NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2009-2010

CV2101 – Mechanics of Materials

November - December 2009 **Time Allowed: 2½ hours**

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FIVE (5)** pages.
2. Answer **ALL FOUR (4)** questions.
3. All questions carry equal marks.
4. An Appendix of **TWO (2)** pages is attached together with this paper.

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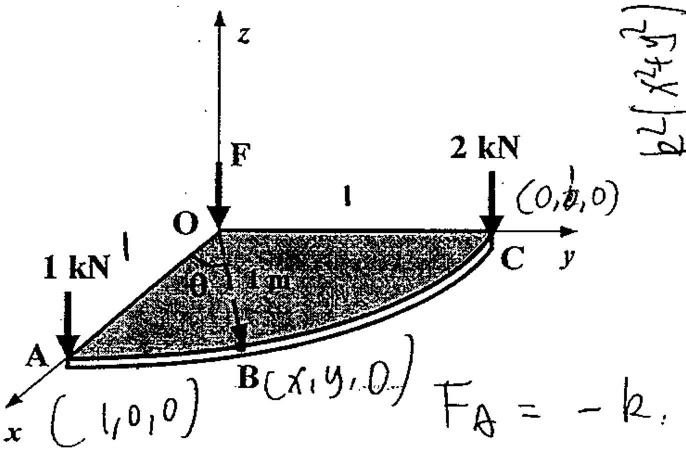
1 (a) The resultant of the three vertical loads of 1 kN at A, 2 kN at C and F kN at O passes through B. Assume the rigid quarter-circular plate ABCO of 1 m radius has negligible weight and thickness. Determine the values of load F and angle θ.

(12 marks)

$k^2(x^2+y^2) = 5$



$y^2k^2 = 4$

$x^2k^2 = 1$

$F_A = -k$

Figure Q1(a)

$F_C = -2k$

$i \quad j \quad k$

$x \quad y \quad 0$

$\overrightarrow{OA} \cdot F_A = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{vmatrix} = +1j$

Note: Question No.1 continues on page 2

$-kyi + xkj$

$k^2y^2 + x^2k^2 = 4+1 = 5$

$ky = 2$

$xk = 1$

$x^2 + y^2 = 1$

$2 \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{vmatrix} = -2i$

$-2i + j$

(b) As shown in Figure Q1(b), a weight of 100 N is supported by a cable EDF and a structure ABCD which is pinned at points A, B and C. Assume a frictionless pulley at point D. Determine the reactions at points A and B.

(13 marks)

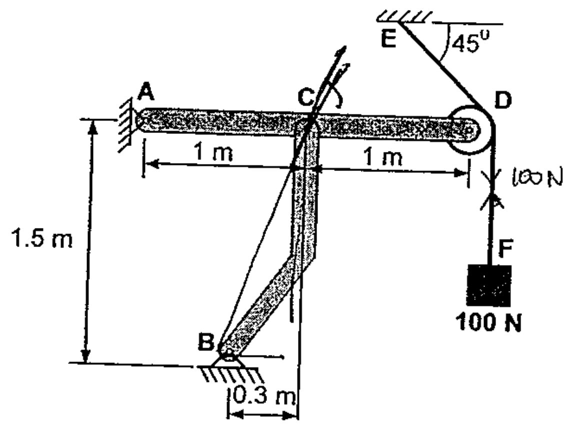


Figure Q1(b)

2. (a) The compound beam ABCDE shown in Figure Q2(a) consists of two beams (AD and DE) joined by a hinged connection at D. The hinge can transmit a shear force but not a bending moment. A force P acts upward at A and a uniform load of intensity q acts downward on beam DE.

$PL.$

Draw the shear-force and bending moment diagrams for the compound beam.

$(-qL-P)x+PL.$

(12 marks)

$-qL^2-PL+PL.$

$qL \times L.$

$\boxed{qL-qx}$

$qLx-qx^2.$

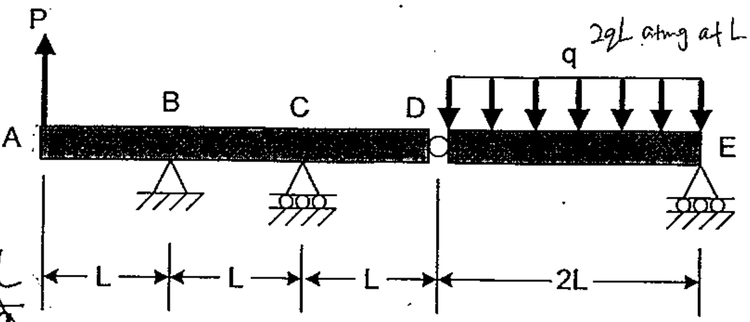


Figure Q2(a)

$qLx-\frac{q}{2}x^2$

$\frac{1}{4}$

$qL^2-\frac{q}{2}L^2.$

$\frac{q}{2}L^2.$

(b) A rigid beam AD as shown in Figure Q2(b) is supported by a smooth pin B and by vertical rods that are attached to the beam at points A and C. The beam is horizontal when only rod (1) is attached and P = 0. Rod (2) was manufactured δ = 0.5 mm too short.

Determine the axial stresses σ_1 and σ_2 induced in the respective support rods after rod (2) is attached to the upper bracket and the load P = 20 kN is applied.

It is known that the cross sectional area of the two rods are $A_1 = A_2 = 200\ mm^2$. The modulus of elasticity of the two rods are $E_1 = 70$ GPa and $E_2 = 100$ GPa. Assume that θ is small and neglect the weight of the beam.

(13 marks)

$(-P-qL)x+PL.$

$-PL-qL^2.$

qL

qL^2

$qLx-\frac{q}{2}x^2.$

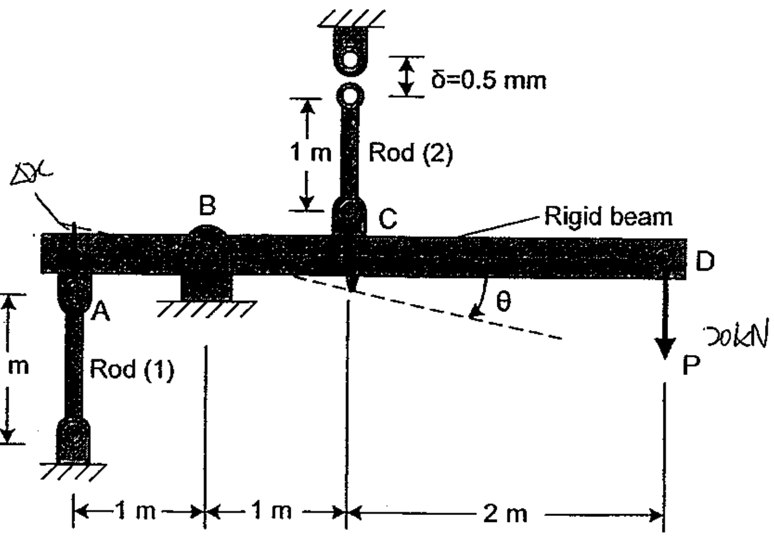


Figure Q2(b)

3. (a) A two element aluminum shaft AC is subjected to external torques at sections B and C as shown in Figure Q3(a). The element AB is a solid segment while the element BC is a hollow one with an inner diameter of d_i. The outer diameter of the shaft is d = 50 mm. The modulus of rigidity of the aluminum material is G = 26 GPa.

Determine the inner diameter d_i such that the maximum shear stress in the shaft does not exceed the allowable shear stress τ_{allow} = 35 MPa and the total angle of twist does not exceed φ_{allow} = 0.08 rad.

(12 marks)

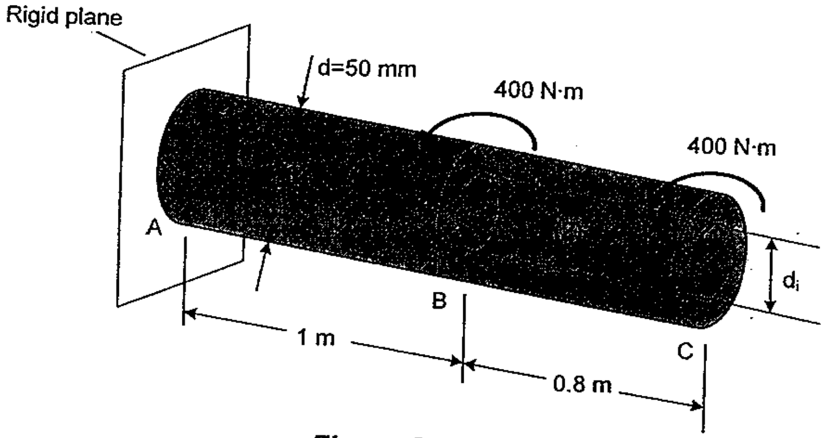


Figure Q3(a)

(b) A steel/wood sandwich beam is fabricated from two 80 mm × 160 mm wooden beams and a 10 mm × 160 mm steel plate. The cross section of the sandwich beam is illustrated in Figure Q3(b). The ratio of the elastic moduli of steel and wood is $E_s / E_w = 20$.

If the allowable stresses in the steel and wood are $(\sigma_{allow})_s$ = 120 MPa and $(\sigma_{allow})_w$ = 8 MPa, respectively, determine the maximum moment M_z that can be safely applied to the sandwich beam.

(13 marks)

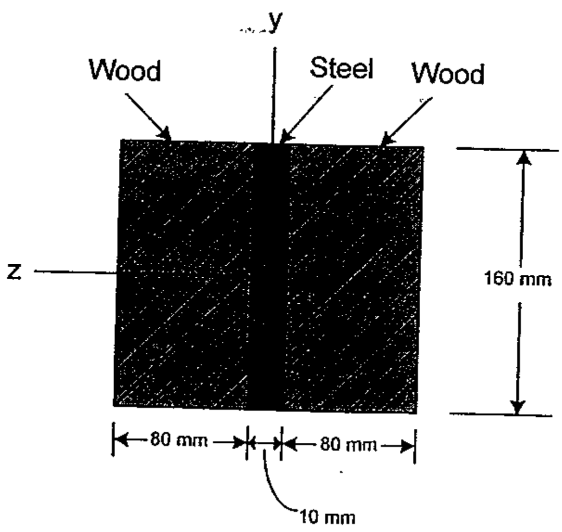


Figure Q3(b)

4. (a) Member AB as shown in Figure Q4(a) has a uniform thickness of 10 mm. A 9 kN force is applied to point G at the middle of the member.

(i) Determine the normal and shearing stresses at point H and point K.
(ii) Determine the principal stresses at point H and point K.
(iii) Determine the maximum shearing stress at point H and point K.

(15 marks)

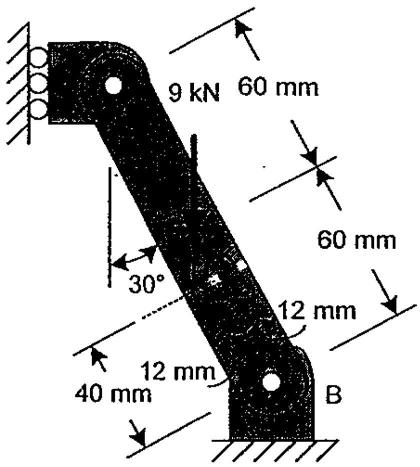


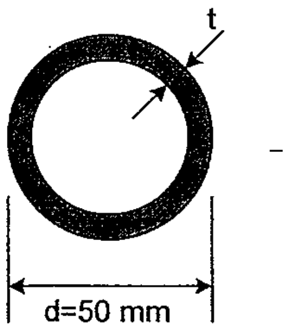
Figure Q4(a)

4.5

(b) A pinned-end strut of aluminum with length L = 1.8 m is constructed of a circular tube with an outer diameter of d = 50 mm as seen in Figure Q4(b). The modulus of elasticity E of the aluminum material is 72 GPa. The strut must resist an axial load P = 18 kN with a factor of safety n = 2.0 with respect to the critical load.

Determine the required thickness t of the tube.

(10 marks)



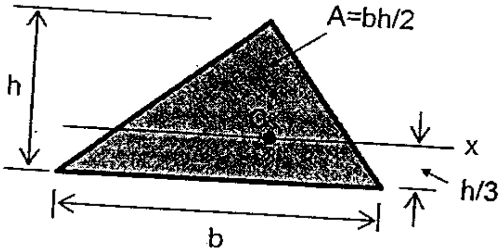
END OF PAPER

1. Equilibrium

Particle $\sum F_x = 0$, $\sum F_y = 0$, $\sum F_z = 0$

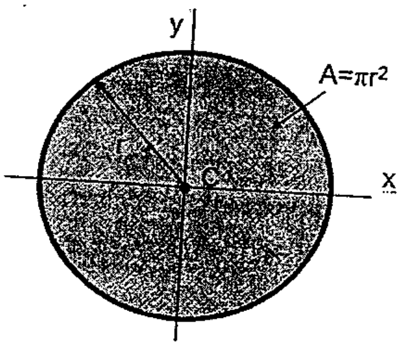
Rigid Body – Two dimensions $\sum F_x = 0$, $\sum F_y = 0$, $\sum M_o = 0$

2. Geometric properties of area elements



Triangular

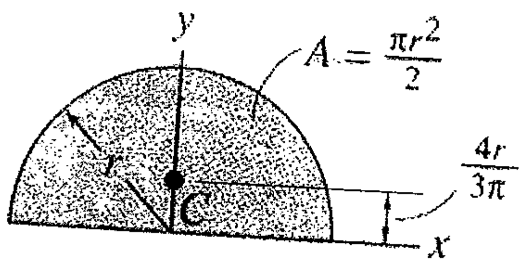
$$I_x = \frac{1}{36} bh^3$$



Circular area

$$I_x = \frac{1}{4}\pi r^4$$

$$I_y = \frac{1}{4}\pi r^4$$



Semicircular area

$$I_x = \frac{1}{8}\pi r^4$$

$$I_y = \frac{1}{8}\pi r^4$$

Parallel-Axis Theorem $I = \bar{I} + Ad^2$

3. Axial load

Normal Stress $\sigma = \dfrac{P}{A}$

Displacement $\delta = \sum \dfrac{PL}{EA}$, $\delta_T = \alpha \Delta T L$

4. Torsion

Shear Stress in Circular Shaft $\tau = \dfrac{T\rho}{J}$

where $J = \dfrac{\pi}{2}c^4$ solid cross section; $J = \dfrac{\pi}{2}(c_o^4 - c_i^4)$ tubular cross section

Angle of Twist $\phi = \sum \dfrac{TL}{GJ}$

5. Bending

Normal Stress $\sigma = \dfrac{My}{I}$

Unsymmetric Bending $\sigma = -\dfrac{M_z y}{I_z} + \dfrac{M_y z}{I_y}$, $\tan\alpha = \dfrac{I_z}{I_y}\tan\theta$

6. Shear

Average Direct Shear Stress $\tau_{ave} = \dfrac{V}{A}$

Transverse Shear Stress $\tau = \dfrac{VQ}{It}$

Shear Flow $q = \tau \cdot t = \dfrac{VQ}{I}$

7. Stress Transformation Equations

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Principal Stress,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Maximum In-Plane Shear Stress,

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}, \quad \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

8. Buckling

Critical Axial Load, $P_{cr} = \dfrac{\pi^2 EI}{(KL)^2}$

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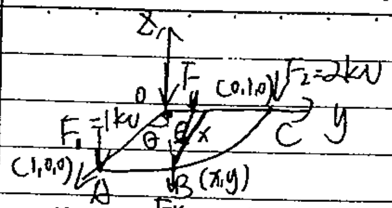
1(a)



From the Figure, we know

$\vec{F_1} = -1k \quad F_2 = -2k$

$M_0 = \vec{OA}\cdot\vec{F_1} + \vec{OC}\cdot\vec{F_2} = -2i + j$

the resultant force should create the same M_0

We assume Fr acting at point B (x, y)

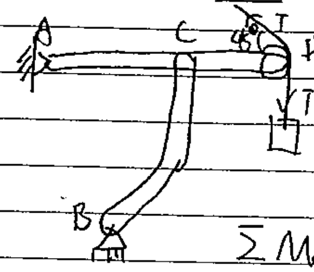
$M_0 = -Fr\cdot y\,i + x\,k\,j = -2i + j$

$\Rightarrow \begin{cases} -Fr\cdot y = -2 \\ x k = 1 \end{cases}$

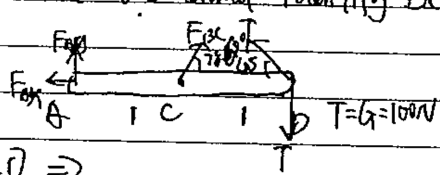
$x^2 + y^2 = 1$ (coz point (x,y) at the quarter-circular plate)

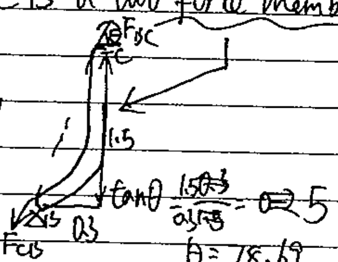
$\Rightarrow Fr = \sqrt{5}$ (direction: down) $\quad x = \frac{1}{\sqrt{5}} \quad y = \frac{2}{\sqrt{5}} \quad \theta = \tan^{-1}\frac{y}{x} = \tan^{-1}2 = 63.43^\circ$

1(b)



First we should identify BC is a two force member.





$\tan\theta = \frac{1.5}{0.3} = 5$

$\theta = 78.69$

$\Sigma M_A = 0 \Rightarrow$

$1 \times F_{BC}\cdot\sin 78.69 + T\cdot\sin 45 \times 2 - T \times 2 = 0$

$\Rightarrow F_{BC} = 59.74\,N.$

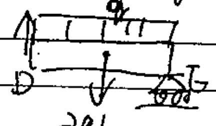
$\Sigma F_y = 0 \Rightarrow F_{Ay} + F_{BC}\cdot\sin 78.69^\circ + T\cdot\sin 45 = T$

$\Rightarrow F_{Ay} = -29.3$ (meaning the force is towards down)

$\Sigma F_x = 0 \Rightarrow F_{Ax} + F_{BC}\cdot\cos 78.69 - T\cdot\cos 45 = 0$

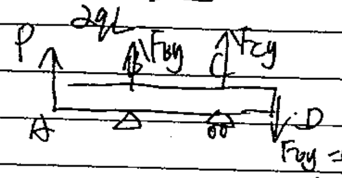
$\Rightarrow F_{Ax} = 59\,N$ (right)

2(a) For this question, we may need to look at DE first.



$\Sigma M_D = 0 \Rightarrow 2qL \times L = F_{Ey} \times 2L$

$\Rightarrow F_{Ey} = qL \quad \Rightarrow F_{Dy} = qL.$



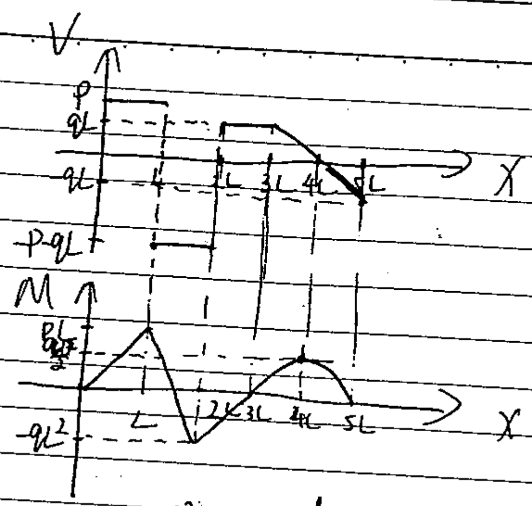
$\Sigma M_B = 0 \Rightarrow -P\cdot L + F_{cy} \times L - \cancel{qL}\,2F_{Dy} \times 2L = 0$

$\Rightarrow F_{cy} = \frac{P\cdot L + qL\,2L}{L} = P + 2qL$

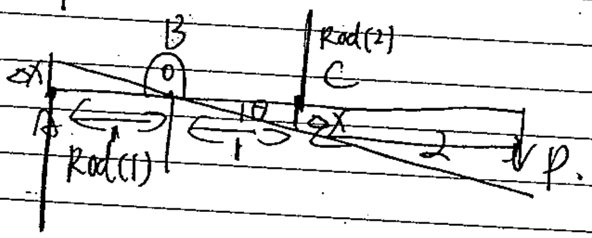
$\Sigma F_y = 0 \Rightarrow F_{By} = -2P - qL$

Now we know all the force acting on those point, we can start to construct the diagram.

2(a)



2(b)



θ is very small.

For this question, the most important thing is know what are the increased part for rod(1), due to the tension, it increases by ΔX
for rod(2), it increases by $\Delta X + 0.5\times10^{-3}$

$\Rightarrow F_{rod(1)} = \dfrac{\Delta X E_1 \cdot A_1}{L_1}$ $\quad F_{rod(2)}\ \dfrac{(\Delta X + 0.5\times10^{-3})E_2 A_2}{L_2}$

$\Sigma M_B = 0 \Rightarrow F_{rod(1)} \times 1 + F_{rod(2)} \times 1 - P\times 3 = 0$

$\Rightarrow \dfrac{\Delta X E_1 A_1}{L_1} + \dfrac{(\Delta X + 0.5\times10^{-3})E_2 A_2}{L_2} = P\times 3$

We know $P = 20\,kN$ $\quad A_1 = A_2 = 200\,mm^2 = 200\times10^{-6}m^2$ $\quad E_1 = 70\,GPa$ $\quad E_2 = 100\,GPa$

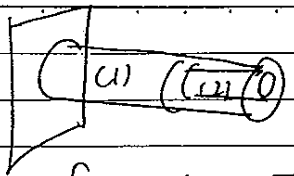
$14000\,\Delta X + 20000(\Delta X + 0.5\times10^{-3}) = 60\,N$

$\Rightarrow \Delta X = 1.47\,mm$

$\Rightarrow \delta_{rod_1} = \Delta X = 1.47\,mm$ $\qquad \delta_{rod_2} = \Delta X + 0.5\,mm = 1.97\,mm$

Date No.

3(a)



for (2). $T_2 = 400\ N\cdot m$ $J_2 = \frac{\pi}{2}(C_o^4 - C_i^4) = \frac{\pi}{2}\left(0.025^4 - \left(\frac{d_i}{2}\right)^4\right)$

$\tau = \frac{T_2 \cdot C_o}{J} = \frac{400 \times 0.025}{\frac{\pi}{2}\left(0.025^4 - \left(\frac{d_i}{2}\right)^4\right)} \leq 35MPa$

$\Rightarrow d_i \leq 42.75\ mm$

$\phi = \frac{T_1 \cdot L_1}{G_1 J_1} + \frac{T_2 \cdot L_2}{G_2 J_2} \leq 0.08$

$G_1 = G_2 = 26GPa$ $J_1 = \frac{\pi}{2}(0.025^4) = 6.136 \times 10^{-7} m^4$ $L_1 = 1m$ $L_2 = 0.8$
$T_1 = 800N\cdot m$

$\Rightarrow \phi = \frac{800 \times 1}{26 \times 10^9 \times 6.136 \times 10^{-7}} + \frac{400 \times 0.8}{26 \times 10^9 \times J_2} \leq 0.08$

$\Rightarrow \frac{400 \times 0.8}{26 \times 10^9 \times \frac{\pi}{2}\left(0.025^4 - \left(\frac{d_i}{2}\right)^4\right)} \leq 0.03$

$\Rightarrow d_i \leq 37.9\ mm$

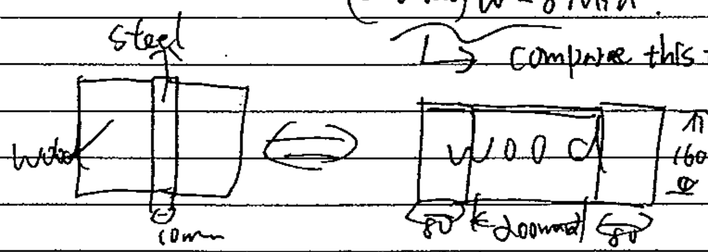
In ~~sto~~ order to satisfy two conditions $\Rightarrow d_i \leq 37.9\ mm$.

3(b)

$n = \frac{E_s}{E_w} = 20$. $(\sigma_{allow})_s = 120MPa \Rightarrow (\sigma_{allow})'_w = \frac{(\sigma_{allow})_s}{n} = \frac{120}{20} = 6MPa$

$(\sigma_{allow})_w = 8MPa$.

compare this two $\sigma_{allow} \leq 6MPa$.



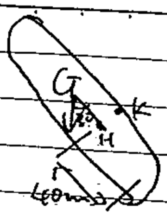
$\Rightarrow I = \frac{1}{12} 360 \times 160^3 = 122.88 \times 10^{-6} m^4$

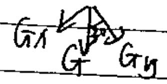
$M_{max} = \frac{\sigma_{allow} \cdot I}{C} = \frac{6MPa \times 122.88 \times 10^{-6}}{0.08} = 9216\ N\cdot m$

$\Rightarrow M_{max} = 9216\ N\cdot m$

Date No.

4(a)





$G_y = 9kN \cdot \cos 30 = 7.79\,kN$ $G_x = 9kN \cdot \sin 30 = 4.5\,kN$

$\sigma_{normal} = \frac{G_y}{A} = \frac{7.79\,kN}{0.024 \times 0.01} = -32.4\,MPa$.

$\sigma_K' = \frac{M \cdot K}{I} = \frac{4.5kN \times 0.02 \times 0.012}{1.3824 \times 10^{-7}} = 7.8\,MPa$, $\sigma_H' = 0$.

$I = \frac{1}{12} bh^3 = \frac{1}{12} 0.01 \times 0.024^3 = 1.3824 \times 10^{-7}\,m^4$

$\tau_H = \frac{3}{2} \cdot \frac{G_x}{A} = -28.1\,MPa$. $\tau_k = 0$.

$\Rightarrow$ $\sigma_K = \sigma_{normal} + \sigma_K' = -24.6\,MPa$ $\tau_k = 0$

$\sigma_H = \sigma_{normal} + \sigma_H' = -32.4\,MPa$ $\tau_H = -28.1$

$\sigma_{max\,1,2\,H} = -16.2 \pm 32.4\,MPa$ $\tau_{max\,H} = 32.4\,MPa$.

$\sigma_{max\,1,2\,K} = 0, 24.6\,MPa$. $\tau_k = 12.3\,MPa$.

(b)

factor of safty $n = 2$.

$\Rightarrow P_{cr} = 2P_{load} = 2 \times 18\,kN = 36\,kN$

$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 E \frac{1}{4}\pi (r_0^4 - (r_0 - t)^4)}{L^2} = 36\,kN$

$L = 1.8\,m$ $r_0 = 0.025\,m$ $E = 72\,GPa$

$\Rightarrow t = 4.4\,mm$

NST Good Luck