CV1012 Semester 1 2022-2023 Examination

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1\. (a) $\rho_w g d_1 + \rho_w g (0.1) - \rho_e g (0.9) = 0$

$$d_1 = \frac{\rho_e}{\rho_w}(0.9) - 0.1$$

$$d_1 = 0.89 \text{ m}$$

(b) By Archimedes' principle,

$$\boxed{\text{weight of buoy} = \text{weight of water displaced}}$$

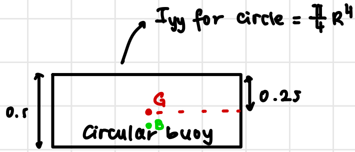
$$1000 = \rho_w g V_{displaced}$$

$$\cancel{1000} = \cancel{1000}\, g V_{displaced}$$

$$V_{displaced} = 0.102 \text{ m}^3$$

$$BM = \frac{I_{yy}}{V_{displaced}} = \frac{\pi R^4}{4} \times \frac{1}{V_{displaced}}$$

$$\boxed{BM = \frac{\pi \times 0.6^4}{4 \times 0.102} = 0.997}$$



$$\text{Submerged height} = \frac{0.102}{\pi \times 0.6^2} = 0.0902 \text{ m}$$

$$\boxed{BG = 0.25 - \frac{0.0902}{2} = 0.2049}$$

$\therefore$ Since $BM > BG$, the buoy is stable.

(c) Weight of buoy + $W_1 = \rho_w g V_{displaced}$

$$1000 + 2000 = (1000)(9.81)(V_{displaced})$$

$$3000 = (9810)(V_{displaced})$$

$$V_{displaced} = 0.3058 \text{ m}^3$$

$$BM = \frac{\pi R^4}{4} \times \frac{1}{V_{displaced}} = \frac{\pi (0.6)^4}{4} \times \frac{1}{0.3058}$$

$$\boxed{BM = 0.3329}$$

Calculating centre of gravity of the circular buoy + weight W_1:

$$CG = \frac{(1000)(0.25) + 2000(0.5 + 0.05)}{1000 + 2000} = 0.45 \text{ m (from base)}$$

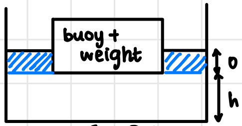
$$\text{Submerged height} = \frac{V_{displaced}}{\text{Base area}} = \frac{0.3058}{\pi \times 0.6^2} = 0.27 \text{ m}$$

$$\boxed{BG = 0.45 - \frac{0.27}{2} = 0.315}$$

$\therefore$ Since $BM > BG$, the buoy with the weight on top is stable.

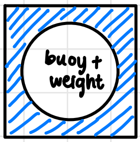
Initial volume of water in 1(a) $= 2 \times 2 \times d_1 = 3.56 \text{ m}^3$

We know that the submerged height of the buoy is 0.27 m. Therefore, we can calculate the volume of water from the surface of the water down to a depth of 0.27 m, which is the same height as the base of the buoy.



SIDE VIEW

→ we can calculate the amount of water on this level



TOP VIEW

$$V_{0-2.7} = (\text{Water surface area})(0.27)$$
$$= (4 - \pi \times 0.6^2)(0.27)$$
$$= 0.775 \text{ m}^3$$

Since the volume of water does not change, the volume of water below the depth of 2.7 m is $(3.56 - 0.775)\,\text{m}^3$

$$3.56 - 0.775 = \text{Tank base area} \times h$$
$$2.785 = 4h$$
$$h = 0.696 \text{ m}$$

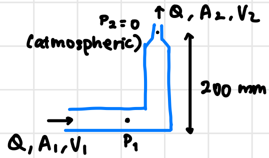
$$\therefore d_3 = h + 0.27 = 0.966 \text{ m}$$

To calculate H_3 and L_3:

$$\rho_w g d_3 + \rho_w g(0.1 + \Delta y) - \rho_e g(0.8 + \Delta y + 0.1 + \Delta y)$$
$$9466.8 + 980 + 9810\Delta y - 9702 - 21560\Delta y = 0$$
$$11750\Delta y = 744.8$$
$$\Delta y = 0.063 \text{ m}$$

$$\therefore H_3 = 0.8 + \Delta y = 0.863 \text{ m}$$
$$L_3 = 0.1 + \Delta y = 0.163 \text{ m}$$

② (a)



By Continuity Equation:

$$A_1 V_1 = A_2 V_2$$

$$\cancel{\pi}(50)^2 \, V = \cancel{\pi}(15)^2 \, V_2$$

$$V_2 = \frac{100}{9} V$$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

$$\frac{10^4}{9800} + \frac{V^2}{2g} + 0 = 0 + \frac{\left(\frac{100}{9}V\right)^2}{2g} + 0.2$$

$$6.241 V^2 = 0.8204$$

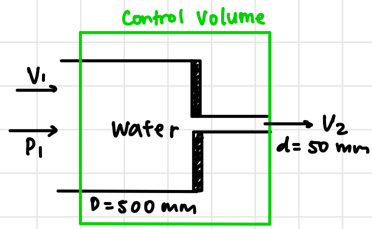
$$V = 0.3626 \text{ m/s}$$

$$Q = \pi(0.015)^2 \left(\frac{100}{9}\right)(0.3626) = 0.0284 \text{ m}^3/\text{s}$$

(b) $Q = 0.1 \text{ m}^3/\text{s}$

$$V_1 = \frac{Q}{\frac{\pi}{4}D^2} = 0.509 \text{ m/s}$$

$$V_2 = \frac{Q}{\frac{\pi}{4}d^2} = 50.93 \text{ m/s}$$



$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z$$

$$\frac{P_1}{9800} + \frac{0.509^2}{2g} + 0 = 0 + \frac{50.93^2}{2g} + 0$$

$$P_1 = 1.295 \times 10^6 \text{ Pa}$$

$$F - P_1\left(\pi \times \frac{D^2}{4}\right) = \rho Q (V_2 - V_1)$$

$$F = (1295000)\left(\pi \times \frac{0.5^2}{4}\right) + 1000(0.1)(50.93 - 0.509)$$

$$F = 259 \text{ kN}$$

③ (a) By Froude's Number Similarity,

$$\frac{V_m}{\sqrt{g L_m}} = \frac{V_p}{\sqrt{g L_p}}$$

$$\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}}$$

By Reynold's Number Similarity,

$$\frac{V_m L_m}{\nu_m} = \frac{V_p L_p}{\nu_p}$$

$$\frac{\nu_m}{\nu_p} = \frac{V_m L_m}{V_p L_p}$$

For the model study to satisfy both the Reynold's and Froude's number similarity, we use the velocity scale obtained from Froude's number in the fluid kinematic viscosity scale obtained using the Reynold's number similarity.

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Therefore, $\frac{\forall_m}{\forall_p} = \sqrt{\frac{L_m}{L_p}} \times \frac{L_m}{L_p}$

$$\frac{\forall_m}{\forall_p} = \left(\frac{L_m}{L_p}\right)^{3/2} \quad \text{(PROVEN)}$$

(b) By Froude Number's Similarity, $\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}}$

$$\frac{Q_m}{Q_p} = \frac{A_m V_m}{A_p V_p} = \left(\frac{L_m}{L_p}\right)^2 \left(\frac{V_m}{V_p}\right)$$

$$= \left(\frac{L_m}{L_p}\right)^2 \left(\sqrt{\frac{L_m}{L_p}}\right)$$

$$= \left(\frac{L_m}{L_p}\right)^{5/2}$$

Since $\frac{L_m}{L_p} = \frac{1}{25}$, $\frac{Q_m}{Q_p} = \left(\frac{1}{25}\right)^{5/2}$

$$Q_m = \left(\frac{1}{25}\right)^{5/2} (200)$$

$$Q_m = 0.064 \ m^3/s$$

(c) (i) At (b) the pump was operating at speed of 1000 rpm when $Q_{(c)} = \frac{1}{2} Q_{(b)}$ and the same pump is used $(D_{(b)} = D_{(c)})$

$$\frac{Q_{(c)}}{\omega_{(c)} \cancel{D_{(c)}^3}} = \frac{Q_{(b)}}{\omega_{(b)} \cancel{D_{(b)}^3}}$$

$$\frac{\frac{1}{2}\cancel{Q_{(b)}}}{\omega_{(c)}} = \frac{\cancel{Q_{(b)}}}{1000}$$

$$\omega_{(c)} = 500 \text{ rpm}$$

(ii) $$\frac{\cancel{g} h_{(b)}}{\omega_{(b)}^2 \cancel{D_{(b)}^2}} = \frac{\cancel{g} h_{(c)}}{\omega_{(c)}^2 \cancel{D_{(c)}^2}}$$

$$h_{(c)} = \left(\frac{\omega_{(c)}}{\omega_{(b)}}\right)^2 h_{(b)}$$

$$h_{(c)} = \left(\frac{500}{1000}\right)^2 h_{(b)}$$

$$h_{(c)} = \frac{1}{4} h_{(b)}$$

$\therefore$ At the new speed of 500 rpm, the operating head of the pump will be $\frac{1}{4}$ of the operating head when the speed is 1000 rpm. This result is obtained with the assumption that gravity remains constant during both situations.

④ (a) $\varepsilon = 0.2 \text{ mm} \qquad D = 0.5 \text{ m}$

$$\frac{\varepsilon}{D} = \frac{0.0002}{0.5} = 4 \times 10^{-4}$$

↳ From the Moody Diagram, for wholly turbulent flow regime $\boxed{f = 0.016}$ when $\frac{\varepsilon}{D} = 4 \times 10^{-4}$

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_f + h_{entr.} + h_{exit}$$

$$0 + 0 + 38 = 0 + 0 + 8 + \frac{fLV^2}{2gD} + 0.5\frac{V^2}{2g} + \frac{V^2}{2g}$$

$$30 = \frac{V^2}{2g}\left(\frac{fL}{D} + 1.5\right)$$

$$30 = \frac{V^2}{2g}\left(\frac{(0.016)(200)}{0.5} + 1.5\right)$$

$$V = 8.6273 \text{ m/s}$$

$$Q = \frac{\pi}{4}D^2 V = 1.69 \text{ m}^3/\text{s}$$

(b) $8 + H = 38 + h_f$

$$8 + 40 - 24Q^2 = 38 + \frac{8fLQ^2}{g\pi^2 D^5}$$

$$10 = Q^2\left(24 + \frac{8fL}{g\pi^2 D^5}\right)$$

$$Q^2 = 0.30798$$

$$Q = 0.55 \text{ m}^3/\text{s}$$

(c)