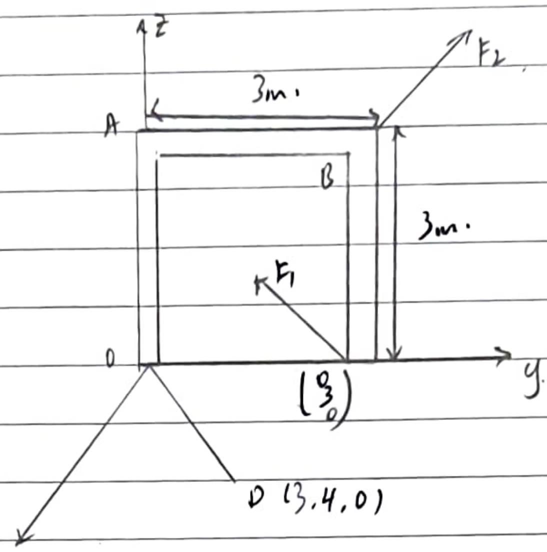
Jervis Teo

2021 - 2022 Sem 2 (CV1011) Date 24/11/2022 No.

1a) $\vec{F_1} = \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix}$

$\vec{F_2} = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$



$\vec{M}_{F_1} = \vec{r} \times \vec{F_1}$

$= \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} \times \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix}$

$\vec{M}_{F_2} = \vec{r} \times \vec{F_2}$

$= \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} \times \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} -6 \\ 6 \\ -6 \end{pmatrix}$

$\vec{M}_{F_1} = (6\underset{\sim}{i} + 3\underset{\sim}{k})$ Nm. $\quad \vec{M}_{F_2} = (-6\underset{\sim}{i} + 6\underset{\sim}{j} - 6\underset{\sim}{k})$ Nm

1b) $|\vec{OD}| = \sqrt{3^2 + 4^2 + 0^2}$

$= \sqrt{9+16}$

$= 5$

$\hat{OD} = \frac{1}{5}\begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$

$\vec{M}_{F_1}$ about OD $= \vec{M}_{F_1} \cdot \hat{OD}$

$= \frac{1}{5}\begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$

$= \frac{1}{5}(18)$

$= 3.6$ Nm.

$\vec{M}_{F_2}$ about OD $= \vec{M}_{F_2} \cdot \hat{OD}$

$= \frac{1}{5}\begin{pmatrix} -6 \\ 6 \\ -6 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$

$= \frac{1}{5}(-18 + 24)$

$= 1.2$ Nm.

1c) $\vec{F_1} = \begin{pmatrix} -1 \\ -2 \\ a \end{pmatrix} \quad \vec{F_2} = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$

$\vec{F_1} \cdot \vec{F_2} = 0$.

$\begin{pmatrix} -1 \\ -2 \\ a \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix} = 0$.

$-2 - 8 + 2a = 0$

$2a = 10$

$a = 5$.

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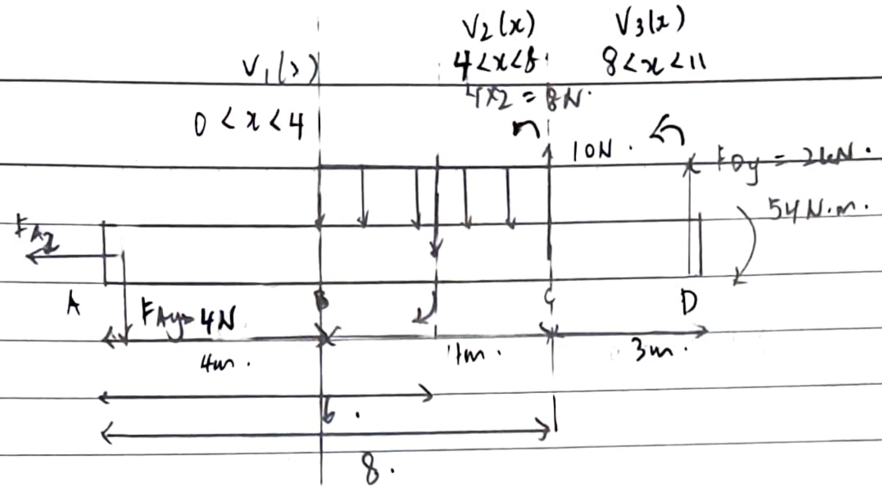
2a)



$\Sigma F_x = 0$ $\qquad$ $\Sigma F_y = 0$

$F_{Ax} = 0$, $\qquad$ $-F_{Ay} - 8 + 10 + F_{Dy} = 0$

$-F_{Ay} + F_{Dy} = -2$

$+\circlearrowleft \Sigma M_A = 0$

$-54 + 11D_y + 10(8) - 8(6) = 0$

$11D_y = 22$

$D_y = 2N$

when $D_y = 2$,

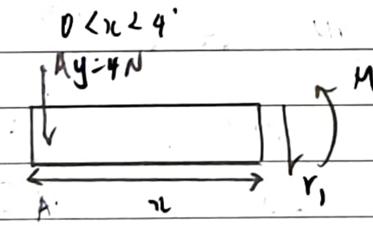
$-F_{Ay} + 2 = -2$

$-F_{Ay} = -4$

$F_{Ay} = 4N$

$\therefore F_{Ay} = 4N$, $F_{Dy} = 2N$

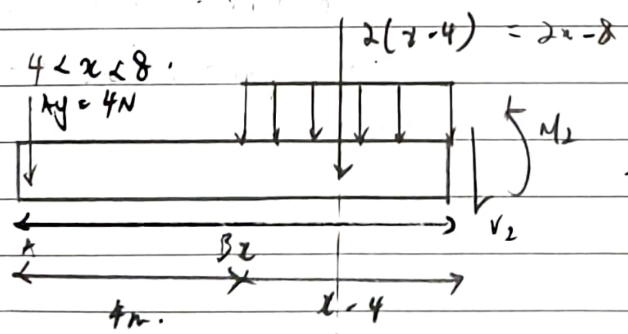
2b)



$+\uparrow \Sigma V_1 = 0$

$-4 - V_1(x) = 0$

$V_1(x) = -4N$



$+\uparrow \Sigma V_2 = 0$

$-4 - 2(x-4) - V_2(x) = 0$

$V_2(x) = 4 - 2x$

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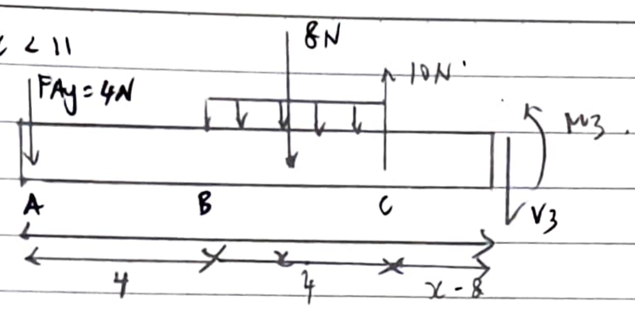
$8 < x < 11$



$$+\uparrow \Sigma V_3 = 0.$$

$$-4 - 8 + 10 - V_3(x) = 0.$$

$$-2 - V_3(x) = 0$$

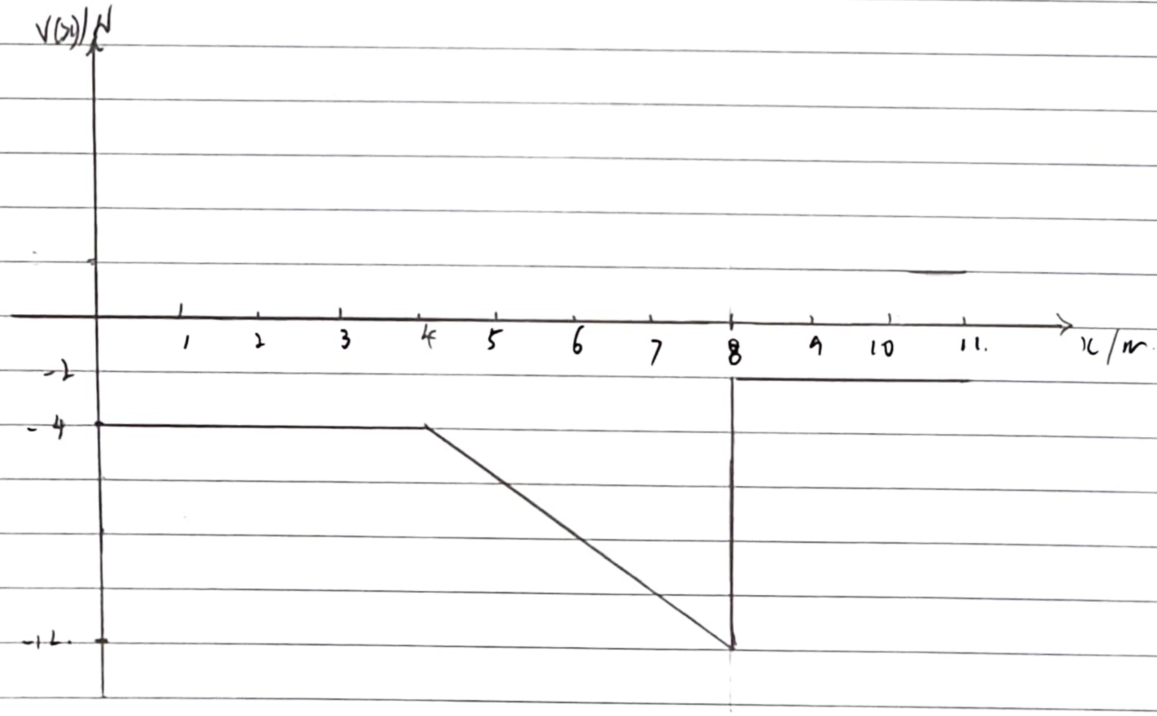
$$V_3(x) = -2.$$

$4 < x < 8$

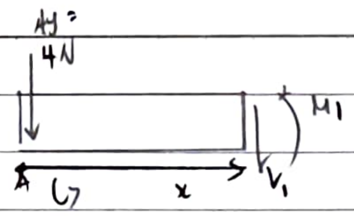
$$V_2(x) = 4 - 2x$$

$$V_2(4) = 4 - 2(4)$$
$$= -4N$$

$$V_2(8) = 4 - 2(8)$$
$$= -12N$$



2c)



$\overset{+}{\circlearrowleft} \sum M_1 = 0$,

$M_1(x) + 4x = 0$

$M_1(x) = -4x$

When $x = 4$,

$M_1(4) = -4(4)$

$= -16 \text{ Nm}$.

3a(i) $\rho = 7850 \, kg/m^3$

Cross sectional area, $A = 16000 \, mm^2$

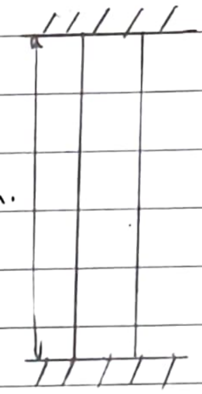
$= 0.016 \, m^2$

Young Modulus, $E = 200 \times 10^9 \, pa.$

Thermal expansion, $\alpha = 1 \times 10^{-5} \, /°C.$

Gravity $= 9.81 \, m/s^2.$

3m.



$$\delta_T = \alpha \Delta T L$$
$$= 1.0 \times 10^{-5} \times (2) \times (3)$$
$$= 0.00006$$

Bar expand but pushed back by the wall.

$$\delta_M = \frac{PL}{EA}$$
$$= P\left(\frac{3}{200 \times 10^9 \times 0.016}\right)$$
$$= 9.375 \times 10^{-10} P.$$

$$\delta_T - \delta_M = 0.$$
$$0.00006 = 9.375 \times 10^{-10} P$$
$$P = 64000 \, N.$$

$$\sigma = \frac{64000}{0.016} = 4 \times 10^6 \, pa$$
$$= 4MPa. \quad \text{(compressive)}$$

3a(i) Mass, $m = \rho V$
$= 7850 \times (0.016 \times 3)$
$= 376.8 \, kg$

$W = mg$
$= 376.8 \times 9.81$
$= 3716.028 \, N$

$$\sigma = \frac{P}{A}$$
$$= \frac{3716.028}{0.016}$$
$$= 232251.75 \, pa = 0.23225 \, MPa.$$

Max stress in column $= 4 + 0.23225$
$= 4.23 \, MPa$ (3sf)
(occurs at the neutral axis aka centroid that is 1.5m from the base).

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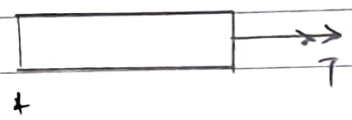
$\longrightarrow$ +ve.

3b) Section AB

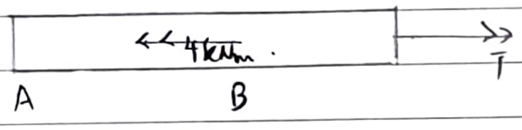


$G_{AC} = 100\,GPa$
$= 100 \times 10^9\,Pa$

$\Sigma T = 0$
$T - 0 = 0$
$T = 0\ kNm$.

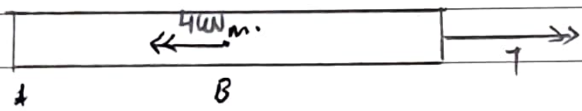
$G_{CD} = 80\,GPa$
$= 80 \times 10^9\,Pa$

Section AC.

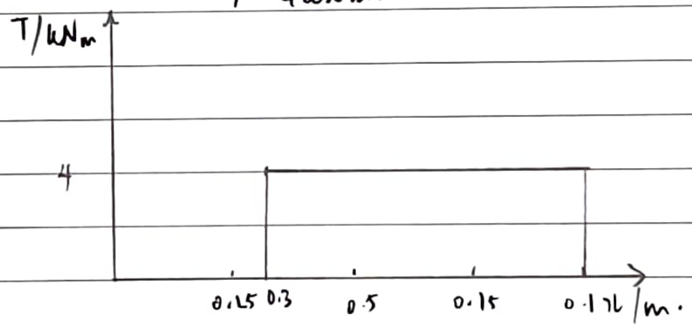


$\Sigma T = 0$.
$T - 4 = 0$
$T = 4\,kNm$.

Section AD



$\Sigma T = 0$
$T - 4 = 0$
$T = 4\,kNm$.



$J = \frac{\pi}{2}(c)^4$
$= \frac{\pi}{2}(100 \times 10^{-3})^4$
$= 0.000157079\ m^4$

$\phi = \Sigma \frac{TL}{GJ}$
$= \frac{(4 \times 10^3) \times (0.2)}{(100 \times 10^9) \times (0.000157079)} + \frac{(4 \times 10^3) \times (0.5)}{(80 \times 10^9) \times (0.000157079)}$

$\pi\ rad = 180°$.
$1\,rad = \frac{180}{\pi}$

$= 0.000050929 + 0.000159155$
$= 0.000210084\ rad$
$= 0.000210084 \times \frac{180}{\pi}$
$= 0.012°$

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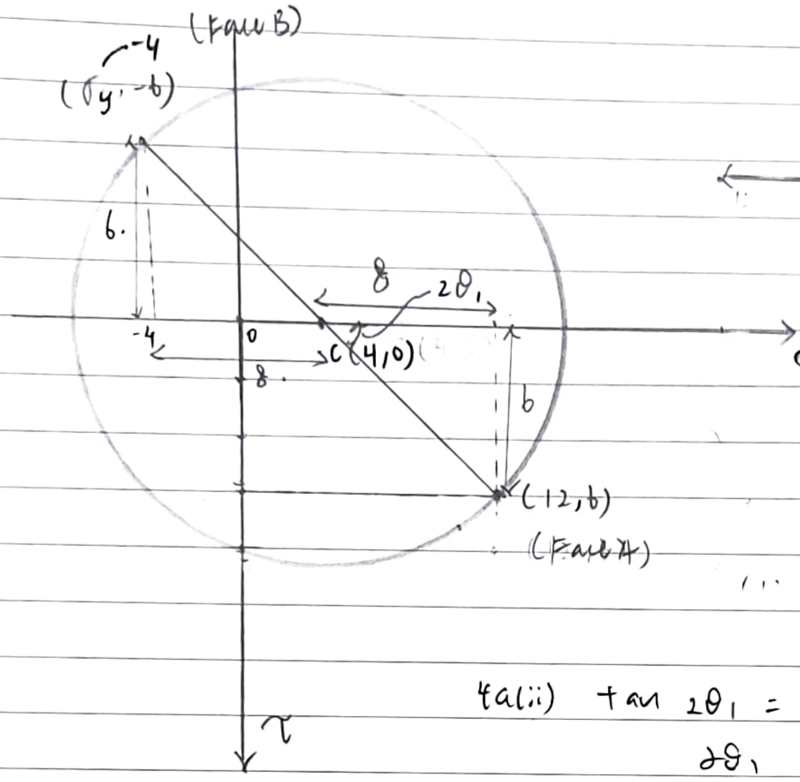
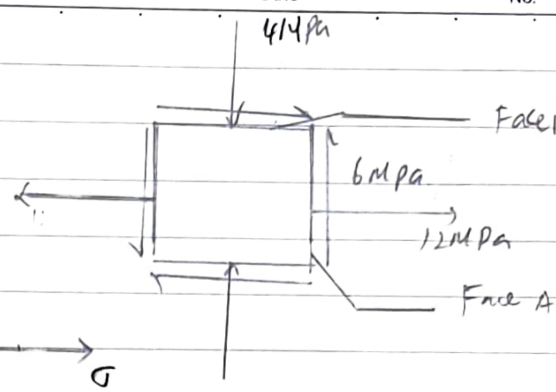
4a(i)





$$\sigma_{avg} = \frac{12+\sigma_y}{2} = 4$$

$$12+\sigma_y = 8$$

$$\sigma_y = -4$$

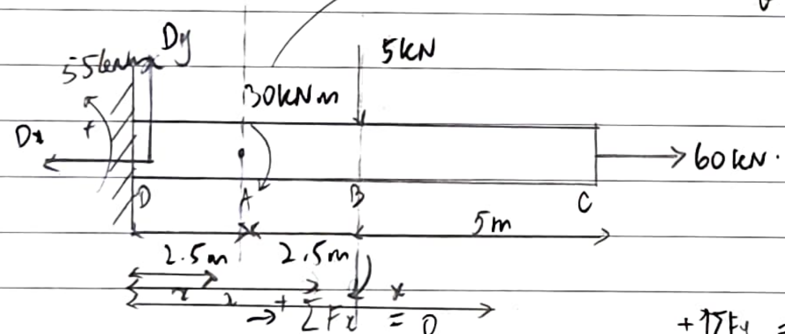
4a(ii) $\tan 2\theta_1 = \frac{6}{8}$

$$2\theta_1 = \tan^{-1}\left(\frac{3}{4}\right)$$

$$2\theta_1 = 36.8699^\circ$$

$$\theta_1 : 18.4^\circ \text{ (1dp.)}$$

4b)



$\xrightarrow{+} \Sigma F_x = 0$

$$-Dx + 60 = 0$$

$$-Dx = -60$$

$$Dx = 60 \text{ kN}$$

$+\uparrow \Sigma F_y = 0.$

$$Dy - 5 = 0$$

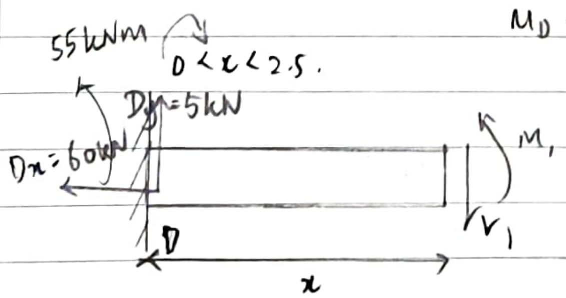
$$Dy = 5\text{kN}.$$

$\circlearrowleft + \; \Sigma M_D = 0$

$$M_D - 30 - (5.0)(5) = 0.$$

$$M_D = 55 \text{ kNm}.$$

$0 < x < 2.5.$

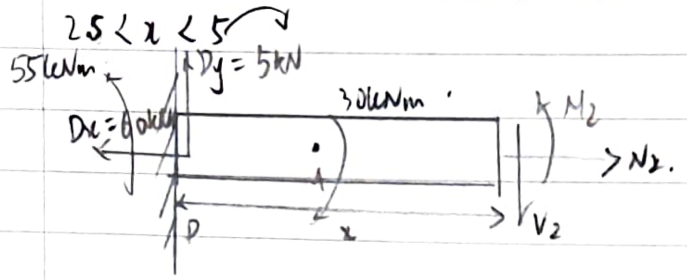


$\circlearrowleft + \; \Sigma M_1 = 0.$

$$M_1(x) - 5x + 55 = 0$$

$$M_1(x) = 5x - 55$$

$2.5 < x < 5$

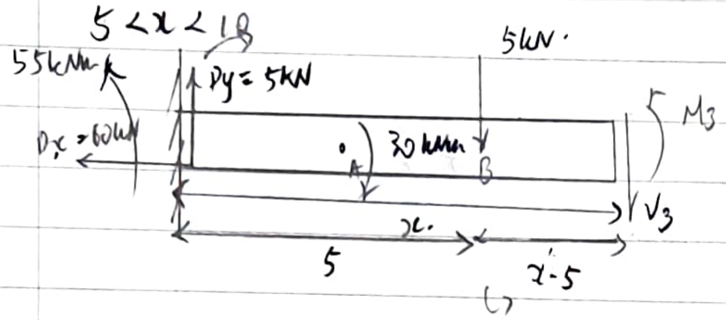


$+\circlearrowleft \sum M_2 = 0.$

$M_2(x) - 30 - 5x + 55 = 0$

$M_2(x) = 5x - 25$

$5 < x < 10$



$+\circlearrowleft \sum M_3 = 0.$

$M_3(x) - 5x - 30 + 5(x-5) + 55 = 0$

$M_3(x) = 5x + 30 - 5x + 25 - 55$

$= 0 \text{ kNm}.$

$0 < x < 2.5.$

$M_1(x) = 5x - 55$

when $x = 0$,

$M_1(0) = 5(0) - 55$

$= -55 \text{ kNm}.$

when $x = 2.5$

$M_1(2.5^-) = 5(2.5) - 55$

$= -42.5 \text{ kNm}.$

$2.5 < x < 5$

$M_2(x) = 5x - 25$

when $x = 2.5$

$M_2(2.5) = 5(2.5) - 25$

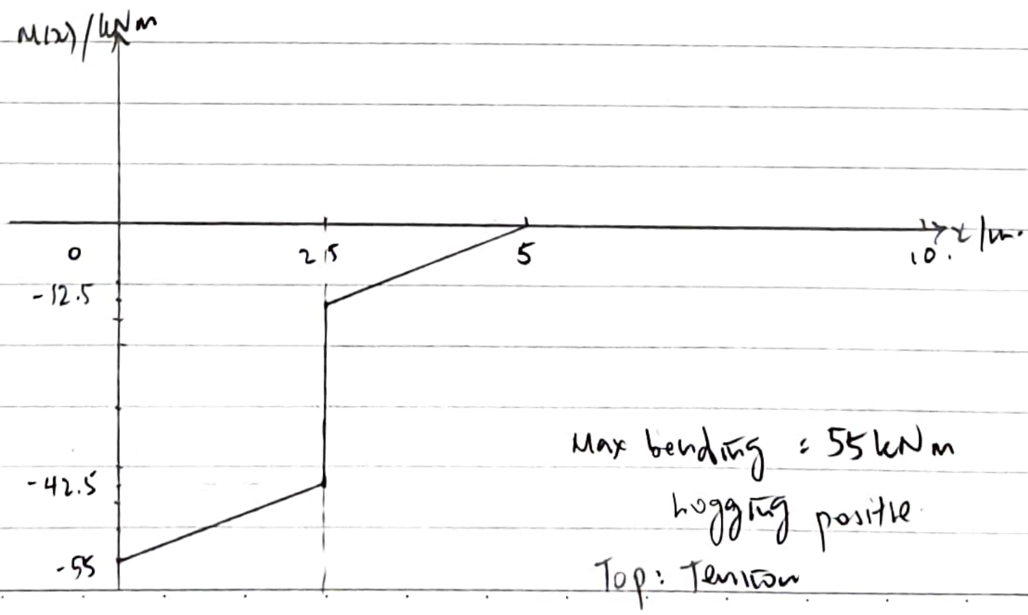
$= -12.5 \text{ kNm}.$

$M_2(5) = 5(5) - 25$

$= 0. \text{ kNm}.$

$5 < x < 10.$

$M_3(x) = 0 \text{ kNm}.$



Max bending = 55 kNm

Hogging positive

Top: Tension

Bottom: Compression.

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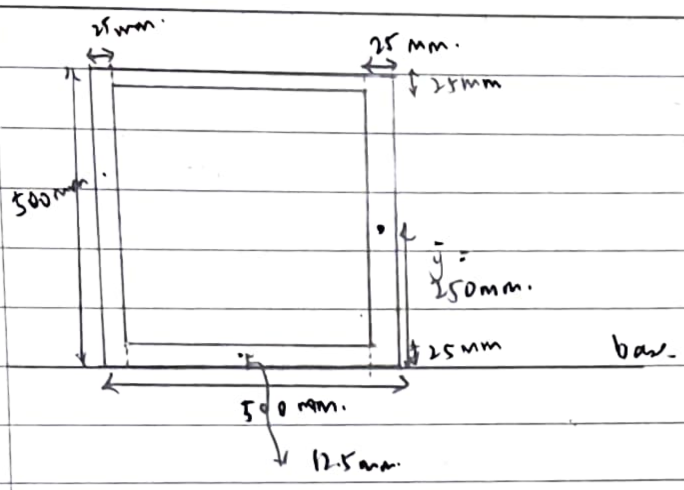


$$I = \frac{1}{12}(500\times10^{-3})\times(500\times10^{-3})^3 - \left[\frac{1}{12}(450\times10^{-3})(450\times10^{-3})^3\right]$$
$$= \frac{1}{192} - 0.003417187$$
$$= 0.001791145\ m^4$$

$$\sigma = \frac{P}{A}$$
$$= \frac{60\times10^3}{\left(\frac{19}{400}\right)}$$
$$= 1.2632\text{MPa}$$

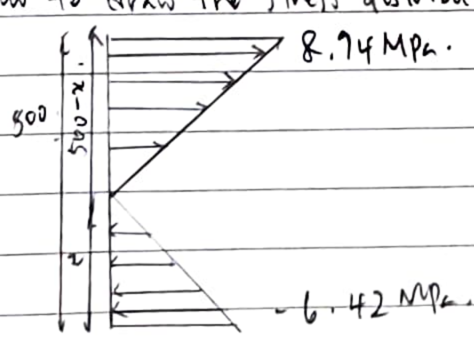
cross sectional area

$$= (500\times10^{-3})^2 - (450\times10^{-3})^2$$
$$= \frac{1}{4} - \frac{81}{400}$$
$$= \frac{19}{400}\ m^2$$

$$\sigma_{top} = \frac{My}{I} + \frac{P}{A}$$
$$= \frac{(55\times10^3)(250\times10^{-3})}{0.001791145} + 1.2632\times10^{-6}$$
$$= 8.94\ \text{MPa (3sf)}.$$

$$\sigma_{bottom} = -\frac{My}{I} + \frac{P}{A}$$
$$= -\frac{(55\times10^3)(250\times10^{-3})}{0.001791145} + (1.2632\times10^{-6})$$
$$= -6.42\ \text{MPa (3sf)}.$$

How to draw the stress distribution diagram?



$$\frac{8.9399}{500-x} = \frac{6.4135}{x}$$

$$8.9399x = 3206.75 - 6.4135x$$

$$15.3534x = 3206.75$$

$$x = 209\text{mm (3sf)}.$$

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