Hoang Thu Minh

1.

(a) Since the specific values of wave heights are not given, one can use any value within the given range to calculate H_{rms}.

Use the mid-value of each range as the representative of that range.

Range of Wave height [m]	Mid-value	Number of waves
0.00 - 0.12	0.06	x
0.12 - 0.24	0.18	57
0.24 - 0.36	0.30	72
0.36 - 0.48	0.42	50
0.48 - 0.60	0.54	42
0.60 - 0.72	0.66	28
0.72 - 0.84	0.78	19
0.84 - 0.96	0.90	11
0.96 - 1.08	1.02	0

$$H_{100} = \frac{0.06x + 0.18\times57 + 0.30\times72 + 0.42\times50 + 0.54\times42 + 0.66\times28 + 0.78\times19 + 0.90\times11}{x + 57 + 72 + 50 + 42 + 28 + 19 + 11 + 0}$$

$$0.40 = \frac{0.06x + 118.74}{x + 279}$$

$$\Rightarrow x = 21$$

Total number of observations = 21 + 279 = 300

$$H_{rms} = \sqrt{\frac{0.06^2\times21 + 0.18^2\times57 + 0.30^2\times72 + 0.42^2\times50 + 0.54^2\times42 + 0.66^2\times28 + 0.78^2\times19 + 0.90^2\times11}{300}}$$

$$= 0.455\,m$$

	Number of observations	Range of wave heights
H_1	1% × 300 = 3	0.84 - 0.96
H_{10}	10% × 300 = 30	0.72 - 0.96
H_{33}	33% × 300 = 100	0.48 - 0.96
H_{50}	50% × 300 = 150	0.36 - 0.96

To calculate H_1, we need 3 largest wave heights from range 0.84 - 0.96. Since this range has 11 observed waves, estimate $H_1 = 0.96\,m$ to be conservative.

To calculate H_{10}, we need all 11 wave heights from range 0.84 - 0.96 and all 19 wave heights from range 0.72 - 0.84. Since more waves are in range 0.72 - 0.84, estimate $H_{10} = 0.82\,m$

To calculate H_{33}, we need 100 waves from range 0.48 - 0.96. Since nearly half of these waves are from range 0.48 - 0.72, estimate $H_{33} = 0.64\,m$

To calculate H_{50}, we need 150 waves from range 0.36 - 0.96. Since most of these waves are from range 0.36 - 0.60, estimate $H_{50} = 0.50\,m$

(b)

(i) If this wave is fetch - limited:

From CEM Fig II - 2 - 23 : $\left.\begin{array}{l} F = 50\,km \\ U = 7.5\,m/s \end{array}\right\} \Rightarrow H = 0.70\,m$

$\Rightarrow$ higher than the estimated $H_s = 0.64\,m$ above

Hence, this wave is duration - limited

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From CEM Fig II - 2 - 25 : $\left.\begin{array}{l} H = 0.64\text{ m} \\ U = 7.5\text{ m/s} \end{array}\right\} \Rightarrow D = 8.5\text{ hrs}$

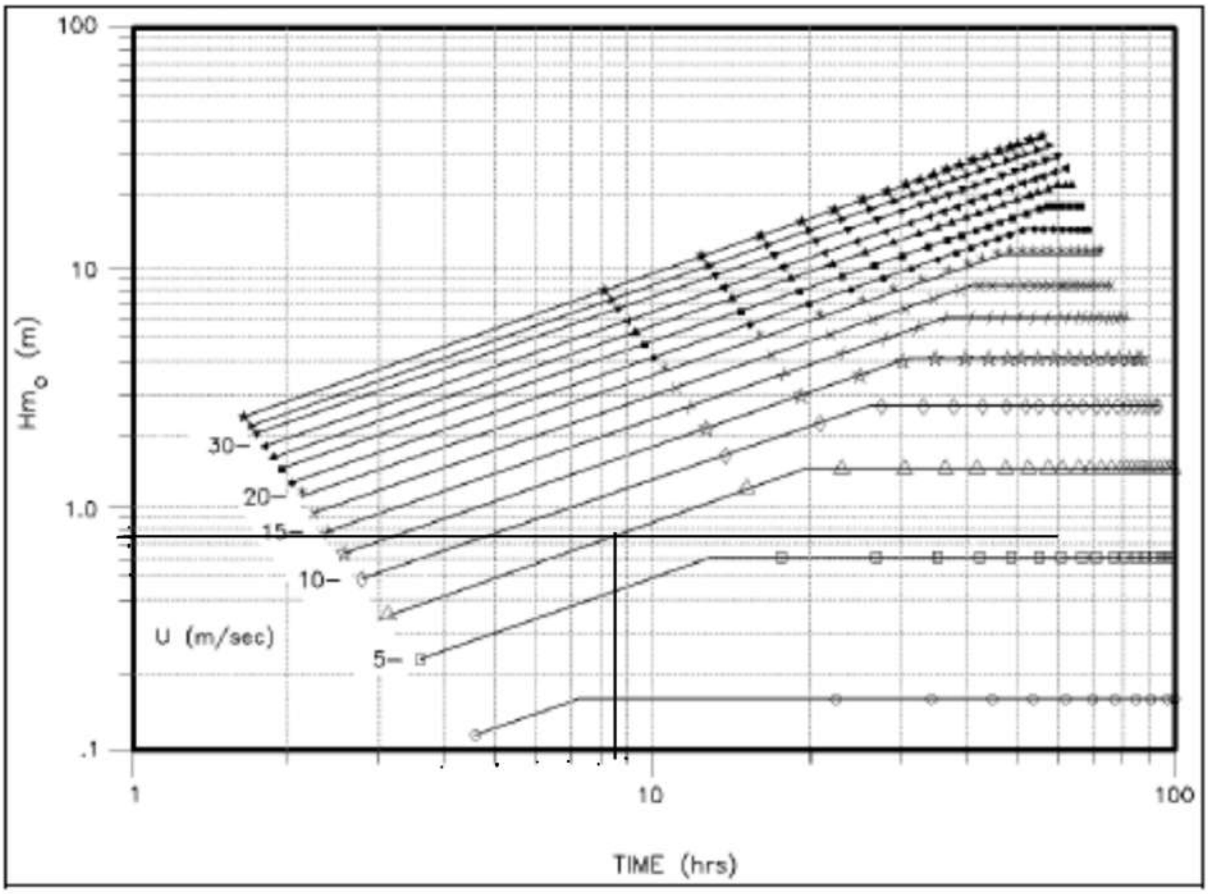


Figure II-2-25. Duration-limited wave heights (wind speeds are plotted in increments of 2.5 m/s)

(ii) From CEM Fig II - 2 - 26 : $\left.\begin{array}{l} D = 8.5\text{ hrs} \\ U = 7.5\text{ m/s} \end{array}\right\} \Rightarrow T = 3.4\text{ s}$

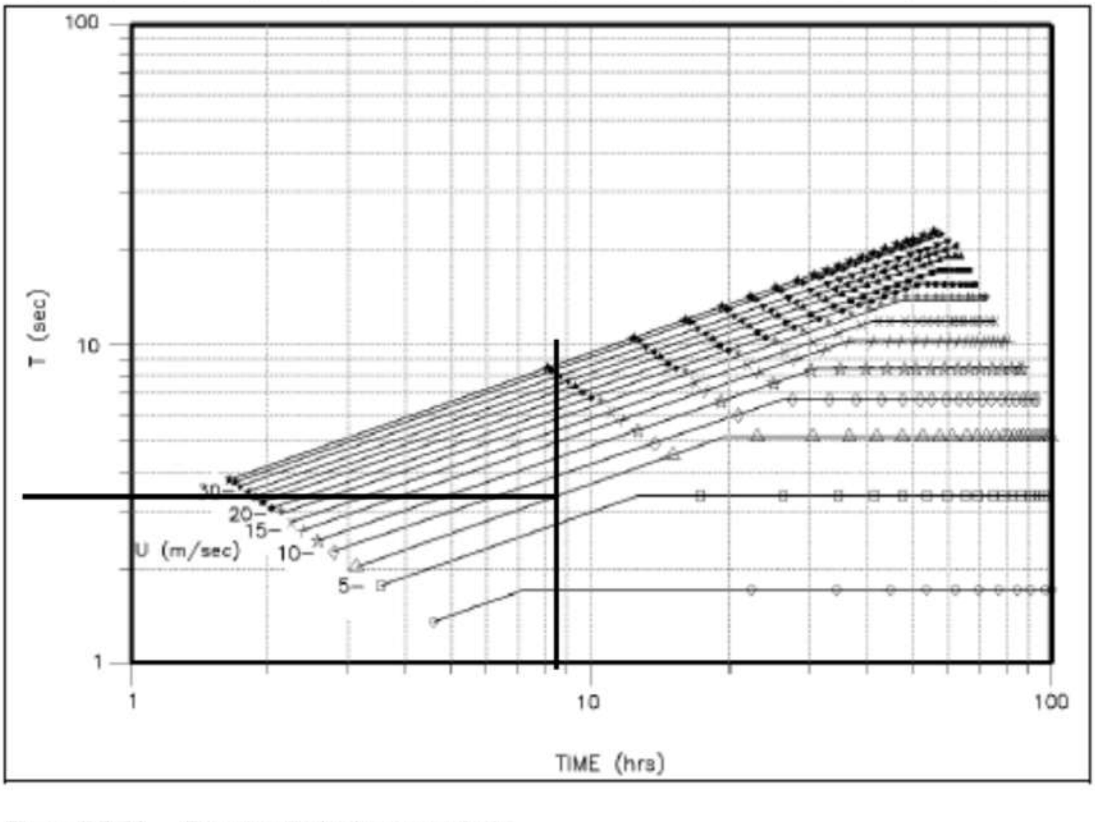


Figure II-2-26. Duration-limited wave periods

(c) $L_o = \frac{gT^2}{2\pi} = \frac{9.81 \times 3.4^2}{2\pi} = 18 \text{ m}$

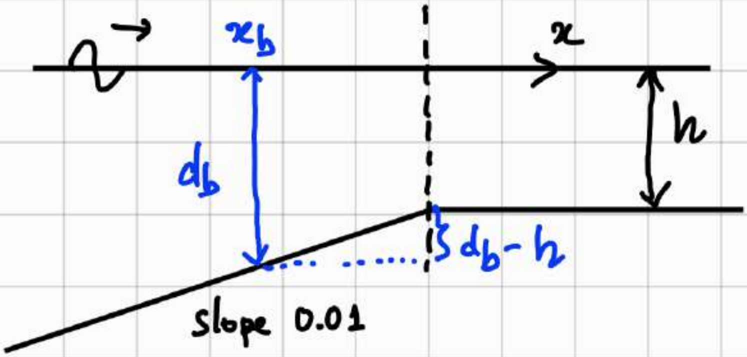
Deep water: $d/L > \frac{1}{2}$

$\Rightarrow d_{min} = \frac{1}{2} \times 18 = 9 \text{ m}$

$C_{g_o} = \frac{gT}{4\pi} = \frac{9.81 \times 3.4}{4\pi} = 2.65 \text{ m/s}$

2.

$T = 10 \text{s}$
$h = 1.5 - 2.0 \text{ m}$
$H_b / d_b = 0.78$



(a) $H_o = 2.14 \text{m}$

Since the wave propagates perpendicular towards the shore, there is no refraction effect (assume that bottom contours are parallel to the shore)

$$H_b = 0.78 d_b = K_s H_o$$

$$\Rightarrow 0.78 d_b = \left[\tanh(kd_b)\left\{1 + \frac{2kd_b}{\sinh(2kd_b)}\right\}\right]^{-0.5} H_o \qquad (1)$$

Dispersion relationship: $\left(\frac{2\pi}{T}\right)^2 = gk\tanh(kd_b)$

$$\Rightarrow \left(\frac{2\pi}{T}\right)^2 d_b = g\,kd_b \tanh(kd_b) \qquad (2)$$

Substitute (2) into (1) and let $\beta = kd_b$:

$$0.78\,\frac{g\beta\tanh(\beta)}{(2\pi/T)^2} = \left[\tanh(\beta)\left\{1+\frac{2\beta}{\sinh(2\beta)}\right\}\right]^{-0.5} H_o$$

With $T = 10\,s$, $H_o = 2.14\,m$ and $g = 9.81\,m/s^2$:

$$19.38\,\beta\tanh(\beta) = \left[\tanh(\beta)\left\{1+\frac{2\beta}{\sinh(2\beta)}\right\}\right]^{-0.5} \times 2.14$$

Solve for $\beta \Rightarrow \beta = 0.37$

$$\Rightarrow d_b = \frac{9.81 \times 0.37\tanh(0.37)}{(2\pi/10)^2} = 3.25\,m$$

At low tide $h = 1.5m$: $x_b = -\dfrac{3.25-1.5}{0.01} = -175\,m$

At high tide $h = 2.0m$: $x_b = -\dfrac{3.25-2.0}{0.01} = -125\,m$

$\therefore$ Region of wave breaking is from $x = -125m$ to $x = -175m$

(b) Breaker height index $\Omega_b = \dfrac{H_b}{H_o} = 0.56\left(\dfrac{H_o'}{L_o}\right)^{-1/5}$

No refraction $\Rightarrow H_o' = H_o$

$$\Rightarrow \frac{H_b}{H_o} = 0.56\left(\frac{H_o}{L_o}\right)^{-1/5} \Rightarrow \frac{H_b^5}{H_o^5} = 0.56^5 \times \frac{L_o}{H_o}$$

$$\left.\begin{aligned} \Rightarrow H_o &= \sqrt[4]{\frac{H_b^5}{0.56^5 L_o}} \\ L_o &= \frac{gT^2}{2\pi} = \frac{9.81 \times 10^2}{2\pi} = 156\,m \end{aligned}\right\} \Rightarrow H_o = 0.584\,H_b^{5/4}$$

At low tide $d = 1.5m$ $\Rightarrow H_b = 0.78 \times 1.5 = 1.17\,m$

$\Rightarrow H_o = 0.584 \times 1.17^{5/4} = 0.71\,m$

At high tide $d = 2.0m$ $\Rightarrow H_b = 0.78 \times 2.0 = 1.56\,m$

$\Rightarrow H_o = 0.584 \times 1.56^{5/4} = 1.02\,m$

$\therefore$ Maximum possible H_o for no wave breaking $= 0.71m$

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(c) $H_0 = 0.71$ m

Horizontal velocity of sediments:

$$u_{s,max} = \left[8\left(\gamma_s/\gamma - 1\right) g\, d_{50}\right]^{0.5}$$

$$= \left[8\left(2570/1000 - 1\right) \times 9.81 \times 1.5 \times 10^{-2}\right]^{0.5}$$

$$= 0.736 \text{ m/s}$$

- At low tide $d = 1.5$ m

Wave length: $L = \dfrac{gT^2}{2\pi} \tanh\left(\dfrac{2\pi d}{L}\right)$

Solve for L with $d = 1.5$m and $T = 10$s $\Rightarrow L = 38$ m

$\Rightarrow k = \dfrac{2\pi}{L} = 0.165$

$$K_s = \left[\tanh\left(0.165 \times 1.5\right)\left\{1 + 2 \times 0.165 \times 1.5 / \sinh(0.165 \times 1.5)\right\}\right]^{-0.5}$$

$$= 1.18$$

$H = K_s H_0 = 1.18 \times 0.71 = 0.84$ m

$d/L = 1.5/38 = 0.04 < 0.05 \Rightarrow$ shallow water

Horizontal velocity of water particles:

$$u_{w,max} = \frac{H}{2}\sqrt{\frac{g}{d}} = \frac{0.84}{2}\sqrt{\frac{9.81}{1.5}} = 1.07 \text{ m/s}$$

- At high tide $d = 2.0$m

Wave length $L = \dfrac{gT^2}{2\pi} \tanh\left(\dfrac{2\pi d}{L}\right) \Rightarrow L = 43.7$ m

$k = 2\pi / L = 0.144$

$K_s = 1.10$

$H = K_s H_0 = 0.78$ m

$d/L = \dfrac{2}{43.7} = 0.046 < 0.05$

$\Rightarrow$ shallow water

Horizontal velocity of water particles:

$$u_{w,max} = \frac{H}{2}\sqrt{\frac{g}{d}} = \frac{0.78}{2}\sqrt{\frac{9.81}{2}} = 0.864 \text{ m/s}$$

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Since sediment velocity is smaller than water velocity in both cases, the particles are not stable throughout the tidal fluctuations.

3.

$H_o = 0.8m$

$T = 5s$

(a) $d = 7.5m$

From CEM Fig II-3-6 : $\left.\begin{array}{l} \dfrac{d}{gT^2} = \dfrac{7.5}{9.81 \times 5^2} = 0.0306 \\ \theta_o = 45° \end{array}\right\} \Rightarrow K_R K_S = 0.87$

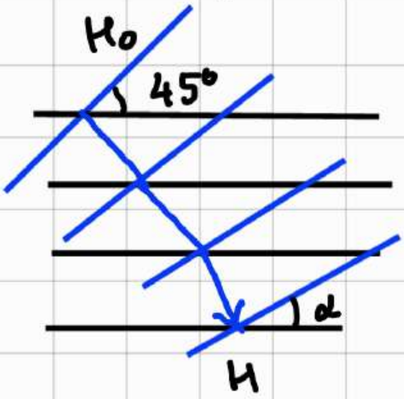
$\Rightarrow H = K_R K_S H_o = 0.87 \times 0.8 = 0.696 m$

(b) From CEM Fig II-3-6: $\left.\begin{array}{l} \dfrac{d}{gT^2} = 0.0306 \\ \theta_o = 45° \end{array}\right\} \Rightarrow K_R = 0.95$

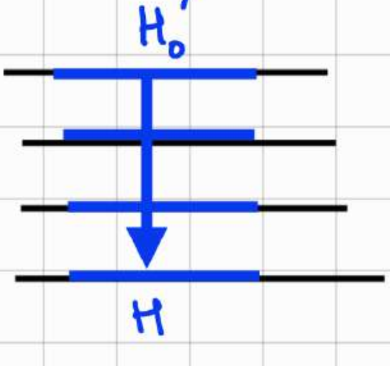
$\Rightarrow H_o' = K_R H_o = 0.95 \times 0.8 = 0.76 m$

Non-refracted deep water wave height H_o' is a hypothetical value taking away the refraction effect. The direction of non-refracted wave is perpendicular to the shore.

True Deepwater H_o



Non-refracted Deepwater H_o'



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(c) Piles are rigid structure ⇒ use $H_{1/100}$ for design

$H = H_{1/100} = 1.67 H_s \approx 1.67 \times 0.696 = 1.16\ m$

Check for breaking: $\frac{H}{d} = \frac{1.16}{7.5} < 0.78$ ⇒ non-breaking

$H_b = 0.78 d = 0.78 \times 7.5 = 5.85\ m$

Using Dean stream theory:

$$\left.\begin{array}{l} W = \frac{C_M D}{C_D H} = \frac{1.15 \times 0.5}{1.0 \times 1.16} = 0.50 \\ \frac{d}{gT^2} = 0.0306 \\ \frac{H}{gT^2} = \frac{1.16}{9.81 \times 5^2} = 0.0047 \end{array}\right\}$$

⇒ From Fig VI - 5 - 133 ⇒ $\varphi_M = 0.17$

∴ Maximum horizontal force $F_M = \varphi_m\, w\, C_D H^2 D$

$= 0.17 \times 10^4 \times 1.0 \times 1.16^2 \times 0.5$

$= 986\ N$

(d) Since the location is shielded by some islands, there would not be much wind blowing into this region. Swells are generated from wind elsewhere and have longer wave periods and larger wave fetch than wind waves. Although swells have smaller energy than wind waves, in this case swell component would be comparable with wind wave component as there is not much wind in the region.

4.

(a) Revetment : slope 1V: 3H, 2 units thick

$d = 3m \pm 0.75m$ (tidal)

$H_s = 1.2m$, $T = 8s$

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$\rho_s = 2600\ kg/m^3$
$\rho_w = 1025\ kg/m^3$

(i) Revetment is a flexible structure ⇒ use $H_{1/10}$ for design

$H = H_{1/10} = 1.27 H_s = 1.27 \times 1.2 = 1.52m$

Check for breaking:

- At low tide : $d = 3 - 0.75 = 2.25m$

$$\frac{H}{d} = \frac{1.52}{2.25} = 0.68 < 0.78 \rightarrow \text{non-breaking}$$

- At high tide : $d = 3 + 0.75 = 3.75m$

$$\frac{H}{d} = \frac{1.52}{3.75} = 0.41 < 0.78 \rightarrow \text{non-breaking}$$

(ii) From CEM Table VI-5-22 : $K_D = 4.0$ for rough angular stone with random placement, no damage

Hudson's formula:

$$M = \frac{\rho_s H^3}{K_D\left(\frac{\rho_s}{\rho_w} - 1\right)^3 \cot\alpha} = \frac{2600 \times 1.52^3}{4.0\left(\frac{2600}{1025} - 1\right)^3 \times 3} = 210\ kg$$

(iii) Since the revetment protects a power plant, run-up should be avoided ⇒ use $R_{2\%}$

$$L_0 = \frac{gT^2}{2\pi} = \frac{9.81 \times 8^2}{2\pi} = 99.9m$$

$$\xi_{om} = \tan\beta\left(\frac{H_s}{L_0}\right)^{-1/2} = \frac{1}{3}\left(\frac{1.2}{99.9}\right)^{-1/2} = 3.04$$

From CEM Fig VI-5-12 ⇒ $\frac{R_{2\%}}{H_s} = 1.95$

$R_{2\%} = 1.95 \times 1.2 = 2.34\ m$

Crest elevation $= 3 + 0.75 + 2.34 = 6.09\ m$

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(iv) Damage = 10 – 15%

From CEM Fig VI-5-22 $\Rightarrow K_D = 6.6$

Reduction in stone mass $= 1 - \frac{4.0}{6.6} = 0.39 = 39\%$

Reduced amount $= 39\% \times 210 = 83$ kg

(b)

(i) $P_e = 1 - \left(1 - \frac{1}{T_r}\right)^L$

$L = 60$ years

$P_e \leq 5\%$

$$\Rightarrow 1 - \left(1 - \frac{1}{T_r}\right)^{60} \leq 0.05$$

$$\Rightarrow 1 - \frac{1}{T_r} \geq 0.9991$$

$$\Rightarrow T_r \geq 1170$$

Return period = 1170 years

(ii) - Sea level rise : Increased water depth leads to increased crest elevation

- Extreme weather events such as floods, storms may require more conservative design