NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2010-2011

CV2101 – MECHANICS OF MATERIALS

December 2010 **Time Allowed: 2½ hours**

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FOUR (4)** pages.
2. Answer **ALL FOUR (4)** questions.
3. All questions carry equal marks.
4. An Appendix of **TWO (2)** pages is attached together with this paper.

---

1. (a) The T-frame shown in Figure Q1(a) is subjected to three forces and one couple. Replace the force-couple system with a single force, and determine the two locations on the T-frame with respect to point O where the line of action of the single force intersects.

(13 marks)

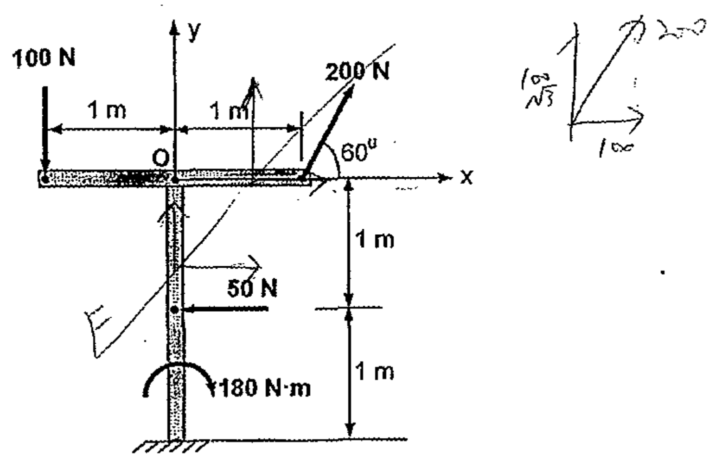


Figure Q1(a)

Note: Question No. 1 continues on page 2

(b) As shown in Figure Q1(b), beam AB and beam BC are connected by a pin at B. The roller at C is resting on a smooth surface. Determine the support reactions at A and C.

(12 marks)

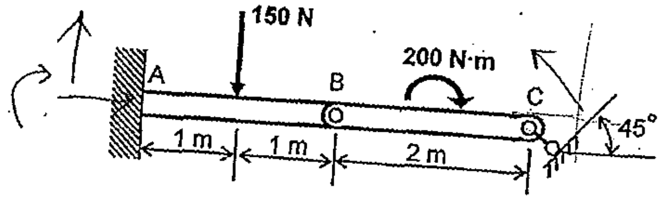


Figure Q1(b)

2. (a) The beam ABC shown in Figure Q2(a) is subjected to a clockwise moment of 10 kN·m at A and a uniform distributed loading of 5 kN/m between B and C. Draw the shear and moment diagrams for the beam shown in Figure Q2(a).

(12 marks)

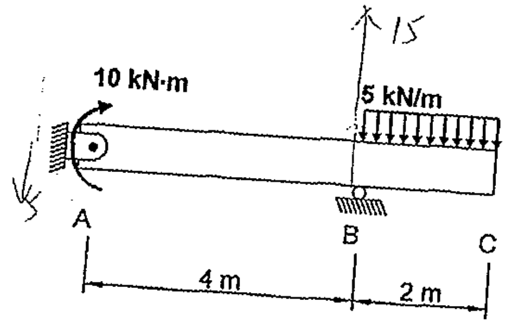


Figure Q2(a)

(b) A rigid beam ABCD is supported by a hinge at A and steel links 1 and 2 at C and D, respectively, and subjected to a force of P kN at B. Links 1 and 2 have the same length and sectional area. Determine the reaction force at A and axial forces of links 1 and 2 as a function of P.
axial load.

(13 marks)

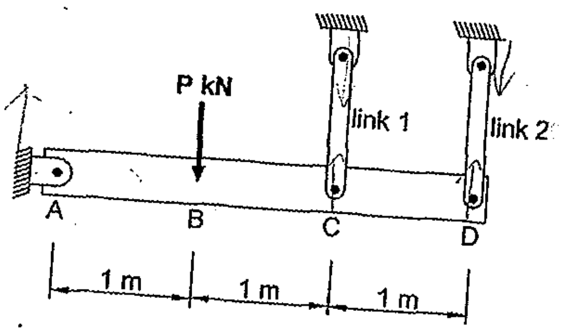


Figure Q2(b)

3. (a) The shaft shown in Figure Q3(a) has a diameter of 0.05 m and is subjected to a distributed torsion of 200 N·m/m between A and C and a concentrated torsion of 250 N·m at C. The shaft is fixed to the wall at A. The shear modulus G = 75 GPa. Determine

(i) the maximum shear stress in the shaft
(ii) the relative angle of twist of the shaft between A and C
(iii) the relative angle of twist of the shaft between C and B

(13 marks)

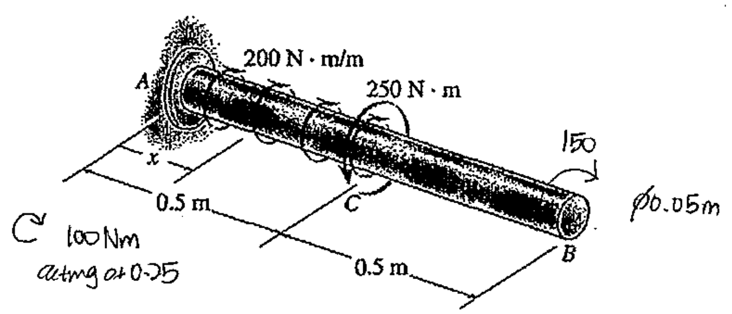
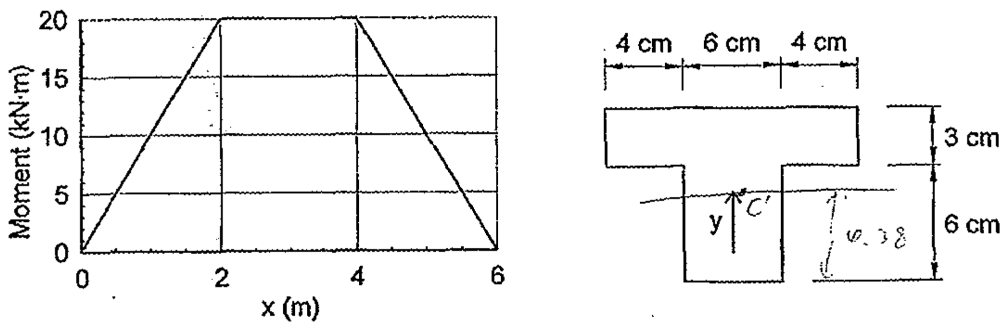


Figure Q3(a)

(b) A beam has a span of 6 meters. The cross section and moment diagram of the beam are given in Figure Q3(b). Given that the shear diagram of the beam is continuous throughout the entire span, determine the maximum shear stress of the cross section in the y direction (see panel 2 of Figure Q3(b)) at

(i) x = 1 m of the span
(ii) x = 2.5 m of the span

(12 marks)



Panel 1: Moment diagram of the beam

Panel 2: Cross section of the beam

Figure Q3(b)

4. (a) The state of stress at a point is shown on the element in panel 1 of Figure Q4(a).

   (i) Draw Mohr's circle for the case of the element in panel 1 of Figure Q4(a).

   (ii) The element in panel 2 of Figure Q4(a) is for the same point as that in panel 1 but in a different orientation. Plot the state of stress for the case of panel 2 (don't plot your answer on this exam paper).

   (iii) Determine the principal stresses and absolute maximum shear stress for this case.

(12 marks)

Panel 1: The state of stress for (i)

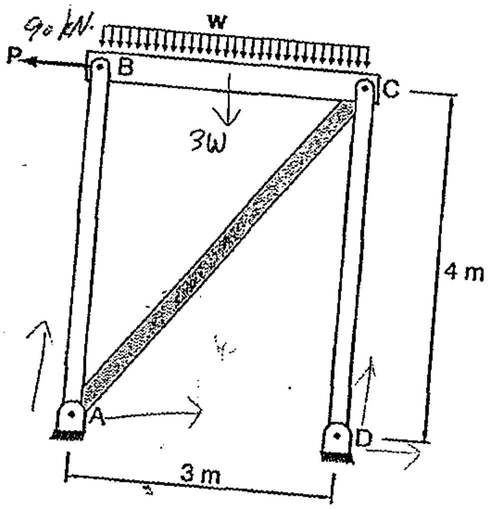
Panel 2: The state of stress for (ii)

Figure Q4(a)

(b) The frame ABCD shown in Figure Q4(b) is braced by a solid circular steel bar AC. The bar is pin supported at its ends. The frame ABCD is subjected to an uniform loading w on beam BC and a lateral load P of 90 kN at B. Assume the columns and beam of the frame do not fail when subjected to w and P. For the design of bar AC, use a factor of safety with respect to buckling (i.e., the critical buckling load to the axial force of bar AC due to P and w) of 3, E_{st} = 200 GPa and σ_Y = 250 MPa. Determine

   (i) the minimum diameter of bar AC if w = 1 kN/m.
   (ii) the minimum diameter of bar AC if w = 10 kN/m.

(13 marks)



$A = \pi r^2$
$= \frac{1}{4}\pi d^2$

Figure Q4(b)

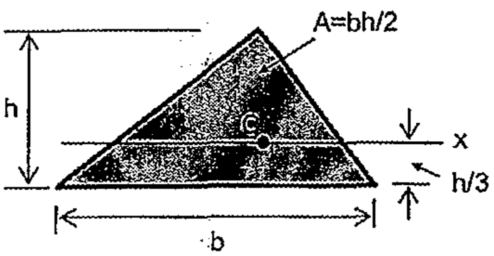
END OF PAPER

1. Equilibrium

   Particle $\sum F_x = 0$, $\sum F_y = 0$, $\sum F_z = 0$

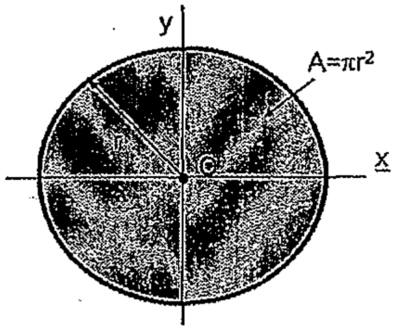
   Rigid Body – Two dimensions $\sum F_x = 0$, $\sum F_y = 0$, $\sum M_o = 0$

2. Geometric properties of area elements



Triangular

$$I_x = \frac{1}{36} b h^3$$



Circular area

$$I_x = \frac{1}{4} \pi r^4$$

$$I_y = \frac{1}{4} \pi r^4$$

Parallel-Axis Theorem $I = \bar{I} + Ad^2$

3. Axial load

   Normal Stress $\sigma = \frac{P}{A}$

   Displacement $\delta = \sum \frac{PL}{EA}$, $\delta_T = \alpha \Delta T L$

4. Torsion

   Shear Stress in Circular Shaft $\tau = \frac{T\rho}{J}$

   where $J = \frac{\pi}{2} c^4$ solid cross section; $J = \frac{\pi}{2}(c_o^4 - c_i^4)$ tubular cross section

   Angle of Twist $\phi = \int_{L_1}^{L_2} \frac{T(x)}{GJ(x)} dx$, $\phi = \sum \frac{TL}{GJ}$

5. Bending

Normal Stress $\sigma = \frac{My}{I}$

Unsymmetric Bending $\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$, $\tan\alpha = \frac{I_z}{I_y}\tan\theta$

6. Shear

Average Direct Shear Stress $\tau_{ave} = \frac{V}{A}$

Transverse Shear Stress $\tau = \frac{VQ}{It}$

Shear Flow $q = \tau \cdot t = \frac{VQ}{I}$

7. Stress Transformation Equations

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Principal Stress,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \underbrace{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}_{R}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Maximum In-Plane Shear Stress,

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}, \quad \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

8. Buckling

Critical Axial Load, $P_{cr} = \frac{\pi^2 EI}{(KL)^2}$.

$$F_S = \frac{P_{cr}}{N_{AC}} = 3$$

CV2101 Mechanics of Materials 2010–2011 Semester 1

1.(a) $F_x = 200 \cdot \cos 60^\circ - 50 = 50\ N \longrightarrow$

$F_y = 200 \cdot \sin 60^\circ - 100 = 100\sqrt{3} - 100 = 73.21\ N \uparrow$

$|F_R| = \sqrt{F_x^2 + F_y^2} = 88.65\ N$

$\tan\alpha = \frac{73.21}{50} \qquad \alpha = 55.67^\circ$

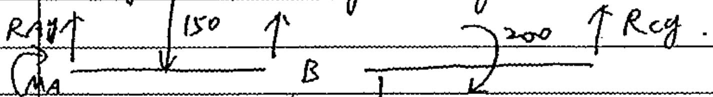
$\Sigma M_O = -100 \times 1 - 100\sqrt{3} \times 1 + 50 \times 1 + 180 = -43.2\ N\text{-}m$

on the y axis, $F_{Rx} \cdot d = 43.2 \quad d = 0.86\ m$ from O

on X axis, $F_{Ry} \cdot d = 43.2, \quad d = 0.59\ m$

1(b) $\theta = 45^\circ, \quad R_{cx} = R_{cy}$

$\Sigma F_y = 0, \quad R_{Ay} + R_{cy} - 150 = 0 \qquad \Sigma F_x = 0, \ R_{cx} = R_{Ax}$



$\Sigma M_B = 0, \quad 200 - R_{cy} \times 2 = 0 \ \therefore R_{cy} = 100\ N$

$\therefore R_{Ay} = 50\ N$

$\Sigma M_B = 0, \quad 150 \times 1 + M_A - 50 \times 2 = 0 \qquad M_A = 50\ N \cdot m$

$R_{cx} = R_{cy} = 100\ N$

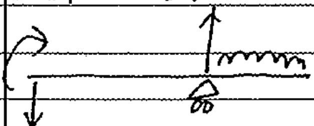
$R_{Ax} = R_{cx} = 100\ N$

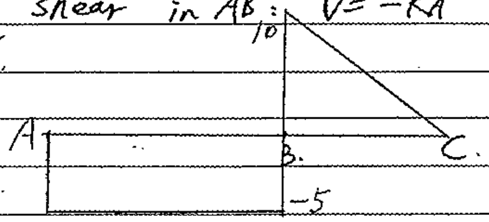
2.(a) $R_A + R_B - 5 \times 2 = 0$

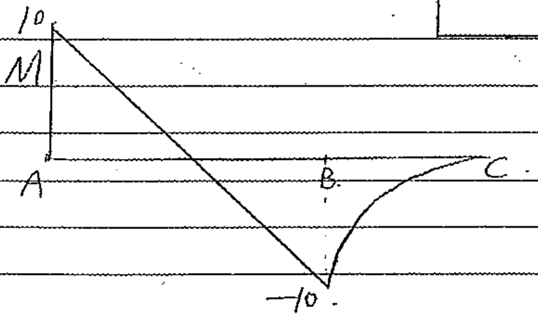
$\Sigma M_A = 0, \quad 10 + 10 \times 5 - R_B \times 4 = 0 \quad \therefore R_B = 15\ kN$

$R_A = -5\ kN \qquad$ shear in AB: $V = -R_A \uparrow$

shear jump at B







2(b) $R_A + R_C + R_D = P \qquad \Sigma M_A = 0, \ P - R_C \times 2 - R_D \times 3 = 0$

$\delta = \frac{PL}{AE} \qquad L_1 = L_2, \ A_1 = A_2$

$\frac{\delta_1}{\delta_2} = \frac{2}{3} \qquad \therefore \frac{P_1}{P_2} = \frac{2}{3}, \quad P_1 = \frac{2}{3}P_2, \quad P_2 = \frac{3}{13}P, \ P_1 = \frac{2}{13}P, \ R_A = \frac{8}{13}P$

NANYANG TECHNOLOGICAL UNIVERSITY

$d=0.05$; $r=0.025$

Date No.

3(a) $\Sigma M_A=0$, $200\times0.5+T-250=0$ $T=150N$ $G=75\times10$

$J=\frac{\pi}{2}c^4=\frac{\pi}{2}\times0.025^4=6.136\times10^{-7}$

$\phi_{AC}=\int_0^{0.5}\frac{200x}{GJ}dx=\frac{1}{GJ}\left[100x^2\right]_0^{0.5}=5.43\times10^{-4}$

$\phi_{CB}=\frac{150\times0.5}{GJ}=1.63\times10^{-3}$

3(b) $\tau_{max}=\frac{VQ}{It}$ $Q=\bar{y}'A'$ $t=6\ cm$

$\bar{y}=\frac{\int y\,dA}{\int dA}=\frac{6\times6\times6+3\times3\times14}{6\times6+3\times14}$ ~~$=4.38\ cm$~~

∴ the neutral axis is 4.38 cm from the bottom

$\bar{y}'=6-4.38+1.5=3.12\ cm$

$A'=14\times3=42\ cm^2$

$I=I_{flange}+I_{web}$

$=\frac{1}{12}\times14\times3^3+42\times3.12^2+\frac{1}{12}\times6\times6^3+6\times6\times(4.38-3)^2$

$=616.9\ cm^4$

$Q=\bar{y}'\cdot A'=3.12\times42=131.04\ cm^3$

$\frac{Q}{It}=0.0354\ \frac{1}{cm^2}=354\ \frac{1}{m^2}$

at $x=1m$, $M=10\ kN\cdot m=10^4\ Nm$, $V=\frac{M}{x}=10^4\ N$

$\tau_{max}=\frac{VQ}{It}=10^4\times354=3.54\times10^6\ N/m^2$

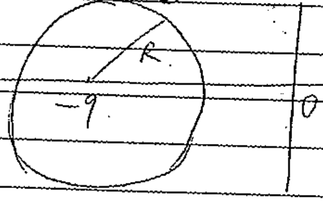
$x=2.5$, $M=20kN\cdot m=2\times10^4\ N\cdot m$

between 2−4 m, M is constant, ∴ V is 0, $\tau=0$.

4(a) $\sigma_x=0$, $\sigma_y=-18$, $\tau_{xy}=45$

Mohr circle: $A(\sigma_x,\tau_{xy})$, $C(\sigma_{ave},0)$

$\sigma_{ave}=\frac{\sigma_x+\sigma_y}{2}=-9$, $R=\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}=45.89$



$\sigma'_x=\frac{\sigma_x+\sigma_y}{2}+\frac{\sigma_x-\sigma_y}{2}\cos2\theta+\tau_{xy}\sin$

$\tau'_{xy}=-\frac{\sigma_x-\sigma_y}{2}\sin2\theta+\tau_{xy}\cos2\theta$

$\theta=-40°$

$\sigma'_x=-9+9\cos80°+45\cdot\sin(-80°)$

$=-51.75$

$\tau'_{xy}=+9\sin80°+45\cdot\cos80°=16.68$

$\sigma_{1,2}=\frac{\sigma_x+\sigma_y}{2}\pm R=-9\pm45.89$

$\tau_{max}=\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}=R=45.89$

NANYANG TECHNOLOGICAL UNIVERSITY

Date No.

4(b) factor of safety $F_s = \frac{P_{cr}}{N_{AC}} = 3$ $P_{cr} = \frac{\pi^2 EI}{(KL)^2}$

AC: both ends pinned $\therefore L_{effective} = L$. $KL = L$

$N = 6A$ $A = \pi r^2 = \frac{1}{4}\pi d^2$

$E = 200 \times 10^9 = 2 \times 10^{11}$ Pa

circular steel bar: $I = \frac{1}{4}\pi r^4$

$F_s = \frac{\pi^2 E \cdot \frac{1}{4}\pi r^4}{L^2 \cdot 6 \cdot \pi r^2} = \frac{\pi^2 E \cdot r^2}{4L^2 6}$

$L = 5m$, $6_y = 250$ MPa $= 2.5 \times 10^8$ Pa.

$w = 1$, $1 \times 3 = 3$ kN

$R_{Ay} + R_{Dy} = 3$, $R_{Ax} + R_{Dx} = 90$

$\Sigma M_A = 0$, $90 \times 4 - 3 \times 1.5 + D_y \times 3 = 0$ $\therefore D_y = -118.5$

$A_y = 121.5$ kN $N_{AC} = \frac{5}{4} A_y = 151.875$ kN

$\therefore P_{cr} = 3 \cdot N_{AC} = 455.625 \times 10^3$ N $= \frac{\pi^2 EI}{L^2}$

$I = 577 \times 10^{-8} = \frac{1}{4}\pi r^4$ $r = 5.2 \times 10^{-2}$ m

$6 = \frac{N}{A} = \frac{151.875 \times 10^3}{\pi \times 5.2^2 \times 10^{-4}} = 1.78 \times 10^7 < 6_y$

$\therefore r = 5.2 \times 10^{-2}$ m, $d = 2r = 10.4$ cm

$w = 10$, $10 \times 3 = 30$ kN.

$R_{Ay} + R_{Dy} = 30$, $R_{Ax} + R_{Dx} = 90$.

$\Sigma M_A = 0$, $90 \times 4 - 30 \times 1.5 + D_y \times 3 = 0$ $D_y = -105$

$A_y = 135$ kN, $N_{AC} = 168.75$ kN

$P_{cr} = 506.25 \times 10^3 N = \frac{\pi^2 EI}{L^2}$ $I = 641.17 \times 10^{-8} = \frac{1}{4}\pi r^4$

$r = 5.35 \times 10^{-2}$ m

$6 = \frac{N}{A} = \frac{168.75 \times 10^3}{5.35^2 \times 10^{-4}} = 5.89 \times 10^7$

$\therefore r = 5.35$ cm, $d = 10.7$ cm