7.85 \times 10^{-9}} + 0 = \text{Normal Stress}$

Normal Stress =  $152.87 \text{ MPa}$

- Shear Stress :

Shear Stress =  $\frac{120 \times 0.01}{1.57 \times 10^{-8}} + \frac{1000 \times 0}{1.57 \times 10^{-8}}$   
 $= 76.43 \text{ MPa}$

- In plane Shear Stress,

$\tau_{\max} = \sqrt{\left(\frac{152.87}{2}\right)^2 + 76.43^2}$   
 $= 108.09 \text{ MPa}$

ii)  $V = 1000 \text{ kN}$

$M_y = 1000 \times 120 \text{ mm} = 120 \text{ Nm}$

$M_x = 1000 \times 120 \text{ mm} = 120 \text{ Nm}$

$M_z = 0 \text{ Nm}$

$N = 0 \text{ N}$

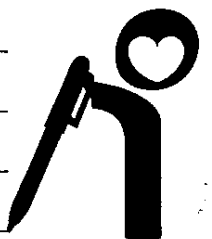
$Q = \frac{4 \times 0.01}{3\pi} \times \frac{\pi \times 0.01^2}{2}$   
 $= 6.67 \times 10^{-7} \text{ m}^3$

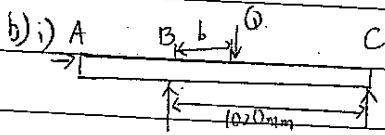
• Normal Stress =  $0 \text{ Pa}$

• Shear Stress =  $\frac{120 \times 0.01}{1.57 \times 10^{-8}} - \frac{1000 \times 6.67 \times 10^{-7}}{7.85 \times 10^{-9} \times 0.01}$   
 $= 67.94 \text{ MPa}$

• In plane Shear Stress,

$\tau_{\max} = \sqrt{\left(\frac{0}{2}\right)^2 + 67.94^2}$   
 $= 67.94 \text{ MPa}$





$$\sum M_B = F_c \times (1020) - Qb = 0$$

$$F_c = \frac{Qb}{1020}$$

$$\sum F_y = -Q + \frac{Qb}{1020} + F_{A_D} = 0$$

$$F_{A_D} = \frac{Q(1020-b)}{1020}$$

$$E = 200 \text{ GPa}$$

$$K_{BD} = 0.7$$

$$L_{BD} = 0.89 \text{ m}$$

$$K_{CE} = 1$$

$$L_{CE} = 1.15 \text{ m}$$

$$I = \frac{1}{12} (0.016)^4$$

$$= 5.46 \times 10^{-9} \text{ m}^4$$

Critical loading for both column BD, CE

$$\rightarrow P_{crit_{BD}} = \frac{\pi^2 \times 200 \text{ G} \times 5.46 \times 10^{-9}}{(0.7 \times 0.89)^2}$$

$$= 27.77 \text{ kN}$$

$$\rightarrow P_{crit_{CE}} = \frac{\pi^2 \times 200 \text{ G} \times 5.46 \times 10^{-9}}{(1 \times 1.15)^2}$$

$$= 8.15 \text{ kN}$$

$$Q_{CE} = 8.15 \text{ k} \times 1020 \times \frac{1}{250}$$

$$= 33.25 \text{ kN}$$

$$Q_{BD} = 27.77 \text{ k} \times 1020 \times \frac{1}{770}$$

$$= 36.79 \text{ kN} > Q_{CE}$$

$\therefore$  The critical load  $Q = 33.25 \text{ kN}$

ii) Q value is maximized when  $Q_{BD} = Q_{CE}$

$$Q_{BD} = Q_{CE}$$

$$27.77 \times 1020 \times \frac{1}{1020-b} = 8.15 \times 1020 \times \frac{1}{b}$$

$$\frac{27.77}{1020-b} = \frac{8.15}{b}$$

$$27.77b = 8313 - 8.15b$$

$$b = 231.43 \text{ mm}$$

$\therefore$  when  $b = 231.43 \text{ mm}$ , Q is maximized

$$Q = 8.15 \times 1020 \times \frac{1}{231.43}$$

$$= 35.92 \text{ kN}$$

