NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2008-2009

CV2101 – Mechanics of Materials

November 2008 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains FOUR (4) questions and comprises FIVE (5) pages.
2. Answer ALL FOUR (4) questions.
3. All questions carry equal marks.
4. An Appendix of TWO (2) pages is attached together with this paper.

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1. (a) A force of magnitude 450 N is applied to member ABC which is supported by a frictionless pin at A and by cable BDC as shown in Figure Q1(a). Since the cable passes over a pulley at D, the tension may be assumed to be the same in portion BD and CD of the cable.

   (i) Find the minimum value of α for the cable to remain taut.

   (ii) If α is 45°, determine the tension in the cable and the reaction at A.

   (13 marks)

   (b) The plane truss is loaded and supported as shown in Figure Q1(b).

   (i) Identify the zero force members.

   (ii) Determine the support reactions at C and F.

   (iii) Determine the force in non-zero force members. State whether the members are in tension (T) or compression (C).

   (12 marks)

Note: Question No. 1 continues on page 2

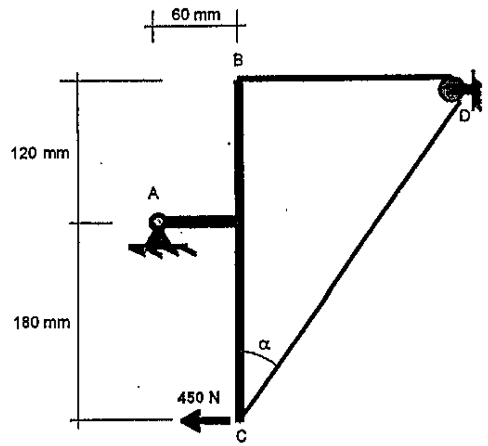


Figure Q1(a)

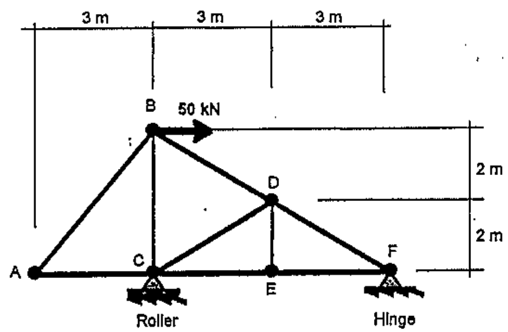


Figure Q1(b)

2. (a) The shaded region shown in Figure Q2(a) lies in the horizontal x-y plane.

Determine:

(i) the location of the centroid of the shaded area.

(ii) the moment of inertia I_x of the shaded area about the x-axis.

(iii) the moment of inertia I_y of the shaded area about the y-axis.

(13 marks)

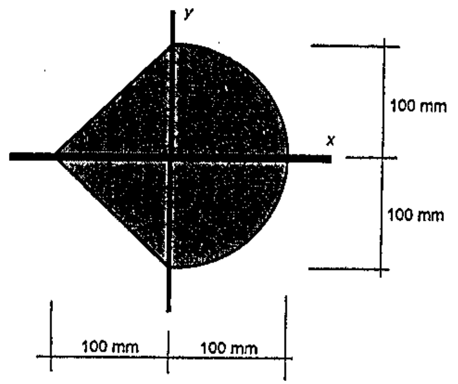


Figure Q2(a)

(b) For the beam ABCD and loading shown in Figure Q2(b),

(i) draw the shear and bending moment diagrams,

(ii) determine the maximum absolute values of the shear and bending moment.

(12 marks)

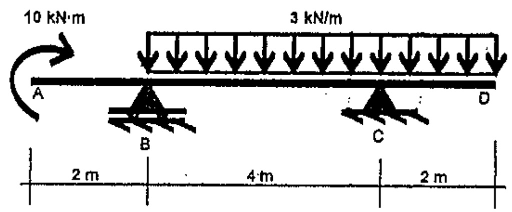


Figure Q2(b)

3. (a) The rigid beam ABC is suspended from two steel rods as shown in Figure Q3(a) and is initially horizontal. The midpoint B of the beam is deflected 10 mm downward by the force P. Knowing that the steel used for the rods has a modulus of elasticity of E = 200 GPa, the cross sectional areas of AD and CE are 400 mm^2 and 500 mm^2, respectively, determine

(i) the required force P.
(ii) the corresponding position of the beam after the force P is applied.

(13 marks)

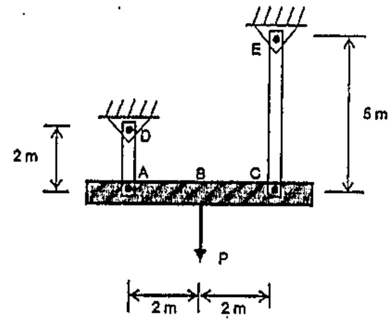


Figure Q3(a)

(b) Two aluminum bars are bonded together to form a layered beam. The cross section of the beam is shown in Figure Q3(b). Knowing that the vertical shear in the beam is 10 kN, determine

(i) the average shear stress at the bonded area.
(ii) the maximum shear stress in the beam.

(12 marks)

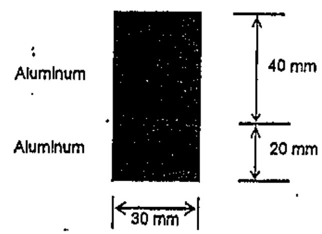


Figure Q3(b)

4 (a) Three forces are applied to the machine component ABD as shown in Figure Q4(a). Knowing that the cross section containing point H is a 20 mm × 40 mm rectangle, determine

(i) the principal stresses and principal directions at point H.
(ii) the maximum shear stress and the associated normal stress at point H.

(13 marks)

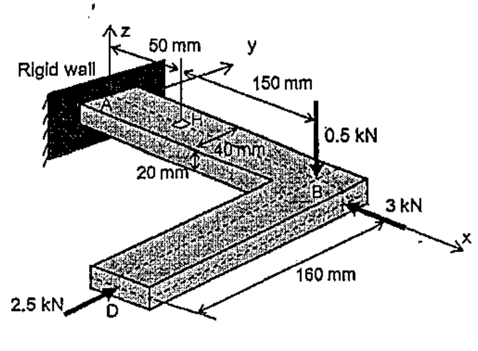


Figure Q4(a)

(b) An aluminum tube CD of circular cross section is pinned at the base and the top with a horizontal beam supporting a load Q=20 kN (see Figure Q4(b)).

Determine the required thickness t of the tube if its outside diameter d is 50 mm and the desired factor of safety with respect to Euler buckling is n=2.5.

Assume that the modulus of elasticity is 70 GPa.

(12 marks)

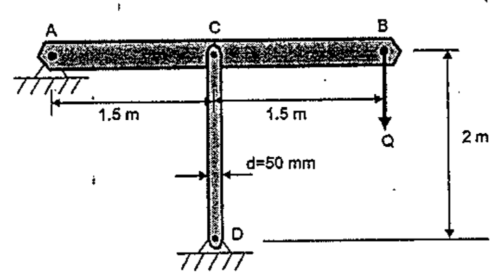
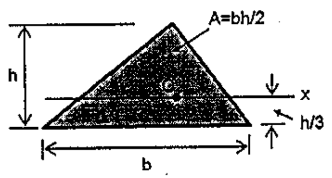


Figure Q4(b)

END OF PAPER

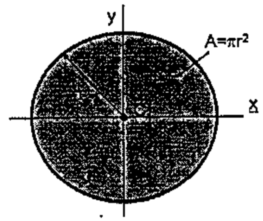
1. Equilibrium
   Particle $\sum F_x = 0,\ \sum F_y = 0,\ \sum F_z = 0$
   Rigid Body – Two dimensions $\sum F_x = 0,\ \sum F_y = 0,\ \sum M_o = 0$

2. Geometric properties of area elements



Triangular

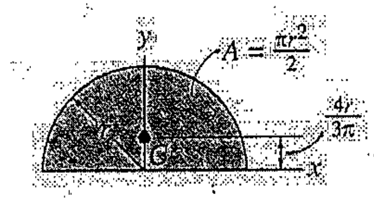
$I_x = \frac{1}{36}bh^3$



Circular area

$I_x = \frac{1}{4}\pi r^4$

$I_y = \frac{1}{4}\pi r^4$



Semicircular area

$I_x = \frac{1}{8}\pi r^4$

$I_y = \frac{1}{8}\pi r^4$

Parallel-Axis Theorem $I = \bar{I} + Ad^2$

3. Axial load
   Normal Stress $\sigma = \frac{P}{A}$
   Displacement $\delta = \sum \frac{PL}{EA}$, $\delta_T = \alpha \Delta T L$

4. Torsion

Shear Stress in Circular Shaft $\tau = \dfrac{T\rho}{J}$

where $J = \dfrac{\pi}{2}c^4$ solid cross section; $J = \dfrac{\pi}{2}(c_o^4 - c_i^4)$ tubular cross section

Angle of Twist $\phi = \sum \dfrac{TL}{GJ}$,

5. Bending

Normal Stress $\sigma = \dfrac{My}{I}$

Unsymmetric Bending $\sigma = -\dfrac{M_z y}{I_z} + \dfrac{M_y z}{I_y}$, $\tan\alpha = \dfrac{I_z}{I_y}\tan\theta$

6. Shear

Average Direct Shear Stress $\tau_{ave} = \dfrac{V}{A}$

Transverse Shear Stress $\tau = \dfrac{VQ}{It}$

Shear Flow $q = \tau \cdot t = \dfrac{VQ}{I}$

7. Stress Transformation Equations

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Principal Stress,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Maximum In-Plane Shear Stress,

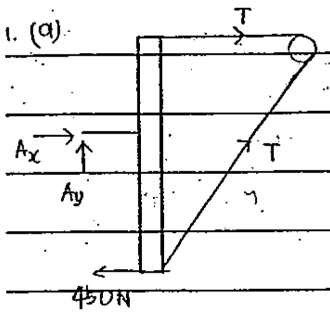
$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}, \quad \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

8. Buckling

Critical Axial Load, $P_{cr} = \dfrac{\pi^2 EI}{(KL)^2}$

1. (a)



(ii) when $\alpha = 45°$,

$\circlearrowleft^{+} \Sigma M_A = 0$;

$-450(1.8) - T(0.12) + T\cos 45°(0.06) + T\sin 45°(0.18) = 0$

$81 = T\cos 45°(0.06) + T\sin 45°(0.18) - 0.12T$

$81 = 0.0497T$

$T = 1620\text{N}$ #

$\Sigma F_y = 0$;

$A_y + T\cos 45° = 0$

$A_y = -1145.5\ \text{N}$

$A_y = 1145.5\ \text{N} \downarrow$ #

$\Sigma F_x = 0$;

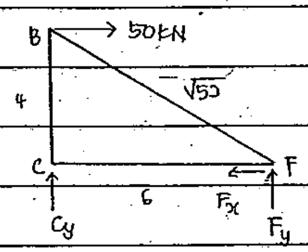
$A_x + T + T\sin 45° - 450 = 0$

$A_x + 1620 + 1620\sin 45° - 450 = 0$

$A_x = 2315.5\ \text{N} \leftarrow$ #

(b) (i) AB, AC, CD and DE are zero force members.

(ii)



$\circlearrowleft^{+} \Sigma M_F = 0$;

$-50(4) - C_y(6) = 0$

$C_y = 33.33\ \text{kN} \downarrow$ #

$\Sigma F_y = 0$;

$F_y - 33.33 = 0$

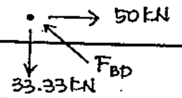
$F_y = 33.33\ \text{kN}$ #

$\Sigma F_x = 0$;

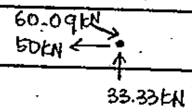
$F_x = 50\ \text{kN}$ #

1.(b) (iii)

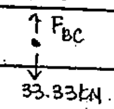
Joint B



Joint F



Joint C



$F_{BD}\left(\frac{6}{\sqrt{52}}\right) = 50$

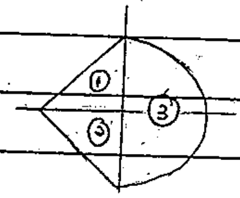
$F_{BD} = 50 \times \frac{\sqrt{52}}{6} = 60.09 \text{ kN (C)}$ ※

$F_{DF} = 60.09 \text{kN (C)}$ ※

$F_{BC} = 33.33 \text{ kN (T)}$ ※

2. (a) (i) Since the region is symmetrical about x-axis, the y-coordinate for its centroid is zero.

segment	A (mm²)	$\tilde{x}$ (mm)	$\tilde{x}A$ (mm³)
1	$\frac{1}{2}\times 100^2 = 5000$	$-\frac{100}{3}$	$-166666\frac{2}{3}$
2	$\frac{1}{2}\times 100^2 = 5000$	$-\frac{100}{3}$	$-166666\frac{2}{3}$
3	$\frac{1}{2}\pi(100)^2 = 15708$	$\frac{4r}{3\pi} = 42.4$	666019.2



Thus, $\bar{x} = \frac{\Sigma \tilde{x}A}{\Sigma A}$

$= \frac{332686}{25708}$

$= 12.94 \text{ mm}$

$\therefore$ coordinate of centroid $= (12.94, 0)$ ※

(ii) For triangle, $I_x = \frac{1}{36} bh^3$

$I_x = \bar{I}_{x'} + A d_y^2$

$= \frac{1}{36}(100)(100^3) + \frac{1}{2}(100)(100)\left(\frac{100}{3}\right)^2$

$= 8.333 \times 10^6 \text{ mm}^4$

For semicircle, $I_x = \frac{1}{8}\pi r^4$

$= \frac{1}{8}\pi(100^4)$

$= 3.927 \times 10^7 \text{ mm}^4$

$\therefore I_x = 2(8.333\times 10^6) + (3.927 \times 10^7) = 5.594 \times 10^7 \text{ mm}^4$ ※

(iii) For triangle, I_x and I_y are the same due to similar geometry with respect to x and y axis.

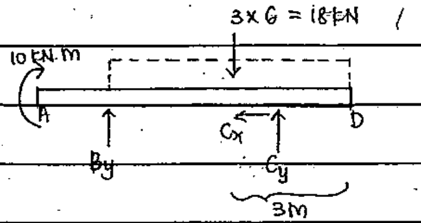
For semicircle, $I_y = \bar{I}_{y'} + A d_x^2$

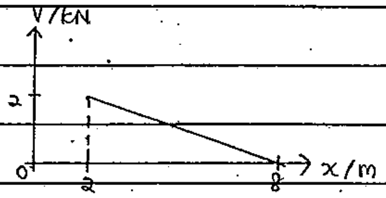
$= \frac{1}{8}\pi(100^4) + \frac{1}{2}\pi(100^2)\left(\frac{4\times 100}{3\pi}\right) = 3.994 \times 10^7 \text{ mm}^4$

$\therefore I_y = 2(8.333\times 10^6) + (3.994 \times 10^7) = 5.661 \times 10^7 \text{ mm}^4$ ※

2. (b)

(i)





$\overset{+}{\circlearrowleft} \Sigma M_C = 0$;

$-10 + 18(1) - B_y(4) = 0$

$B_y = 2\,kN$

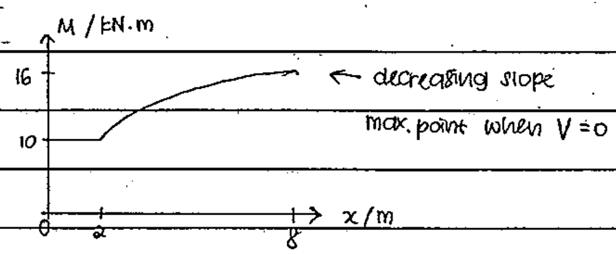
$\Sigma F_y = 0$;

$2 + C_y - 18 = 0$

$C_y = 16\,kN$

$\Sigma F_x = 0$;

$C_x = 0N$



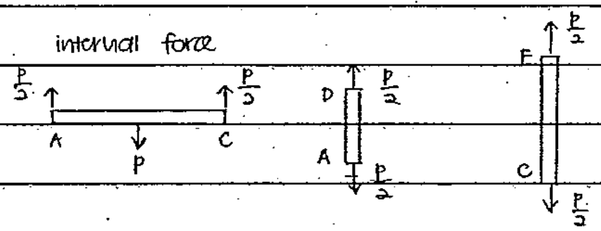
$\Delta M = \int V\,dx = \frac{1}{2}(6)(2) = 6\,kN.m$

$M|_{x=8} = M|_{x=2} + 6 = 18\,kN.m$

(ii) maximum absolute V = 2 kN

maximum absolute M = 16 kN.m

3. (a) (i) internal force



Displacement

Rod AD

$\delta_A = \frac{P_{AD} L_{AD}}{A_{AD} E_{st}}$

$= \frac{\frac{P}{2} \times 2}{400 \times 10^{-6} \times 200 \times 10^{-9}}$

$\delta_A = 1.25 \times 10^{-8}\,P$ m.

Rod CE

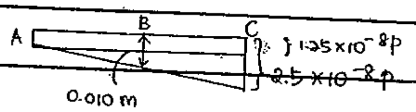
$\delta_C = \frac{P_{CE} L_{CE}}{A_{CE} E_{st}}$

$= \frac{\frac{P}{2} \times 5}{500 \times 10^{-6} \times 200 \times 10^{9}}$

$\delta_C = 2.5 \times 10^{-8}\,P$ m

$\Rightarrow$ to continue

3. (a) (i)



$$\frac{0.01-(1.25\times10^{-8}P)}{2} = \frac{1.25\times10^{-8}P}{4}$$

$$0.04 - 5\times10^{-8}P = 2.5\times10^{-8}P$$

$$7.5\times10^{-8}P = 0.04$$

$$P = 5.33\times10^{5}\ \text{N}$$ ※

(ii)

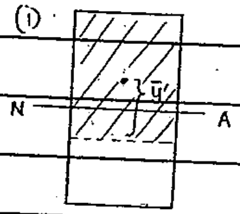
$$\delta_A = 1.25\times10^{-8}\times5.33\times10^{5}$$

$$\delta_A = 0.0067\ \text{m}\quad\text{or}\quad 6.7\ \text{mm}$$ ※

$$\delta_C = 2.5\times10^{-8}\times5.33\times10^{5}$$

$$\delta_C = 0.0133\ \text{m}\quad\text{or}\quad 13.3\ \text{mm}$$ ※

(b) (i)



section properties

$$I = \frac{1}{12}bh^3$$

$$= \frac{1}{12}(0.03)(0.06^3)$$

$$= 5.4\times10^{-7}\ \text{m}^4$$

$$Q = A'\bar{y}'$$

$$= 0.010\times(0.04\times0.03)$$

$$= 1.2\times10^{-5}\ \text{m}^3$$

shear stress

$$\tau = \frac{VQ}{It}$$

$$= \frac{10\times10^{3}\times1.2\times10^{-5}}{5.4\times10^{-7}\times0.03}$$

$$\tau = 7.41\ \text{MPa}$$ ※

(ii) maximum τ occurs at the neutral axis. Since the cross section of the beam is rectangular,

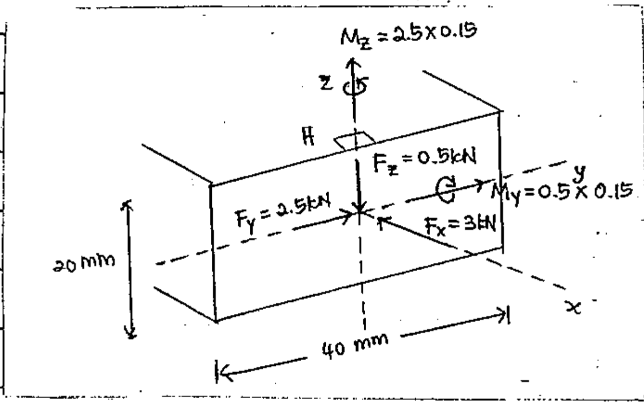
$$\tau_{max} = 1.5\,\frac{V}{A}$$

$$= 1.5\times\frac{10\times10^{3}}{0.03\times0.06}$$

$$= 8.33\ \text{MPa}$$ ※

4. (a)

(i) the internal loadings are as shown.



stress components

① normal force $\sigma_H = \frac{P}{A}$

$= \frac{3\times10^3}{0.04 \times 0.02}$

$\sigma_H = 3.75$ MPa

② shear force due to F_z

As point H is at the top corner of the rectangular cross section, and the shear stress distribution is parabolic with zero shear at both ends and maximum at NA.

$\therefore \tau_H = 0$ Pa due to F_z.

③ shear force due to F_y

$\tau_H = \frac{VQ}{It}$

$I = \frac{1}{12}bh^3 = \frac{1}{12}(0.02)(0.04^3)$ ← be careful of the placement of NA with respect to the force

$= 1.067 \times 10^{-7}$ m^4

$Q = \bar{y}'A' = 0.01 \times (0.02 \times 0.02)$

$= 4 \times 10^{-6}$ m^3

$\tau_H = \frac{2.5\times10^3 \times 4\times10^{-6}}{1.067\times10^{-7} \times 0.02}$

$\tau_H = 4.686$ MPa

④ Bending moment

$\sigma_H = \frac{M_y c}{I}$ with $I = \frac{1}{12}bh^3$

$= \frac{0.5\times10^3 \times 0.15 \times 0.02}{2.667\times10^{-8}}$ $= \frac{1}{12} \times 0.04 \times 0.02^3$

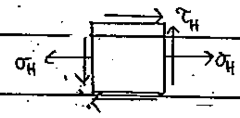
$= 56.25$ MPa $= 2.667 \times 10^{-8}$ m^4

As for M_z component, point H lies on the NA, so the corresponding $\sigma_H = 0$ Pa.

$\therefore$ Total σ_H and τ_H

$\sigma_H = 56.25 - 3.75 = 52.5$ MPa

$\tau_H = 4.686$ MPa

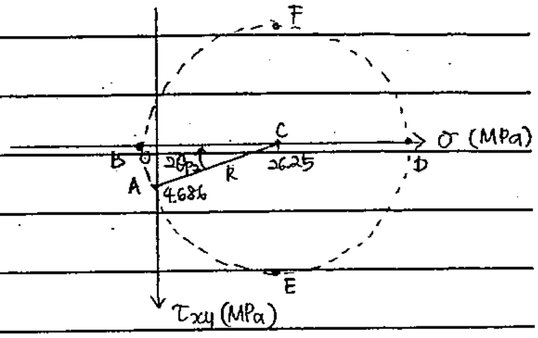


→ to continue.

4. (a) (i)

$\sigma_x = 0$; $\sigma_y = 52.5$ MPa ; $\tau_{xy} = 4.686$ MPa

construction of Mohr's circle



center of circle

$\sigma_{avg} = \frac{0 + 52.5}{2} = 26.25$ MPa

reference point A = (0, 4.686)

radius, R $= \sqrt{26.25^2 + 4.686^2}$

$= 26.66$ MPa

Principal stresses – when $\tau_{xy} = 0$ ⇒ points B and D

$\sigma_1 = 26.25 + 26.66 = 52.91$ MPa ※

$\sigma_2 = 26.25 - 26.66 = -0.41$ MPa ※

$2\theta_{p_2} = \tan^{-1}\frac{4.686}{26.25} = 10.12°$

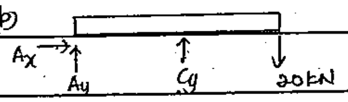
$\theta_{p_2} = -5.06°$ ※

$\theta_{p_1} = 84.94°$ ※

(ii) maximum shear stress, $\tau_{x'y'} = 26.66$ MPa ※

associated normal stress, $\sigma_{avg} = 26.25$ MPa ※

(b)



$P_{cr} = \frac{\pi^2 E I_{cD}}{(K L_{cD})^2}$ K = 1 since both ends are pinned

$\circlearrowleft + \Sigma M_A = 0$;

$100 \times 10^3 = \frac{\pi^2 (70 \times 10^9)(\frac{1}{4}\pi)(0.025 - r_i)^4}{2^2}$

$C_y(1.5) + (-20)(3) = 0$

$C_y = 40$ kN

$0.025 - r_i = \pm 0.0293$

$-r_i = -0.0293 - 0.025$

$P_{cr} = P_{allow} \times F.S$

$r_i = 0.0543$ m

$= 40 \times 2.5$

$\therefore t = 0.0543 - 0.025 = 0.0293$ or 29.3 mm ※

$P_{cr} = 100$ kN