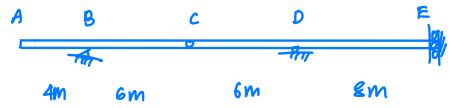
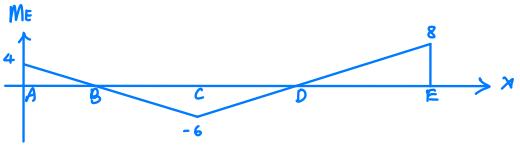
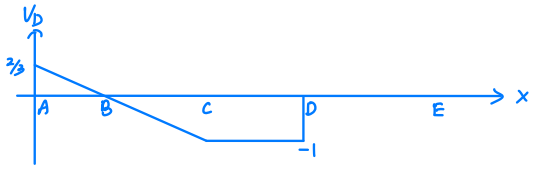
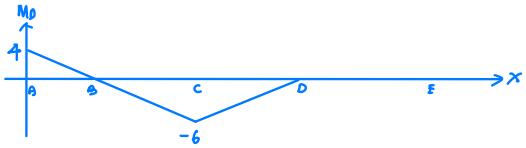
By Tan Yi Xun

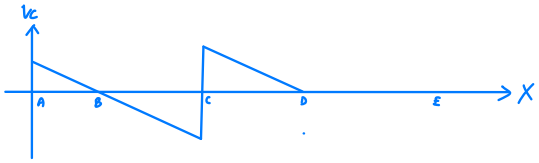
1. (a)

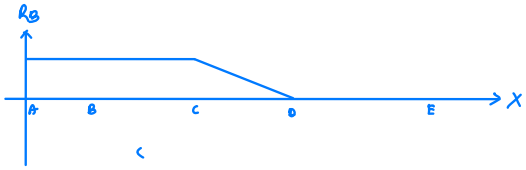




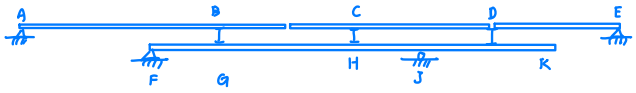


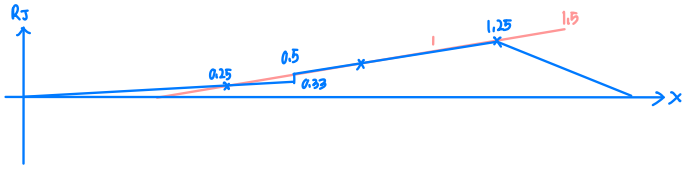


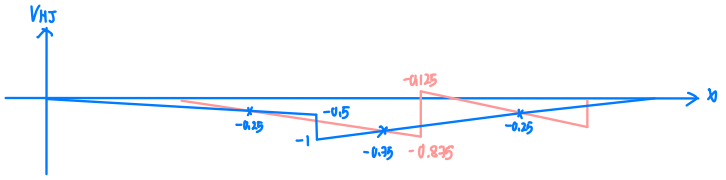


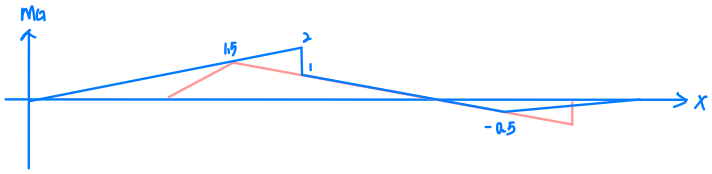


(b)

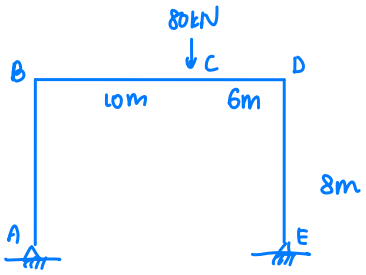




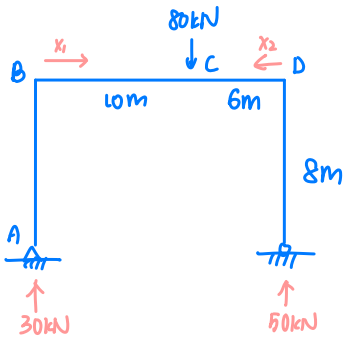


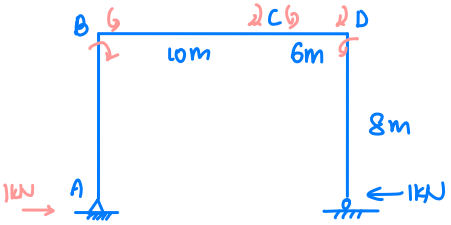


2

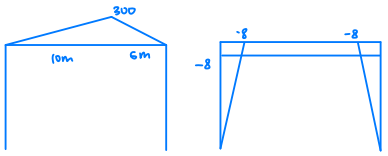


(a)





→ $A_y = 30kN(\uparrow)$, $E_y = 50kN(\uparrow)$ alternative : BMD



→ $M_1 = 30x \qquad m_1 = -8$

$M_2 = 50x \qquad m_2 = -8$

$$\Delta_{EE} = \int_0^{10} -8(30x)\frac{dx}{EI} + \int_0^6 -8(50x)\frac{dx}{EI}$$

$$= \frac{-12000 - 7200}{EI} = \frac{-19200}{EI}$$

from $\int_0^L mm'\,dx$

$\Delta_{EE} = \frac{1}{2}(300)(-8)(10)/EI + \frac{1}{2}(300)(-8)(6)/EI$

$= -19200/EI$

$f_{EE} = (-8)(-8)(16)/EI + 2(\frac{1}{3})(-8)(-8)(8)/EI$

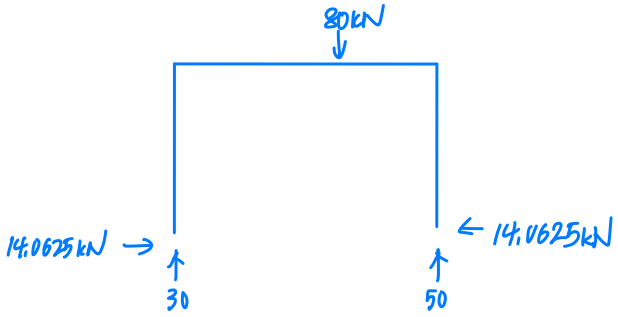
$= 1356.3333/EI$

→ $$f_{EE} = 2\int_0^8 x^2\frac{dx}{EI} + \int_0^{10} 8^2\frac{dx}{EI} + \int_0^6 8^2\frac{dx}{EI}$$

$$= \frac{1356.3333}{EI}$$

→ $\Delta_{EE} + f_{EE} \cdot E_x = 0$

$E_x = 14.0625kN$ (leftward)



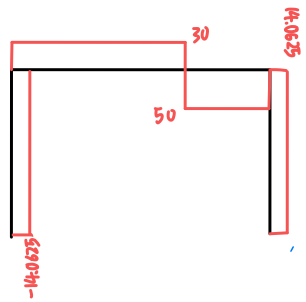
∴ $E_x = 14.0625kN$ (leftward)

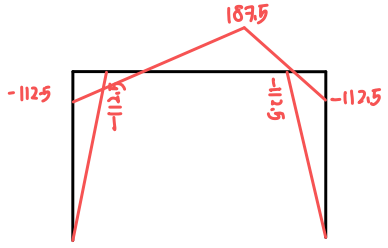
$A_x = 14.0625kN$ (rightward)

$E_y = 50kN$ (upward)

$A_y = 30kN$ (upward)

(b)





3.

pin-supported end span : BD

overhang end span : BC

(a) → $(FEM)_{AB} = -\frac{40(6)^2}{12} = -120\,kNm$

$(FEM)_{BA} = \frac{40(6)^2}{12} = 120\,kNm$

$(FEM)'_{BD} = \frac{-40\left[4x2^2 + \frac{1}{2}(4^2)(2)\right]}{(4+2)^2} = -35.56\,kNm$

$(FEM)'_{BC} = 40(6) = -240\,kNm$

→ $M_{BC} = (FEM)'_{BC} = -240\,kNm$

$M_{AB} = \frac{2EI}{6}(\theta_B) - 120$

$M_{BA} = \frac{2EI}{6}(2\theta_B) + 120$

$M_{BD} = \frac{3EI}{6}\theta_B - 35.56$

$M_{BC} + M_{BA} + M_{BD} = 0$

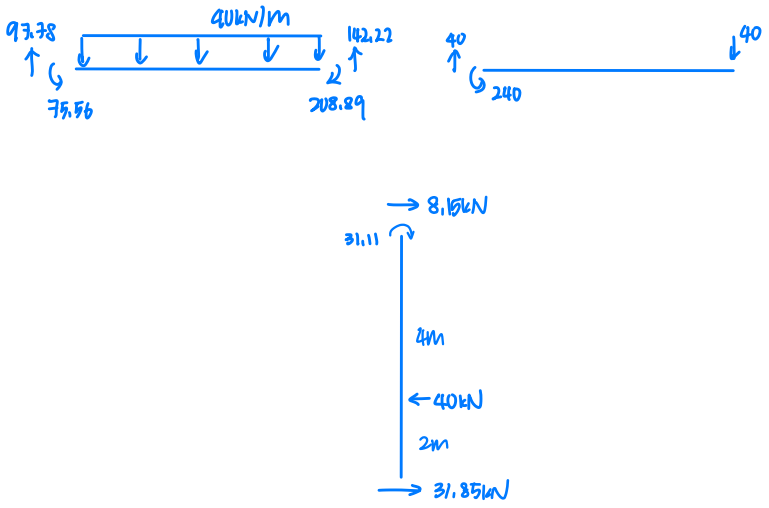
$\theta_B = 133.33/EI$

$M_{AB} = -75.56\,kNm$

$M_{BA} = 208.89\,kNm$

$M_{BD} = 31.11\,kNm$

(b)

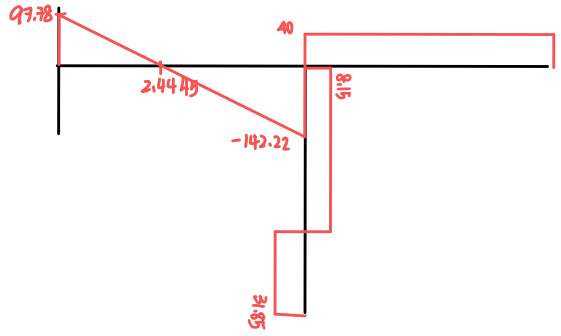


$M_{AB} = -75.56\,kNm$ $M_{BA} = 208.89\,kNm$ $M_{BD} = 31.11\,kNm$

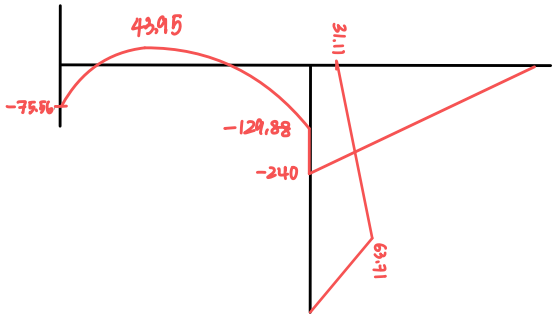
$M_{BC} = -240\,kNm$ $V_{AB} = 97.78kN$ $V_{BA} = 142.22kN$

$V_{BD} = 8.15kN$ $V_{DB} = 31.85kN$ $V_{BC} = 40kN$

(c) SFD



BMD



$L_1 = 6m$, $L_2 = 4m$, $F = 12kN$, $w = 9kN/m$

(a) when $L_1 = 6m$ $12/L^3 = 1/18$ $6/L^2 = 1/6$ $4/L = 2/3$ $2/L = 1/3$

when $L_2 = 4m$ $12/L^3 = 3/16$ $6/L^2 = 3/8$ $4/L = 1$ $2/L = 1/2$

$$\begin{bmatrix} V_{1A} \\ M_{1A} \\ V_{1B} \\ M_{1B} \end{bmatrix} = EI \begin{bmatrix} 1/18 & 1/6 & -1/18 & 1/6 \\ 1/6 & 2/3 & -1/6 & 1/3 \\ -1/18 & -1/6 & \boxed{1/18} & \boxed{-1/6} \\ 1/6 & 1/3 & \boxed{-1/6} & \boxed{2/3} \end{bmatrix} \begin{bmatrix} V'_A \\ \theta'_A \\ V'_B \\ \theta'_B \end{bmatrix} + \begin{bmatrix} P \\ 2PL_1/9 \\ P \\ -2PL_1/9 \end{bmatrix}$$

$P = 12$

$2PL_1/9 = 16$

$wL_2/2 = 18$

$wL_2^2/12 = 12$

$$\begin{bmatrix} V_{2B} \\ M_{2B} \\ V_{2C} \\ M_{2C} \end{bmatrix} = EI \begin{bmatrix} \boxed{3/16} & \boxed{3/8} & -3/16 & 3/8 \\ \boxed{3/8} & \boxed{1} & -3/8 & 1/2 \\ -3/16 & -3/8 & 3/16 & -3/8 \\ 3/8 & 1/2 & -3/8 & 1 \end{bmatrix} \begin{bmatrix} V'_B \\ \theta'_B \\ V'_C \\ \theta'_C \end{bmatrix} + \begin{bmatrix} wL/2 \\ wL^2/12 \\ wL/2 \\ -wL^2/12 \end{bmatrix}$$

(a)
$$\begin{bmatrix} V_A \\ M_A \\ V_B \\ \cancel{M_B}^{\,0} \\ V_C \\ M_C \end{bmatrix} = \begin{bmatrix} 1/18 & 1/6 & -1/18 & 1/6 & 0 & 0 \\ 1/6 & 2/3 & -1/6 & 1/3 & 0 & 0 \\ -1/18 & -1/6 & 35/144 & 5/24 & -3/16 & 3/8 \\ 1/6 & 1/3 & 5/24 & 5/3 & -3/8 & 1/2 \\ 0 & 0 & -3/16 & -3/8 & 3/16 & -3/8 \\ 0 & 0 & 3/8 & 1/2 & -3/8 & 1 \end{bmatrix} \begin{bmatrix} \cancel{V_A}^{\,0} \\ \cancel{\theta'_A}^{\,0} \\ \cancel{V'_B}^{\,0} \\ \theta'_B \\ \cancel{V'_C}^{\,0} \\ \cancel{\theta'_C}^{\,0} \end{bmatrix} + \begin{bmatrix} 12 \\ 16 \\ 30 \\ -4 \\ 18 \\ -12 \end{bmatrix}$$

From Row (4)

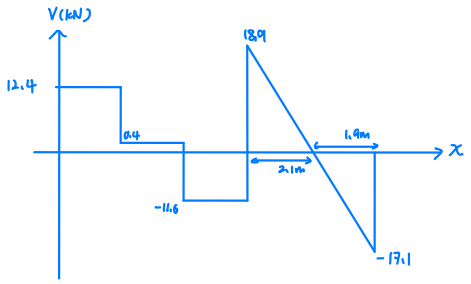
$0 = 5/3\,\theta'_B - 4$

$\theta'_B = 2.4$

From other Rows

$$\begin{bmatrix} V_A \\ M_A \\ V_B \\ V_C \\ M_C \end{bmatrix} = \begin{bmatrix} 12.4kN \\ 16.8kNm \\ 30.5kN \\ 17.1kN \\ 10.8kNm \end{bmatrix}$$

(b) SFD



BMD

$\frac{13.1}{9-x} = \frac{18.9}{x}$

$x = 2.1m$