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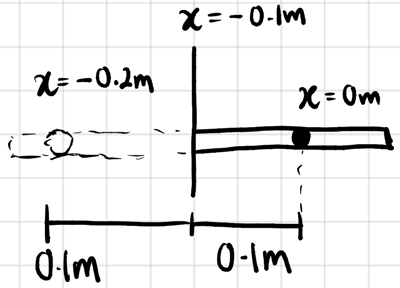
EN3003 AY2021-2022 sem 2

1ai) $x = -0.2m$



1aii) $m = 10^{-5} g = 10^{-8} kg$

$t = 200 \text{ days} = 200 \times 24 \times 60^2 = 1.728 \times 10^7 s$

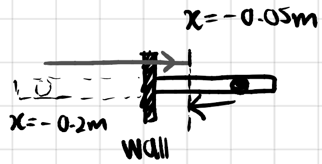
$A = \left(\frac{0.05}{2} \times 10^{-3}\right)^2 \times \pi = 1.96 \times 10^{-9} \text{ kg m}^{-3}$

$M = \frac{10^{-8}}{1.96 \times 10^{-9}} = 5.102 \text{ kg m}^{-2}$

$$c(x,t) = \frac{M}{\sqrt{4\pi Dmt}} e^{\frac{-(x-x_0)^2}{4Dmt}}$$

$4Dmt = 4(10^{-9})(1.728 \times 10^7)$

$= 0.06912$



$C(-0.05, 1.728 \times 10^7) = C_{real} + C_{image}$

$$= \frac{5.102}{\sqrt{0.06912 \times \pi}} e^{-\frac{(0.05-0)^2}{0.06912}} + \frac{5.102}{\sqrt{0.06912 \times \pi}} e^{-\frac{(0.15-0)^2}{0.06912}}$$

$= (10.949 \times 0.96448) + (10.949 \times 0.72215)$

$= 10.56 + 7.9068 = 18.467 \approx 18.5 \text{ kg m}^{-3}$ //

1aiii) Max concentration occurs at $x = 0\,m$

$C(0, 1.728 \times 10^{7}) = 10.949$

Assuming $\rightarrow$ positive

$J_m'{}_{real} = -D_m A \frac{dc}{dx}$

<u>Real</u>

$$\frac{dc}{dx} = \frac{10.56 - 10.949}{-0.05 - 0} = 7.78$$

$$J_m'{}_{real} = -10^{-9}(1.96 \times 10^{-9})(7.78) = -1.5249 \times 10^{-17}$$

<u>Image</u>

$$\frac{dc}{dx} = \frac{7.9068 - 10.949}{-0.05 - (-0.2)} = -20.281$$

$$J_m'{}_{image} = -10^{-9}(1.96 \times 10^{-9})(-20.281) = 3.9751 \times 10^{-17}$$

Overall flux: $10^{-17}(3.9751 + (-1.5249))$

$= 2.4502 \times 10^{-17} \approx 2.45 \times 10^{-17}\ kg\ s^{-1}$

Direction: Mass flux is travelling from left to right.

2bi)

$$L = L_o e^{-\frac{k}{u}x}$$

$$= 500\, e^{-0.2(5)}$$

$$= 183.94 \approx 184 \text{ mg/L}.$$

(2bii) $\frac{k_1}{k_2} = \beta^{(T_1 - T_2)}$

$$\frac{k_1}{0.2} = 1.065^{(25-20)}$$

$$k_1 = 0.27402$$

First 2 days 20°C , next 3 days 25°C .

$$L = L_o e^{-k_2\left(\frac{x}{v}\right)}$$

$$L = 500\, e^{-0.2(2)} = 335.16$$

At start of day 3, $L = 335.16$ mg/l.

$$L = 335.16\, e^{-0.27402(3)} = 147.31$$

2ai)

discharge location $\Rightarrow$ channel

$DO = 1 mg/L$

$BOD = 30 mg/L$

$Q = 0.05 m^3/s$

$DO = 6 mg/L$

$BOD = 20 mg/L$

$Q = 0.5 m^3/s$

$DO_{sat} = 8.4 mg/L$ $u = 0.1 m/s$

$K_1 = 0.15/d$ $K_2 = 0.174/d$

$= 1.7361 \times 10^{-6}$ $= 2.0139 \times 10^{-6}$

$$x_c = \frac{u}{K_2 - K_1} \ln\left\{\frac{K_2}{K_1}\left[1 - \left(\frac{K_2}{K_1} - 1\right)\left(\frac{D_o}{L_o}\right)\right]\right\}$$

$$L_o = \frac{30(0.05) + 20(0.5)}{0.05 + 0.5} = 20.909$$

$$DO_o = \frac{1(0.05) + 6(0.5)}{0.05 + 0.5} = 5.5455$$

$$D_o = DO_{sat} - DO_o = 8.4 - 5.5455 = 2.8545$$

$$\frac{D_o}{L_o} = \frac{2.8545}{20.909} = 0.13652$$

$$\frac{K_2}{K_1} = \frac{2.0139 \times 10^{-6}}{1.7361 \times 10^{-6}} = 1.16$$

$$\frac{u}{K_2 - K_1} = \frac{0.1}{10^{-6}(2.0139 - 1.7361)} = 359971$$

$$x_c = 359971 \ln\{1.16[1 - (1.16 - 1)(0.13652)]\}$$

$$= 359971 \ln(1.1347) = 45489$$

ans: 45489m

(2aii)

$$D = D_o e^{-\frac{k_1}{u}x} + \frac{k_1 L_o}{k_2 - k_1}\left(e^{-\frac{k_1}{u}x} - e^{\frac{-k_2}{u}x}\right)$$

$$\frac{x}{u} = \frac{45489}{0.1} = 454890$$

$$e^{-k_1\left(\frac{x}{u}\right)} = e^{-1.7361\times10^{-6}(454890)} = 0.45397$$

$$e^{-k_2\left(\frac{x}{u}\right)} = e^{-2.0139\times10^{-6}(454890)} = 0.40008$$

$$\frac{k_1 L_o}{k_2 - k_1} = \frac{1.7361\times10^{-6} \times 20.909}{10^{-6}(2.0139 - 1.7361)} = 130.67$$

$$D = 2.8545(0.45397) + 130.67(0.45397 - 0.40008)$$

$$= 8.3377 \text{ (5sf)}$$

$$DO = DO_{sat} - D = 8.4 - 8.3377 = 0.0623 \text{ mg/L}$$ //

(2aiii) $$L = L_o e^{-\frac{k}{u}x} = 20.909 e^{-1.7361\times10^{-6}(454890)}$$

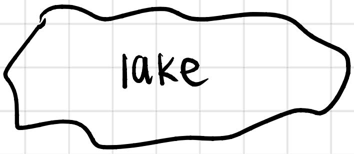
$$= 9.492 \approx 9.49 \text{ mg/L}$$ //

(2bi)

$P^s = 1.25\,g\,s^{-1}$ ↓

$(Q^s \ll Q^\circ)$

$Q^\circ = 1.02\,m^3\,s^{-1}$ →



$P^\circ = 23\,ppb$
$= 23\,Mg/L$
$= 0.023\,g\,m^{-3}$

$SA = 8.9 \times 10^5\,m^2$, depth $= 9m$

$P = 13.2\,ppb = 13.2\,Mg/L$
$= 0.0132\,g\,m^{-3}$

$$P = \frac{Q^\circ P^\circ + Q^s P^s}{Q^\circ + K_s V}$$

$$0.0132 = \frac{1.02(0.023) + 1.25}{1.02 + K_s(8.9 \times 10^5 \times 9)}$$

$0.013464 + 105732 K_s = 0.02346 + 1.25$

$105732\,K_s = 1.26$

$K_s = 1.1917 \times 10^{-5}$

$= 1.19 \times 10^{-5}\,s^{-1}$ //

(2bii) Fraction of P remaining in lake $= \dfrac{1}{1 + K_s\theta}$

$$\theta = \frac{V}{Q^\circ} = \frac{8.9 \times 10^5 \times 9}{1.02} = 7852941$$

$$\frac{1}{1 + K_s\theta} = \frac{1}{1 + (7852941)(1.1917 \times 10^{-5})} = 0.010573$$

Fraction of P removed by lake $= 1 - 0.010573 = 0.98943$

≈ 0.989 //

(Q3a)

case : pulse input without chemical reaction

$$t = \frac{L}{V} = \frac{10(10^3)}{0.1} = 100\,000\,s$$

$$\approx 1.1574 \text{ day} \approx 1.16 \text{ d}$$ //

(Q3b)

$$\frac{C}{C_0} = e^{-\frac{t}{\bar{t}}}$$

$$\bar{t} = \frac{V}{Q} = \frac{40\,000}{0.1} = 400\,000\,s$$

$$\frac{C}{C_0} = 0.1$$

$$0.1 = e^{-t/400\,000}$$

$$t = \ln(0.1)(-400\,000)$$

$$= 921034\,s \approx 10.66 \text{ days} \approx 10.7 \text{ d}$$ //

(Q3c)

In a non-ideal CMFR model, the pollutant molecules do not all have the same residence time due to reasons such as channeling of flow, short-circuiting, dead zones, restricted inlet or outlet effects, which happens in a naturally mixed lake.

(Q4a) $K_p = 8000 \times 1.75 = 14000$ //

(Q4b) $$\frac{1}{F_{solid}} = 1 + \frac{V_W}{V_{solid}} \frac{1}{K_p} + \frac{V_{gas}}{V_{solid}} \frac{H_{cc}}{K_p}$$

$$= 1 + \frac{5\times10^5}{45} \frac{1}{14000} + \frac{5\times10^9}{45} \frac{0.00275}{14000} = 23.619$$

$$F_{solid} = (23.619)^{-1} = 0.042339 \approx 0.0423$$

$$\frac{1}{F_{water}} = \frac{V_{solid} K_p}{V_W} + 1 + \frac{V_{gas} H_{cc}}{V_W} = \frac{45(14000)}{5\times10^5} + 1 + \frac{5\times10^9 \times 0.00275}{5\times10^5}$$

$$= 29.76$$

$$F_{water} = (29.76)^{-1} = 0.033602 \approx 0.0336$$

$$\frac{1}{F_{gas}} = \frac{V_{solid} K_p}{V_{gas} H_{cc}} + \frac{V_W}{V_{gas} H_{cc}} + 1 = \frac{45(14000)}{5\times10^9 \times 0.00275} + \frac{5\times10^5}{5\times10^9 \times 0.00275} + 1$$

$$= 1.0822$$

$$F_{gas} = (1.0822)^{-1} = 0.92404 \approx 0.924$$

$F_{solid} = 0.0423$, $F_{water} = 0.0336$, $F_{gas} = 0.924$

(Q4C)

$$F_{fauna} = \frac{V_{fauna} C_{fauna}}{V_{solid} C_{solid} + V_{water} C_{water} + V_{air} C_{air} + V_{fauna} C_{fauna}}$$

(5a)

1. Water softerner
2. Copper pipe erosion

(5bi) $Sh = \frac{1.09}{\varepsilon} Re^{0.333} Sc^{0.333}$, $Sh = \frac{kd}{D}$

$Sc = \frac{M}{\rho D} = \frac{0.0131}{(1)(5\times10^{-6})} = 2620$

$V = 100 \text{ cm d}^{-1}$
$= 0.0011574 \text{ cm s}^{-1}$

$Re = \frac{\rho \varepsilon v d}{M} = \frac{(1)(0.3)(1.1574\times10^{-3})(1)}{0.0131} = 0.026505$

$Sh = \frac{1.09}{0.3} (0.026505)^{0.333} (2620)^{0.333}$

$= 14.913$

$Sh = \frac{kd}{D}$

$k = \frac{14.913(5\times10^{-6})}{1} = 7.4565\times10^{-5} \text{ cm s}^{-1}$ (shown, k is between $6-8\times10^{-5} \text{ cm s}^{-1}$)

(5bii) specific surface area of the bed considering pore volume only

$$a = \frac{S}{\varepsilon V}$$

160 napthalene spheres per m^3

$1 m^3 \Rightarrow 10^6 cm^3$

$$S = 160 \pi d^2 = 160(\pi)(1)^2 = 502.65$$

$$\varepsilon V = 0.3 \times 10^6 = 300\,000$$

$$a = \frac{502.65}{300\,000} = 0.0016755$$

$$= 0.00168 \ cm^2 / cm^3$$

(5biii)

$$\ln\left(\frac{C_s - C_x}{C_s - C_I}\right) = \frac{-kax}{V} \quad , \quad C_I = 0$$

$$\ln\left(\frac{C_s - C_x}{C_s}\right) = \frac{-7.4565 \times 10^{-5} \times 0.00168 \times 10 \times 100}{0.0011574}$$

$$\frac{C_s - C_x}{C_s} = e^{-0.10823}$$

$$C_s - C_x = 0.89742 C_s$$

$$C_x = C_s - 0.89742 C_s$$

$$C_x = 0.10528 C_s \approx 0.105 C_s$$