Name: Owen

Paper: CV1012 – Fluid Mechanics 2021–2022 Sem 2

1. (a)

Inner velocity, $v_1 = 30$ m/s

Inner diameter, $D_1 = 0.02$ m

Inner area, $A_1 = (\pi/4)(0.02)^2$

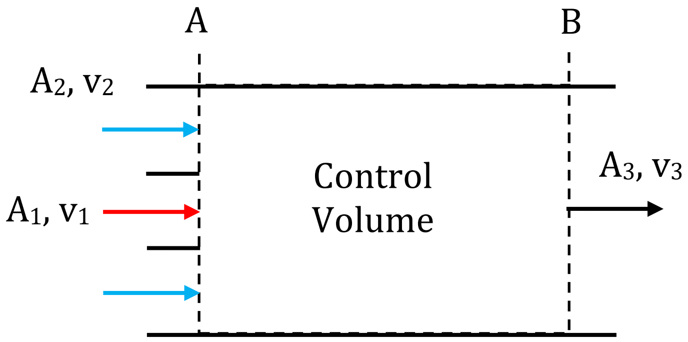
$= 3.1416*10^{-4}$ m^2

Outer velocity, $v_2 = 5$ m/s

Outer diameter, $D_2 = 0.025$ m

Outer area, $A_2 = (\pi/4)(0.05)^2 - A_1$

$= 0.0016493$ m^2



$Q_{total} = \Sigma\ vA$

$= (30)(3.1416*10^{-4}) + (5)(0.0016493)$

$= 0.017671$ m^3/s = Q at B (continuity)

Total area, $A_3 = (\pi/4)(0.05)^2$

$= 0.0019635$ m^2

Velocity at B, $v_3 = Q/A_3$

$= 0.017671/0.0019635$

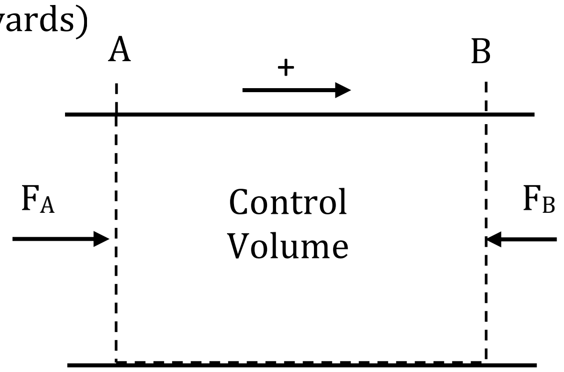
$= 9$ m/s

$F_{net} = M_{out} - M_{in}$

$= [\rho(v_3)^2A_3] - [\rho(v_1)^2A_1 + \rho(v_2)^2A_2]$

$= \rho[(9)^2(0.0019635) - (30)^2(3.1416*10^{-4}) - (5)^2(0.0016493)]$

$= -164.93$ N ($M_{in} > M_{out}$, F_{net} is acting leftwards)



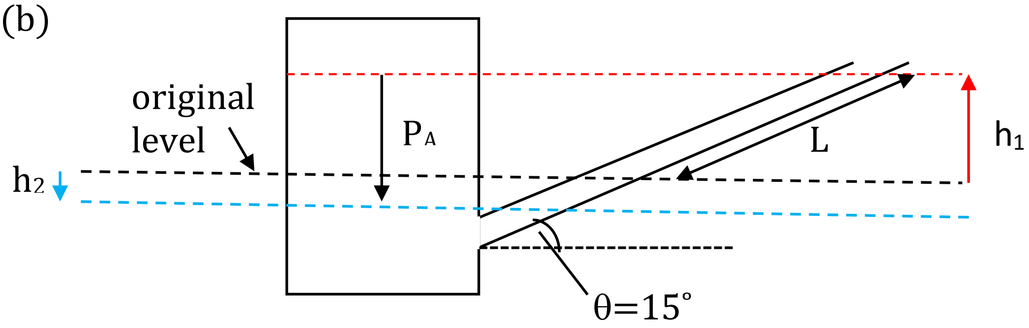
$F_{net} = F_A - F_B$

$= P_A A_3 - P_B A_3$

$$P_A = \frac{-164.93 + (200*10^3)(0.0019635)}{0.0019635}$$

$= \underline{\underline{116\ \text{kPa}}}$

1. (b)



h_1 = increase in tube level = 0.3sin(15°) m

h_2 = decrease in tank level

Total height difference, $H = h_1 + h_2$

Radius of tank = 8R

Radius of tube = R

Volume decrease in tank = Volume increase in tube

$\pi(8R)^2 h_2 = \pi R^2(0.3)$

$h_2 = 0.3/64$ m

$H = 0.3/64 + 0.3\sin(15°)$

Assume $P_{atm} = 0$

$P_A = \rho g H$

$= (1200)(9.81)(0.3/64 + 0.3\sin(15°))$

$= \underline{\underline{969 \text{ Pa}}}$

2. (a)

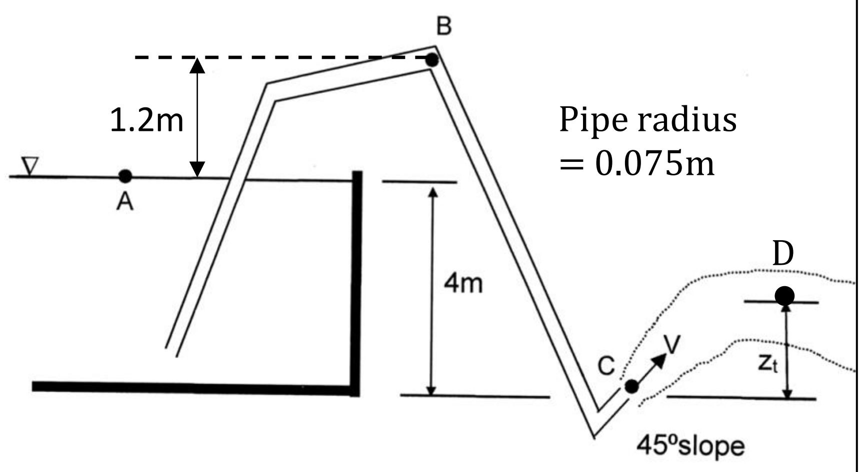


Figure Q2(a)

At A,
$P_A = 0$
$V_A = 0$
$z_A = 4$ m (relative to C)

At B,
$P_B = ?$
$V_B = V$
$z_B = 5.2$ m

At C,
$P_C = 0$
$V_C = V$ (continuity)
$z_C = 0$

Bernoulli's equation A to C:

$$z_A = \frac{V^2}{2g} + 0.1\frac{V^2}{2g} + 1.5\frac{V^2}{2g}$$

$$V = \sqrt{\frac{(4)(2)(9.81)}{1+0.1+1.5}}$$

$$= 5.4941 \text{ m/s}$$

$$Q = AV$$

$$= (\pi/4)(0.15)^2(5.4941)$$

$$= \underline{\underline{0.0971 \text{ m}^3/\text{s}}}$$

Bernoulli's equation A to B:

$$z_A = \frac{P_B}{\rho g} + \frac{V^2}{2g} + z_B + 0.1\frac{V^2}{2g}$$

$$\frac{P_B}{\rho g} = 4 - 5.2 - \frac{(5.4941)^2}{2(9.81)} - 0.1\frac{(5.4941)^2}{2(9.81)}$$

$$= \underline{\underline{-2.89 \text{ m}}}$$

At D,
$P_D = 0$
$V_D = V\cos(45^\circ)$ (horizontal velocity only)
$= 3.8849$ m/s
$z_D = z_t$

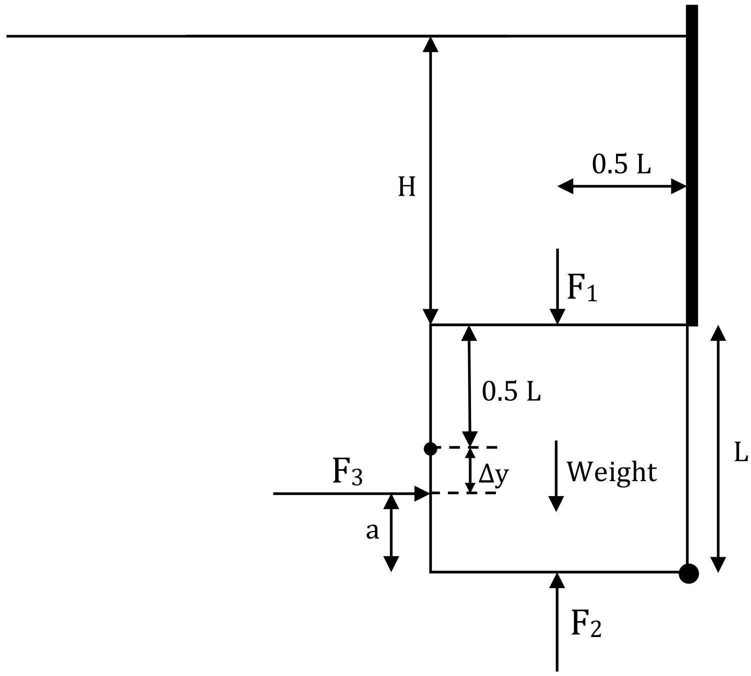
Bernoulli's equation C to D:

$$\frac{V^2}{2g} = \frac{{V_D}^2}{2g} + z_t$$

$$z_t = \frac{(5.4941)^2}{2(9.81)} - \frac{(3.8849)^2}{2(9.81)}$$

$$= \underline{\underline{0.769 \text{ m}}}$$

2. (b)



Weight $= 3000gL^3$

$F_1 + F_2 =$ Upthrust

$= 1000gL^3$

$F_3 = \rho g A h_c$

$= 1000gL^2(H+0.5L)$

$\Delta y = \frac{I_c}{A y_c}$

$= \frac{\frac{1}{12}L^4}{L^2(H+0.5L)}$

$= \frac{L^2}{12(H+0.5L)}$

$a = 0.5L - \Delta y$

$\sum M = 0$

$F_3 a +$ (Upthrust)(0.5L) $=$ (Weight)(0.5L)

$1000gL^2(H+0.5L)(0.5L - \Delta y) + 1000gL^3(0.5L) = 3000gL^3(0.5L)$

$(H+0.5L)(0.5L - \Delta y) + 0.5L^2 = 1.5L^2$

$(H+0.5L)\left(\frac{6L(H+0.5L)-L^2}{12(H+0.5L)}\right) = L^2$

$6LH + 3L^2 - L^2 = 12L^2$

$6H = 10L$

$H = \frac{5}{3}L$

3. (a)

$\mathrm{Re} = \frac{VL}{\upsilon}$

For Re similarity,

$V_p L_p = V_m L_m$, if $\upsilon_p = \upsilon_m$

$\frac{V_p}{V_m} = \frac{L_m}{L_p}$

$V_r = (L_r)^{-1}$

$\mathrm{Fr} = \frac{V}{\sqrt{gL}}$

For Fr similarity,

$\frac{V_p}{\sqrt{L_p}} = \frac{V_m}{\sqrt{L_m}}$ if $g_m = g_p$

$\frac{V_p}{V_m} = \sqrt{\frac{L_p}{L_m}}$

$V_r = (L_r)^{0.5}$

Hence, due to a conflict in the length scale used, it is impossible to satisfy both the Re and Fr similarity simultaneously if the same fluid is used.

3. (b)

$\tau = f(V, y, \mu)$

Their dimensions are:

$\tau = [ML^{-1}T^{-2}]$

$V = [LT^{-1}]$

$y = [L]$

$\mu = [ML^{-1}T^{-1}]$

Using V, y, μ as repeating variables, only 1 ∏ term:

$\prod_1 = \tau V^a y^b \mu^c$

Using MLT system:

$ML^{-1}T^{-2}L^aT^{-a}L^bM^cL^{-c}T^{-c} = M^0L^0T^0$

$1 + c = 0$

$-1 + a + b - c = 0$

$-2 - a - c = 0$

Solving: $c = -1$, $a = -1$, $b = 1$

$\prod_1 = \frac{\tau y}{V\mu}$ = constant, i.e.

$$\underline{\underline{\tau = \text{constant}\,\frac{\mu V}{y}}}$$

3. (c)

Spillway flow is governed by similarity of Froude number.

For Froude number similarity:

$\frac{V_p}{V_m} = \sqrt{\frac{L_p}{L_m}}$ when $g_p = g_m$

$\frac{Q_p}{Q_m} = \frac{V_p {L_p}^2}{V_m {L_m}^2} = \left(\frac{L_p}{L_m}\right)^{\frac{5}{2}}$

$L_p = L_m\left(\frac{Q_p}{Q_m}\right)^{\frac{2}{5}}$

$= 1.0\left(\frac{87.1}{0.1}\right)^{\frac{2}{5}}$

$= \underline{\underline{15.0\text{ m}}}$

For similarity of Drag coefficient:

$\frac{F_p}{0.5\rho_p {V_p}^2 {L_p}^2} = \frac{F_m}{0.5\rho_m {V_m}^2 {L_m}^2}$

$F_p = \left(\frac{{V_p}^2 {L_p}^2}{{V_m}^2 {L_m}^2}\right) F_m$

$= \left(\frac{L_p}{L_m}\right)^3 F_m$

$= \left(\frac{15.0}{1.0}\right)^3 (5)$

$= \underline{\underline{16865\text{ N}}}$

3. (d) (i) (ii)

For laminar flow, use Poiseuille's equation:

$\frac{128\mu L Q_1}{\rho g\pi(D)^4} = \frac{128\mu L(Q_2)}{\rho g\pi(2D)^4}$, μ, L, ρ, g constant

$Q_2 = 16\, Q_1$, $\underline{\underline{\text{1500\% increase}}}$

For turbulent flow, use Darcy-Weisbach equation:

$\frac{8fL{Q_1}^2}{g\pi^2(D)^5} = \frac{8fL{Q_2}^2}{g\pi^2(2D)^5}$, f, L, g constant

$Q_2 = \sqrt{32}\, Q_1$

$= 5.66\, Q_1$, $\underline{\underline{\text{466\% increase}}}$

4. (a)

1. Re $<$ 2100, in laminar flow regime, f is a function of Re only & is independent of ε/D, the equation is f = 64/Re.
2. 2100 $<$ Re $<$4000, in transition range, f is uncertain as flow may be laminar or turbulent.
3. Re $>$ 4000 but not in wholly turbulent flow regime, f depends on both Re and ε/D.
4. Re very large and in the wholly turbulent flow regime, f depends on ε/D only, independent of Re.

4. (b)
Assumptions:
1. Fully developed, steady, incompressible, Newtonian fluid flow.

Using Darcy-Weisbach equation:
$5 = \frac{8f(100)(0.01)^2}{(9.81)\pi^2(0.1)^5}$
f = 0.0605

From Moody diagram, turbulent smooth pipe curve does not reach f = 0.0605, therefore flow must be laminar.

4. (c) (i)

Assuming $P_{atm} = 0$, $v = 0$ at top of reservoir,
$h_A = 100$ m, $h_B = 50$ m, $h_C = 10$ m
Ignoring minor losses,
$h_f = h_1 - h_2$
$= KQ^2$
$|Q| = \sqrt{\frac{|h_1 - h_2|}{K}}$
$Q_1 + Q_2 + Q_3 = 0$ (continuity, take into J as positive and out of J as negative)

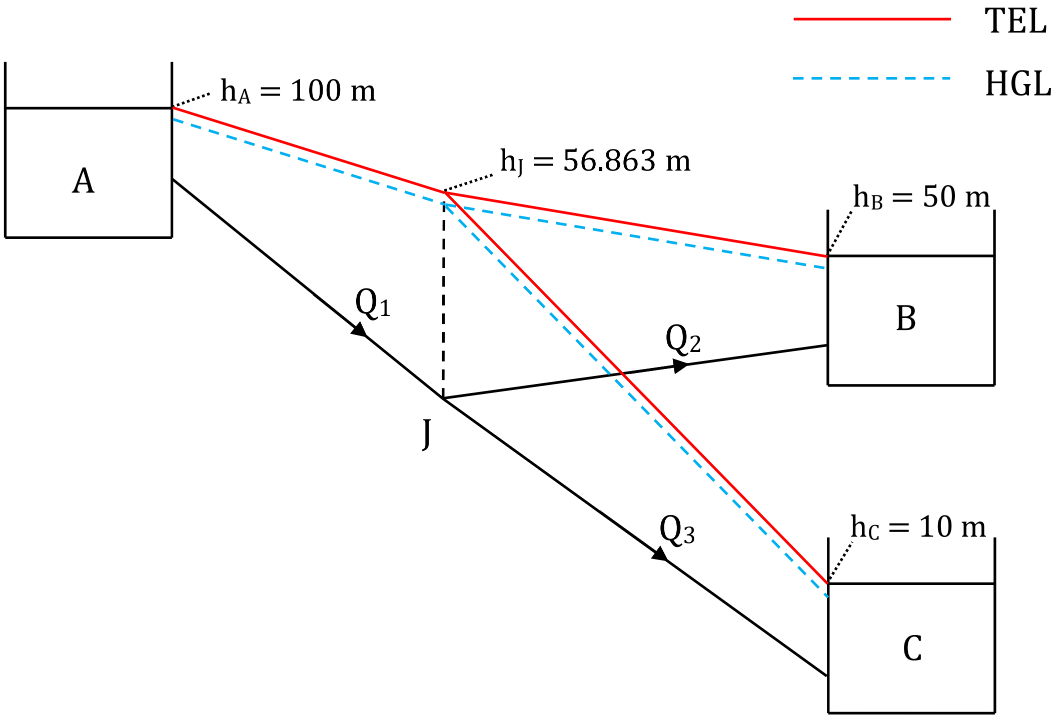
$|Q_1| = \sqrt{\frac{|100 - h_J|}{340}}$
$|Q_2| = \sqrt{\frac{|50 - h_J|}{612}}$
$|Q_3| = \sqrt{\frac{|10 - h_J|}{748}}$

Guess:
Q_1 into J, Q_2 into J, Q_3 out of J
Using G.C., solve for $|Q_1| + |Q_2| - |Q_3| = 0$, $10 < h_J < 50$
No solution for h_J

Guess:
Q_1 into J, Q_2 out of J, Q_3 out of J
Using G.C., solve for $|Q_1| - |Q_2| - |Q_3| = 0$, $50 < h_J < 100$
$h_J = 56.863$ m

Thus,
$Q_1 =$ <u>0.356 m³/s into J</u>
$Q_2 =$ <u>0.106 m³/s out of J</u>
$Q_3 =$ <u>0.250 m³/s out of J</u>

4. (c) (ii)



NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2021-2022

CV1012 - FLUID MECHANICS

April / May 2022 **Time Allowed: 2 hours**

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **SIX (6)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. An **Appendix** of **TWO (2)** pages containing useful data and formulas is attached to the paper.
5. This is a Closed-Book Examination.
6. All answers must be written in the answer book provided. Answer each question beginning on a **FRESH** page of the answer book.

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1. (a) Figure Q1(a) shows the water flow inside a circular duct with 5-cm diameter. At Section A, additional water is injected into the duct through a 2-cm circular nozzle at the centre, and the flow velocity is 30 m/s inside the nozzle, and 5 m/s outside the nozzle. At Section B, the flow becomes uniform and has the same velocity across the cross-section. If the pressure at Section B is equal to 200 kPa, what is the pressure at Section A? You can assume that the pressure is uniform across both Section A and Section B, and that the head loss between the two Sections is negligible. (Hint: use the momentum equation).

(13 Marks)

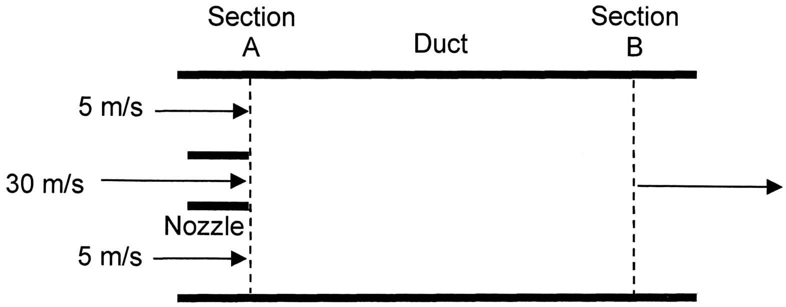


Figure Q1(a)

Note: Question No. 1 continues on page 2.

(b) Figure Q1(b) shows a cylindrical tank of liquid connected to an inclined tube (15º slope) that is open to the atmosphere. The specific density for the liquid is 1.2. The ratio of the tank diameter to the tube diameter is 8. When the air pressure inside the tank is atmospheric, the surface of the liquid inside the tube is at the position of A. When air pressure inside the tank is increased to P_A, the liquid level inside the tank is lower and the liquid level inside the tube moves up a distance of L along the tube to the position of B. Calculate the magnitude of P_A if L = 0.3 m. Assume there is no flow in the tube.

(12 Marks)

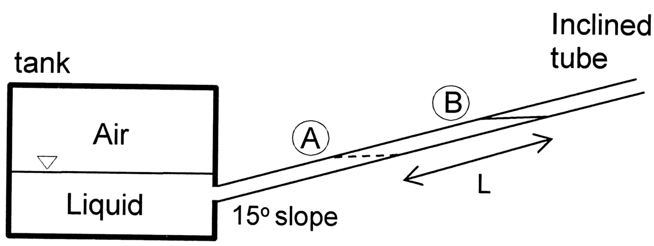


Figure Q1(b)

2. (a) Figure Q2(a) shows a siphon water system with the pipe diameter equal to 15 cm. Point B is at an elevation of 1.2 m above the water surface inside the tank, and Point C is open to the atmosphere with a discharge angle of 45 degree. It is given that the head loss is equal to 0.1 $V^2/2g$ between A and B, and 1.5 $V^2/2g$ between B and C. Calculate the total discharge from the siphon system, the pressure head at Point B, and the height z_t whereby the water can reach after discharging from Point C.

(13 Marks)

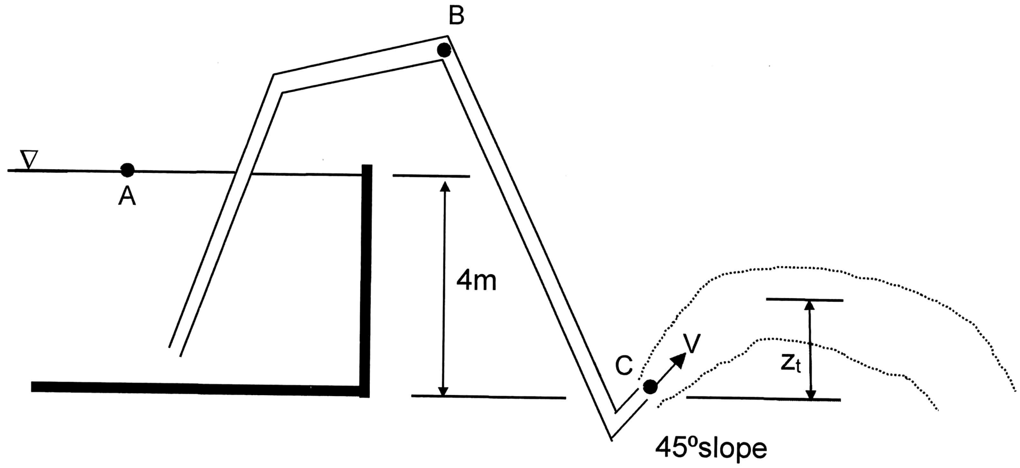


Figure Q2(a)

(b) Figure Q2(b) shows a cube with a side length of L submerged in water at a water depth of H. The cube has a density of 3,000 kg/m³, and it is being used as a gate with a hinge at the bottom as shown in Figure Q2(b). Compute the relationship between H and L so that the moment at the hinge is equal to zero.

(12 Marks)

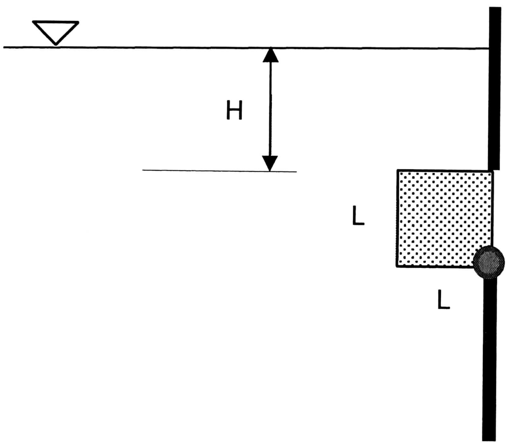


Figure Q2(b)

3. (a) Based on the definitions of the Reynolds (Re) and Froude (Fr) numbers, show why it is impossible to satisfy both the Re and Fr similarity simultaneously in hydraulic modelling if the same fluid is used in both the prototype and model.

(6 Marks)

(b) A plate at a distance y from a fixed plate moves at a velocity V, and has a shear stress τ to maintain this speed. The fluid between the plates has an absolute viscosity of μ. Using dimensional analysis, determine the form of Newton's Law of Absolute Viscosity.

(5 Marks)

(c) For dynamic similarity, a scale model of a dam spillway with a height of 1.0 m is used to study the flow characteristics of the prototype. The peak flood in the prototype is 87.1 m^3/s and the model is tested with a discharge of $Q_m = 0.1\ m^3/s$.

(i) Calculate the height of the prototype spillway.

(ii) On a part of the model, a force of 5 N is measured, calculate the force in the prototype spillway.

(8 Marks)

(d) Water flows with a velocity V in a circular pipe with a diameter D and a length of L. If the head loss, h_f remains constant, what would be the % increase in the discharge (Q) if D is increased by 2 times? Consider flow in the following 2 regimes:

(i) When the flow is in the laminar regime with Reynolds number, Re < 2000.

(ii) When the flow is in the wholly turbulent flow regime where f remains a constant and the Re is very high.

(6 Marks)

4. (a) Explain briefly 3 important characteristics of the Moody Diagram for pipe flow.

(5 Marks)

(b) Assess the type of flow regime (laminar or turbulent) for a particular fluid flow in a **smooth** pipe based on the following data: pipe length, L = 100 m, diameter D = 0.1 m, discharge Q = 0.01 m^3/s, head loss is 5 m. State any assumptions made.

(5 Marks)

(c) Figure Q4 shows reservoirs A, B and C connected to pipes 1, 2 and 3, respectively, with a common junction **J**. The water level elevations at reservoirs A, B, and C are fixed at 100m, 50m and 10m, respectively. The pipe characteristics and K values are given in Table Q4.

(i) Calculate the flow rates in the 3 pipes.

(ii) Sketch the total energy line (TEL) and hydraulic grade line (HGL) for pipeline system.

Note: Neglect all minor losses.

(15 Marks)

Table Q4

Pipeline	Pipe diameter D (m)	Length L (m)	Friction factor f	$K = \frac{8fL}{g\pi^2 D^5}$
1	0.3	500	0.02	340
2	0.3	900	0.02	612
3	0.3	1100	0.02	748

Note: Question No. 4 continues on page 6.

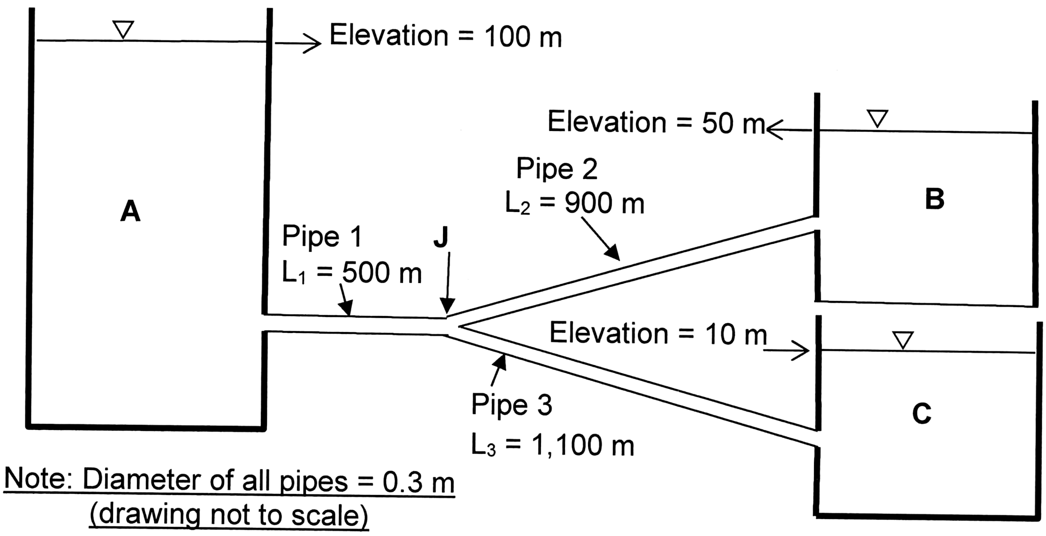


Figure Q4

END OF PAPER

Useful Formulas and Moody Diagram

$\gamma = \rho g$

$SG = \rho/\rho_{water}$

$\tau = \mu(du/dy)$

$\mu = \rho\nu$

$E_V = - dp/ (dV/V) = dp/ (d\rho/\rho)$

$p = p_o + \gamma h$

$p_{atm} \approx 1$ bar = 100 kPa (abs) $\approx$ 760 mm Hg (abs)

$p_{gauge} = p_{abs} - p_{atm}$

$F_R = p_c A = \gamma h_c A$

$y_R = I_{xc}/(y_c A) + y_c$

$I_{xc} = ba^3/12$ (rectangle); $\pi R^4/4$ (circle); $0.11R^4$ (semicircle); $0.055R^4$ (quarter circle)

$I_{xc} = ba^3/36$ (triangle, b = base, a = height)

$F_B = \gamma V_{displaced}$

$BM = I_{yy}/V_{displaced}$

$\Sigma Q_{in} = \Sigma Q_{out}$

$p_1/(\rho g) + V_1^2/(2g) + z_1 = p_2/(\rho g) + V_2^2/(2g) + z_2$

$\Sigma F_x = \Sigma(\rho Q V_x)_{out} - \Sigma(\rho Q V_x)_{in}$

Darcy-Weisbach Eq: $h_f = \frac{f\ L\ V^2}{2\ g\ D} = \frac{8\ f\ L\ Q^2}{g\ \pi^2\ D^5}$

Poiseuille's Eq: $h_f = \frac{32\ \mu\ L\ V}{\rho\ g\ D^2} = \frac{128\ \mu\ L\ Q}{\rho\ g\ \pi\ D^4}$

For laminar flow, $f = \frac{64}{Re}$

Blasius Formula: $f = \frac{0.316}{Re^{0.25}}$,

$K_{expansion} = \left(1 - \frac{A_1}{A_2}\right)^2$, $K_{contraction} = 0.5$, $K_{entrance} = 0.5$, $K_{exit} = 1.0$.

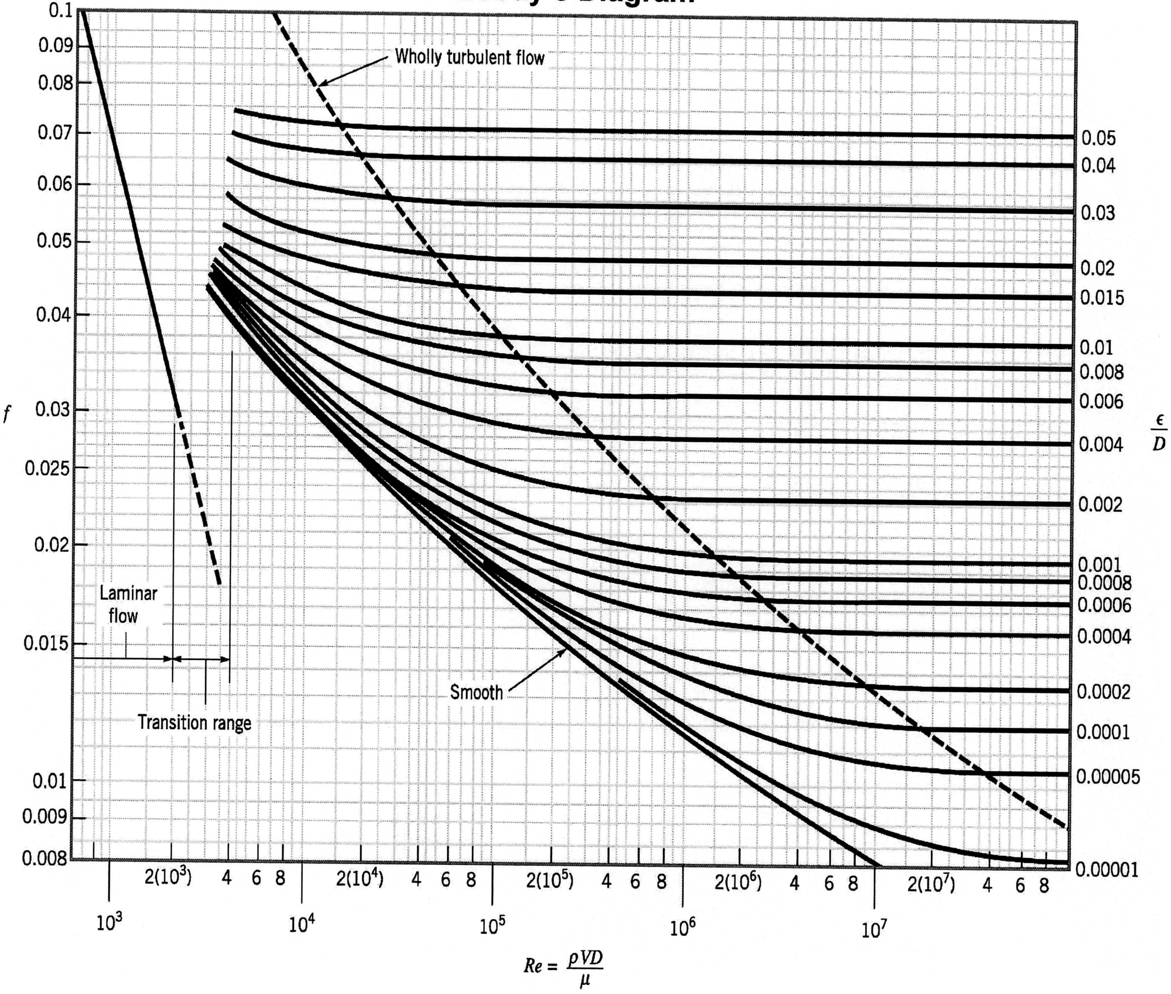
Fluid power = $\rho\ g\ Q\ h_p$

Head rise coefficient = $\frac{gH}{\omega^2 D^2}$

Flow coefficient = $\frac{Q}{\omega D^3}$

Power coefficient = $\frac{P}{\rho\ \omega^3\ D^5}$

Moody's Diagram



CV1012 FLUID MECHANICS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.