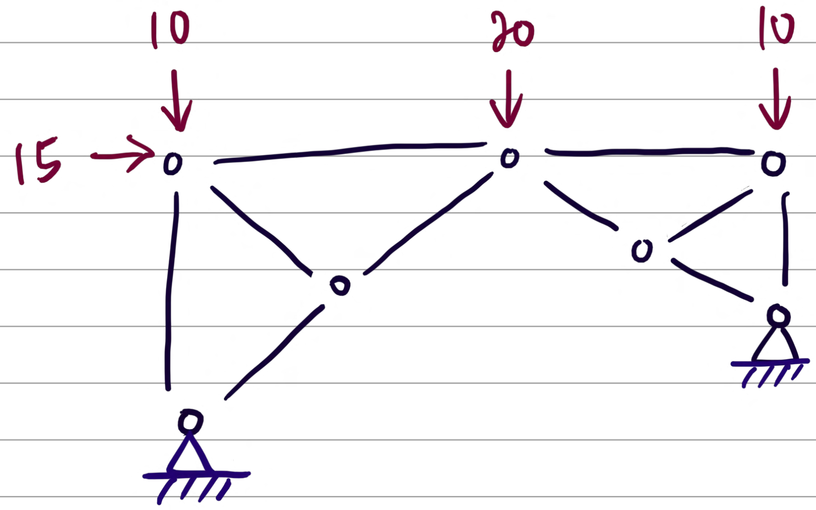
CV2011 AY21/22 Sem 1
Wong Jingheng
jwong125@e.ntu.edu.sg



1a No. of members m $= 10$
No. of rxn forces r $= 4$
No. of joints j $= 7$

$m + r = 2j$
No concurrent, parallel forces
No collapsible mechanism

$\therefore$ Truss is overall statically determinate

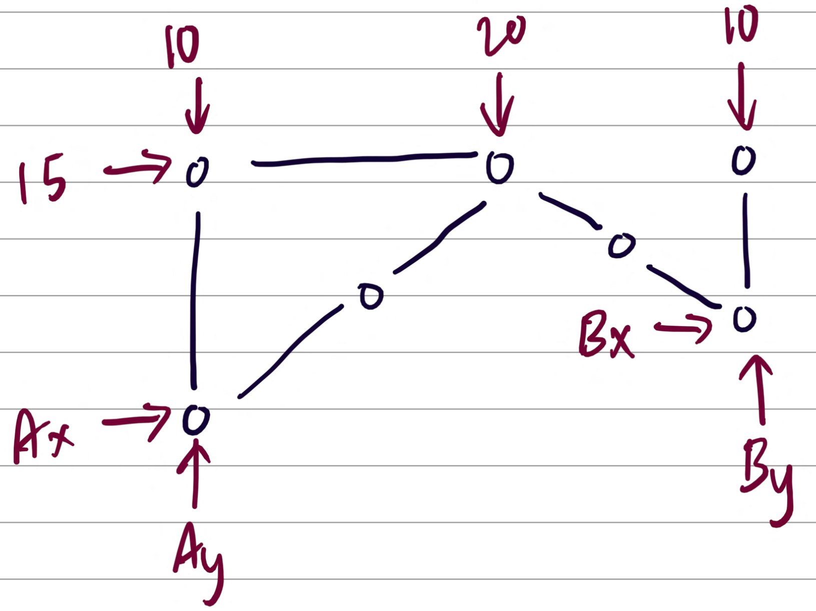
1b $\Sigma F_y = 0$: $A_y + B_y - 10 - 20 - 10 = 0$
$A_y + B_y = 40$

$\Sigma F_x = 0$: $A_x + B_x + 15 = 0$
$A_x + B_x = -15$

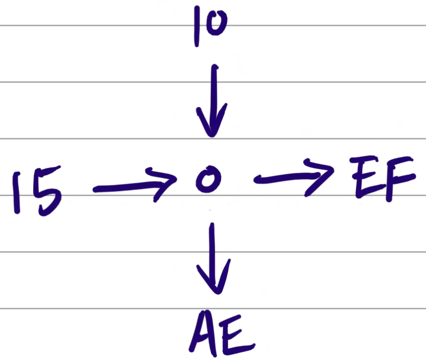
$\Sigma M_A = 0$: $-15(8) - 20(8) - 10(16) - B_x(2) + B_y(16) = 0$
$-2B_x + 16B_y = 440$

4 unknowns, 3 equations. Need 1 more equation

CE, DG and FG are zero-force members



Joint E



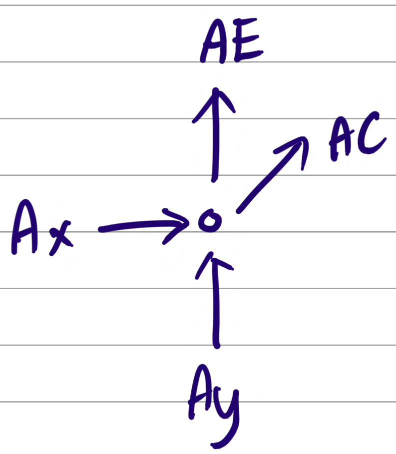
$\Sigma F_x = 0: EF + 15 = 0$

$EF = -15$

$\Sigma F_y = 0: -10 - AE = 0$

$AE = -10$

Joint A



$\Sigma F_x = 0: A_x + AC\left(\frac{1}{\sqrt{2}}\right) = 0$

$AC = -\sqrt{2}A_x$

$\Sigma F_y = 0: A_y + AE + AC\left(\frac{1}{\sqrt{2}}\right) = 0$

$\frac{1}{\sqrt{2}}AC = 10 - A_y$

$AC = 10\sqrt{2} - \sqrt{2}A_y$

Equating AC: $-\sqrt{2}A_x = 10\sqrt{2} - \sqrt{2}A_y$

$A_x = -10 + A_y$

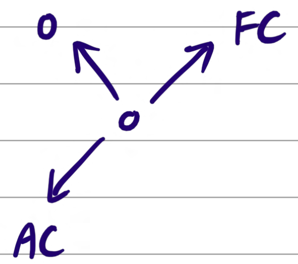
$A_x - A_y = -10$

$$\begin{aligned} A_y \qquad\qquad + B_y &= 40 \\ A_x \qquad + B_x \qquad\qquad &= -15 \\ -2B_x + 16B_y &= 440 \\ A_x - A_y \qquad\qquad &= -10 \end{aligned}$$

Solving simultaneously,

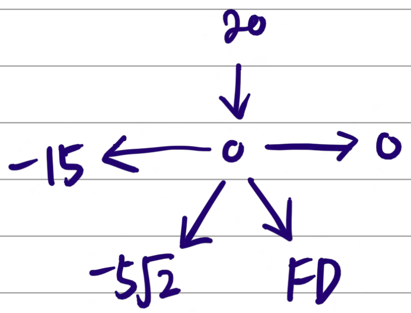
$A_x = 5\,\text{kN} \; (\rightarrow)$
$A_y = 15\,\text{kN} \; (\uparrow)$
$B_x = 20\,\text{kN} \; (\leftarrow)$
$B_y = 25\,\text{kN} \; (\uparrow)$

1c) Joint C



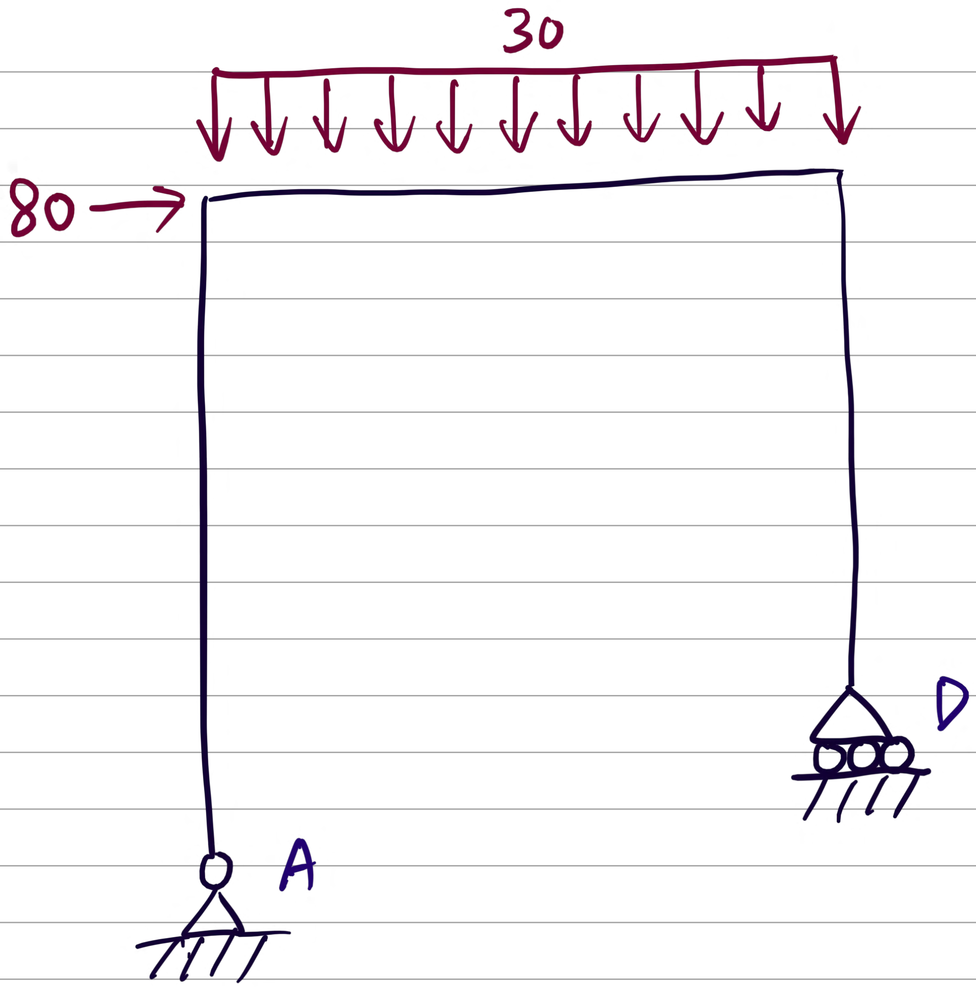
$FC - AC = 0$
$FC = AC = -\sqrt{2}A_x$
$= -5\sqrt{2}$

Joint F



$\Sigma F_x = 0: -(-15) - \left(\frac{-5\sqrt{2}}{\sqrt{2}}\right) + FD\left(\frac{4}{5}\right) = 0$
$-\frac{4}{5}FD = 20$
$FD = -25$

$FC = 7.07\,\text{kN}\,(C)$
$FD = 25\,\text{kN}\,(C)$



1a No. of members $n = 1$
No. of reaction forces $r = 3$

$r = 3n$
No concurrent, parallel forces
No collapsible mechanism

$\therefore$ Frame is statically determinate

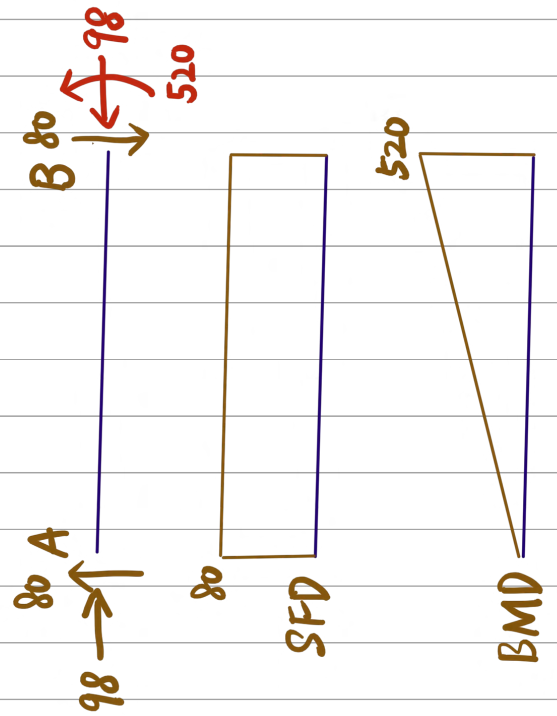
1b $\Sigma F_x = 0$: $A_x + 80 = 0$
$A_x = -80$

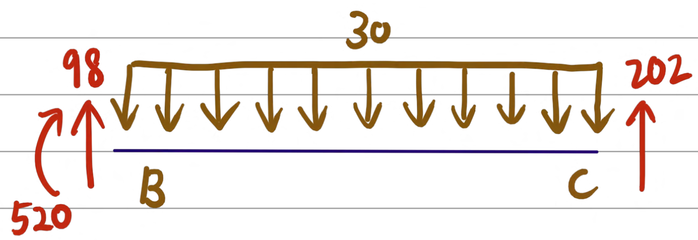
$\Sigma M_A = 0$: $-80(6.5) - 30(10)(5) + D_y(10) = 0$
$10D_y = 2020$
$D_y = 202$

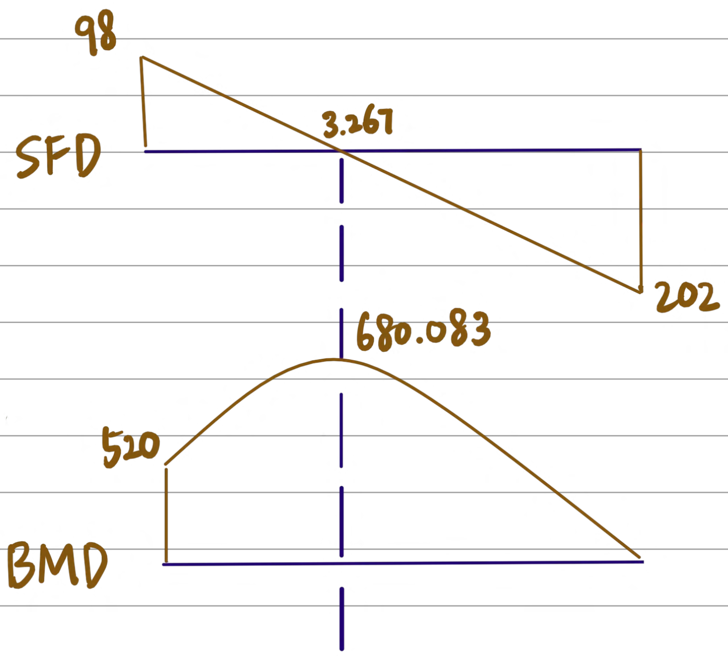
$\Sigma F_y = 0$: $A_y + D_y - 30(10) = 0$
$A_y = 98$

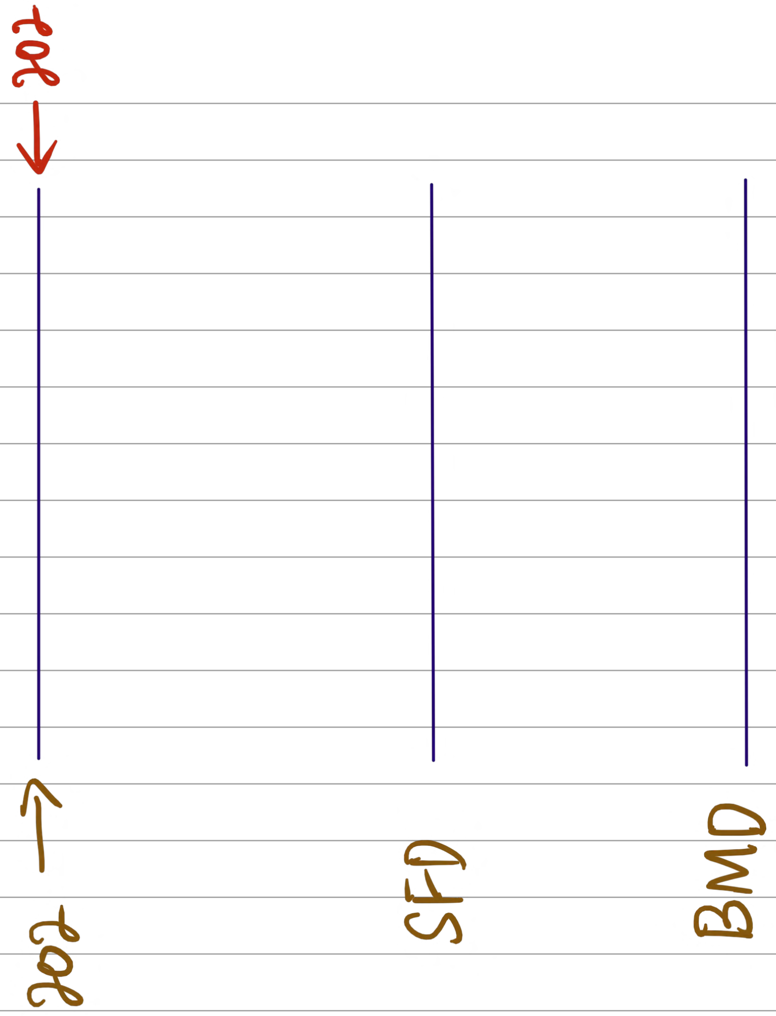
$A_x = 80\,kN\ (\leftarrow)$
$A_y = 98\,kN\ (\uparrow)$
$D_y = 202\,kN\ (\uparrow)$

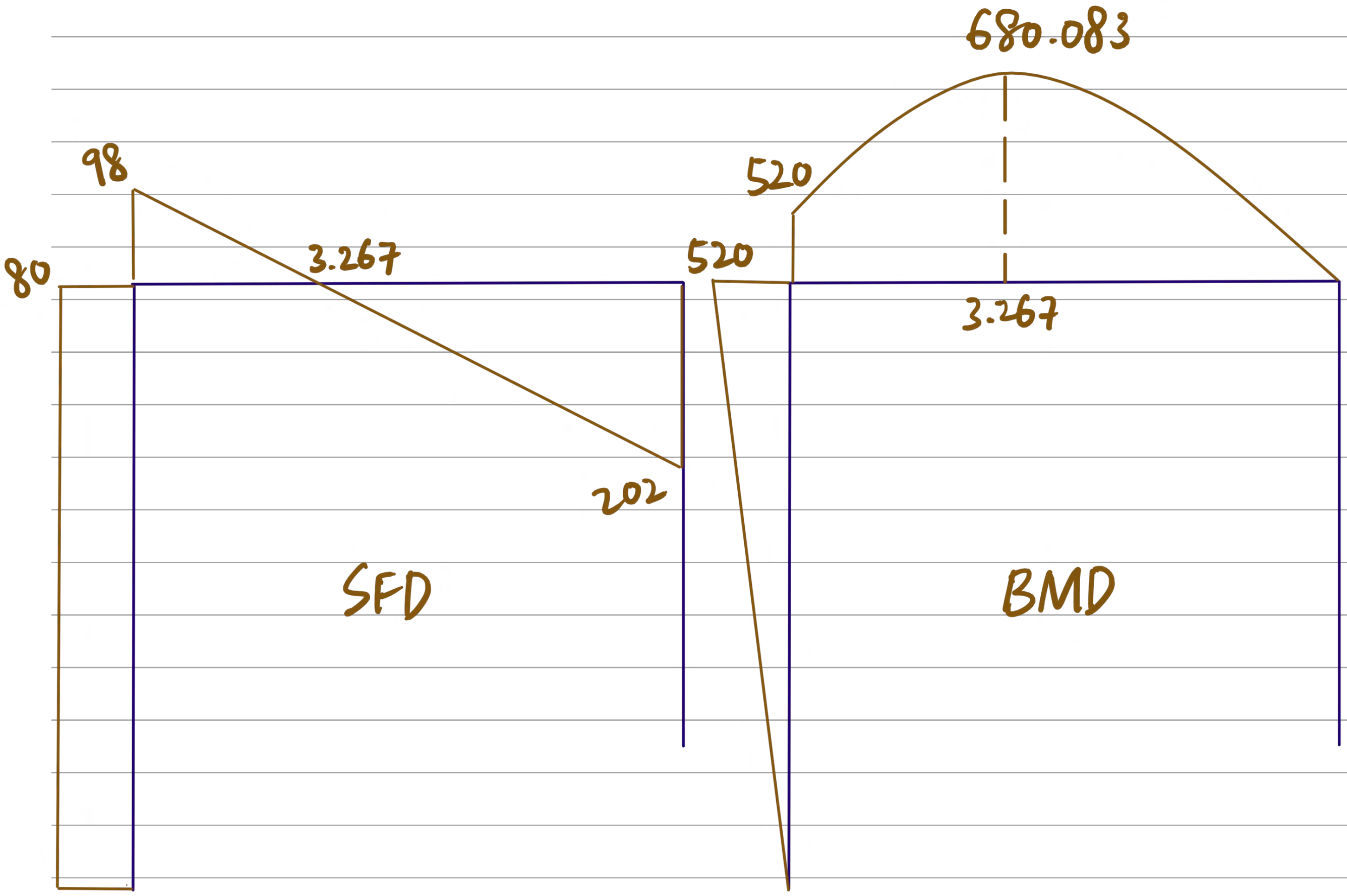
1c

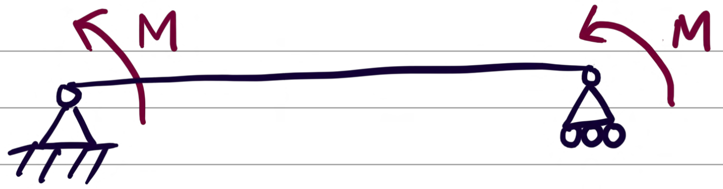







202
202
SFD
BMD


680.083
98
520
80
3.267
520
202
3.267
SFD
BMD



3a

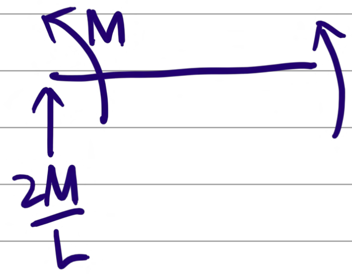
$$\Sigma M_A = 0: \quad M + M + B_y(L) = 0$$
$$LB_y = -2M$$
$$B_y = \frac{-2M}{L}$$

$$\Sigma F_y = 0: \quad A_y + B_y = 0$$
$$A_y = \frac{2M}{L}$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$M(x) = EIv''(x)$$
$$EIv(x) = \iint M(x)$$

$0 \leq x \leq L$



$$M(x) = -M + \frac{2M}{L}(x)$$
$$EIv' = -Mx + \frac{Mx^2}{L} + C_1$$
$$EIv = \frac{-Mx^2}{2} + \frac{Mx^3}{3L} + C_1x + C_2$$

Boundary condition

$v(0)=0$

$$-\frac{M(0)^2}{2}+\frac{M(0)^3}{3L}+C_1(0)+C_2=0$$

$C_2=0$

$v(L)=0$

$$-\frac{ML^2}{2}+\frac{ML^3}{3L}+C_1L=0$$

$$C_1L=\frac{ML^2}{6}$$

$$C_1=\frac{ML}{6}$$

$$v(x)=\frac{-Mx^2}{2EI}+\frac{Mx^3}{3LEI}+\frac{MLx}{6EI}$$

3b $$v'(x)=\frac{-Mx}{EI}+\frac{Mx^2}{LEI}+\frac{ML}{6EI}$$

Max deflection $\rightarrow v'(x)=0$

$$-Mx+\frac{Mx^2}{L}+\frac{ML}{6}=0$$

$$\frac{1}{L}x^2-x+\frac{L}{6}=0$$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(\frac{1}{L})(\frac{L}{6})}}{2(\frac{1}{L})}=\frac{1\pm\sqrt{1-\frac{4}{6}}}{2/L}$$

$$=\left(\frac{1\pm\sqrt{2/6}}{2}\right)L$$

$$x_1 = \left(\frac{1-\sqrt{2/6}}{2}\right)L \qquad x_2 = \left(\frac{1+\sqrt{2/6}}{2}\right)L$$

$$= 0.211L \qquad = 0.789L$$

For x_1, $\max v = \frac{-M(0.211L)^2}{2EI} + \frac{M(0.211L)^3}{3LEI} + \frac{ML(0.211L)}{6EI}$

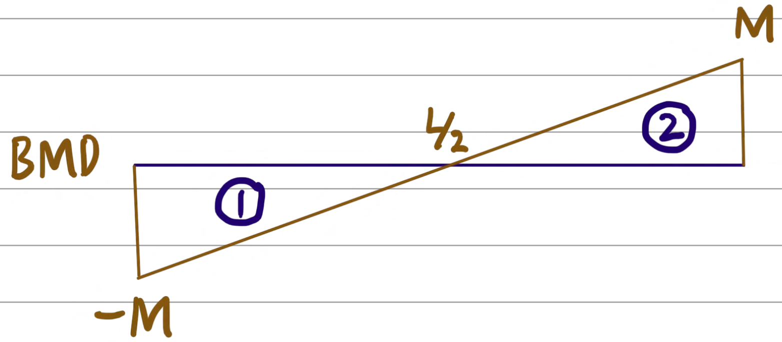
$$= \frac{0.016ML^2}{EI} \ (\uparrow)$$

For x_2, $\max v = \frac{-M(0.789L)^2}{2EI} + \frac{M(0.789L)^3}{3LEI} + \frac{ML(0.789L)}{6EI}$

$$= \frac{-0.016ML^2}{EI}$$

$$= \frac{0.016ML^2}{EI} \ (\downarrow)$$

3c



i	A_i	x_{ci}
1	$\frac{-ML}{4EI}$	$\frac{L}{6}$
2	$\frac{ML}{4EI}$	$\frac{5L}{6}$

$$v_B - v_A - (x_B - x_A)\theta_A = \int_A^B (x_B - x)\frac{M}{EI}\,dx$$

$$= \sum (x_B - x_{ci})A_i$$

$$0 - 0 - (L-0)\theta_A = \left(L-\frac{L}{6}\right)\left(\frac{-ML}{4EI}\right) + \left(L-\frac{5L}{6}\right)\left(\frac{ML}{4EI}\right)$$

$$-L\theta_A = \frac{-5ML^2}{24EI} + \frac{ML^2}{24EI}$$

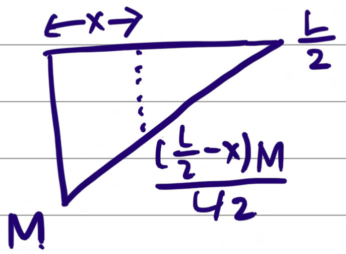
$$\theta_A = \frac{ML}{6EI}$$

Let location of max deflection be c.

$$\theta_c - \theta_A = \int_A^c \frac{M}{EI}\,dx$$

$$= \sum A_i$$

Scenario 1: $0 \le x \le L/2$



$$0 - \left(\frac{ML}{6EI}\right) = \frac{-1}{EI}\left[\left(\frac{1}{2}\right)\left(\frac{L}{2}\right)(M) - \left(\frac{1}{2}\right)\left(\frac{L}{2}-x\right)\left(\frac{(\frac{L}{2}-x)M}{L/2}\right)\right]$$

$$-\frac{ML}{6} = \frac{-ML}{4} + \left(\frac{L}{4}-\frac{x}{2}\right)\left(M - \frac{2Mx}{L}\right)$$

$$\frac{ML}{12} = \frac{ML}{4} - \frac{Mx}{2} - \frac{Mx}{2} + \frac{Mx^2}{L}$$

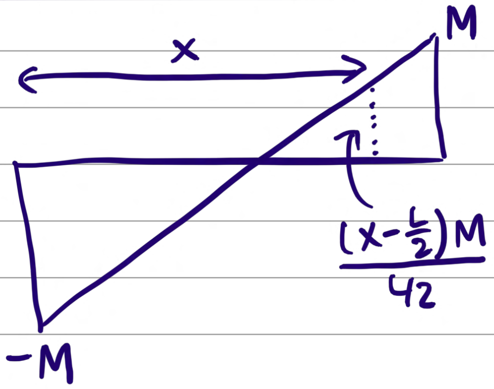
$$\frac{x^2}{L} - x + \frac{L}{6} = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4\left(\frac{1}{L}\right)\left(\frac{L}{6}\right)}}{2\left(\frac{1}{L}\right)}$$

$$= \frac{1 \pm \sqrt{1 - 2/3}}{2/L}$$

$$= 0.211L \text{ or } 0.789L$$ (rej. initially as $0.789L > \frac{L}{2}$)

Scenario 2: $\frac{L}{2} \le x \le L$



$$0 - \left(\frac{ML}{6EI}\right) = \frac{1}{EI}\left[-\left(\frac{1}{2}\right)\left(\frac{L}{2}\right)(M) + \left(\frac{1}{2}\right)\left(x - \frac{L}{2}\right)\left(\frac{\left(x - \frac{L}{2}\right)M}{L/2}\right)\right]$$

$$\frac{-ML}{6} = \frac{-ML}{4} + \left(\frac{x}{2} - \frac{L}{4}\right)\left(\frac{2Mx}{L} - M\right)$$

Same equation as Scenario 1.

$$\therefore x = 0.211L \text{ or } 0.789L$$

For $x = 0.211L$

$$V_C - V_A - (x_C - x_A)\theta_A = \int_A^C (x_C - x)\frac{M}{EI}\,dx = \sum (x_C - x_{ci})A_i$$

$$V_C - 0 - (0.211L - 0)\left(\frac{ML}{6EI}\right) = \left[0.211L - \left(\frac{1}{3}\right)(0.211L)\right]\left[\left(-\frac{1}{2}\right)(0.211L)\left(\frac{0.211M}{0.5EI}\right)\right]$$

$$EIV_C - \frac{0.211ML^2}{6} = (0.4106L)(-0.044521ML)$$

$$V_C = \frac{0.0169ML^2}{EI} \ (\uparrow)$$

For $x = 0.789L$

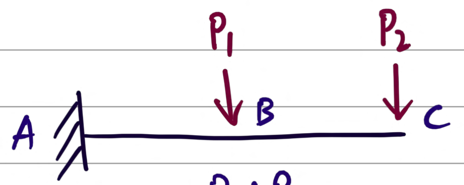
$$v_C - v_A - (x_C - x_A)\theta_A = \int_A^C (x_C - x)\frac{M}{EI}\,dx = \sum (x_C - x_{Ci}) A_i$$

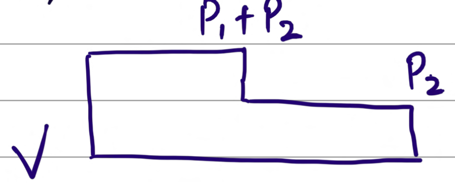
$$v_C - 0 - (0.789L)\left(\frac{ML}{6EI}\right)$$
$$= \left(0.789L - \frac{L}{6}\right)\left(\frac{-ML}{4EI}\right) + \left(\frac{1}{3}\right)(0.289L)\left(\frac{1}{2}\right)(0.289L)\left(\frac{0.289M}{0.5EI}\right)$$

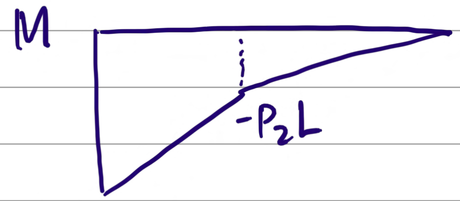
$$EIv_C - 0.1315ML^2 = -0.155583ML^2 + 0.008046ML^2$$

$$v_C = \frac{0.016ML^2}{EI} \;(\downarrow)$$

4ai







$$U_i = \int_A^C \frac{M^2}{2EI}\,dx$$

$$2EIU_i = \int_A^C M^2\,dx$$

Integration tough. Using appendix,

Trapezoid section

$$\int_A^B M^2\,dx = \frac{1}{6}\Big\{(P_1+2P_2)L\big[2(P_1+2P_2)L+P_2L\big]$$

$$+P_2L\big[(P_1+2P_2)L+2P_2L\big]\Big\}L$$

$$=\frac{L}{6}\Big\{2(P_1+2P_2)^2L^2+P_2(P_1+2P_2)L^2$$

$$+P_2(P_1+2P_2)L^2+2P_2^2L^2\Big\}$$

$$=\frac{L}{6}\Big\{2P_1^2L^2+8P_1P_2L^2+8P_2^2L^2+P_1P_2L^2+2P_2^2L^2$$

$$+P_1P_2L^2+2P_2^2L^2+2P_2^2L^2\Big\}$$

$$=\frac{L}{6}\Big\{2P_1^2L^2+10P_1P_2L^2+14P_2^2L^2$$

$$=\frac{P_1^2L^3}{3}+\frac{5P_1P_2L^3}{3}+\frac{7P_2^2L^3}{3}$$

Triangle section

$$\int_B^C M^2\,dx = \frac{1}{3}(P_2L)(P_2L)L$$

$$=\frac{P_2^2L^3}{3}$$

$$2EI\,U_i = \frac{P_1^2L^3}{3}+\frac{5P_1P_2L^3}{3}+\frac{8P_2^2L^3}{3}$$

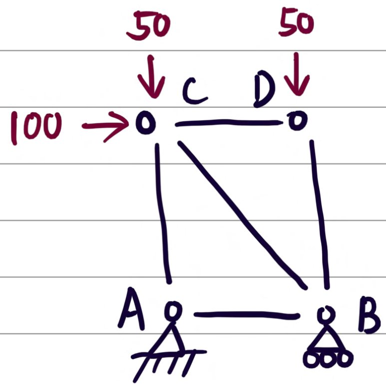
$$U_i = \frac{P_1^2L^3}{6EI}+\frac{5P_1P_2L^3}{6EI}+\frac{8P_2^2L^3}{6EI}$$

4aii By Castigliano's Theorem,

$$\Delta_B = \frac{\partial U_i}{\partial P_1} = \frac{P_1L^3}{3EI}+\frac{5P_2L^3}{6EI}$$

$$\Delta_C = \frac{\partial U_i}{\partial P_2} = \frac{5P_1L^3}{6EI}+\frac{8P_2L^3}{3EI}$$

4bi

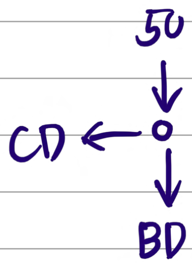


$$\Sigma M_A = 0: \quad -100(4) - 50(3) + B_y(3) = 0$$
$$3B_y = 550$$
$$B_y = \frac{550}{3}$$

$$\Sigma F_y = 0: \quad A_y + B_y - 50 - 50 = 0$$
$$A_y = 100 - \frac{500}{3}$$
$$= -\frac{200}{3}$$

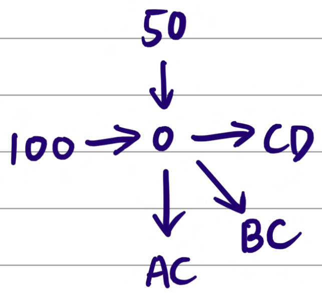
$$\Sigma F_x = 0: \quad 100 + A_x = 0$$
$$A_x = -100$$

Joint D



$$\Sigma F_x = 0: \quad -CD = 0$$
$$CD = 0 \quad \text{(2FM)}$$
$$\Sigma F_y = 0: \quad -50 - BD = 0$$
$$BD = -50$$

Joint C

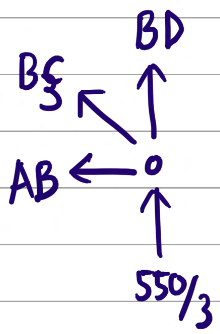


$$\Sigma F_x = 0: \quad 100 + CD + BC\left(\frac{3}{5}\right) = 0$$
$$\frac{3}{5}BC = -100$$
$$BC = \frac{-500}{3}$$

$$\Sigma F_y = 0: \quad -50 - AC - BC\left(\frac{4}{5}\right) = 0$$

$$AC = -50 + \left(\frac{500}{3}\right)\left(\frac{4}{5}\right)$$
$$= \frac{250}{3}$$

Joint B



$\Sigma F_y = 0: \quad BD + BC\left(\frac{4}{5}\right) + \frac{550}{3} = 0$

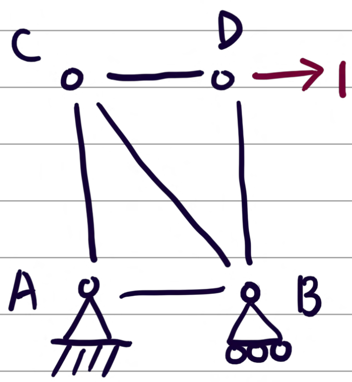
$BD = \left(\frac{500}{3}\right)\left(\frac{4}{5}\right) - \frac{500}{3}$

$= \frac{-100}{3}$

$\Sigma F_x = 0: \quad -AB - BC\left(\frac{3}{5}\right) = 0$

$AB = \frac{3}{5}\left(\frac{500}{3}\right)$

$= 100$



$\Sigma M_A = 0: \quad -1(4) + B_y(3) = 0$

$3B_y = 4$

$B_y = 4/3$

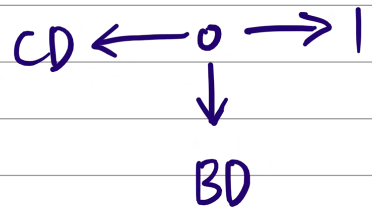
$\Sigma F_y = 0: \quad A_y + B_y = 0$

$A_y = -4/3$

$\Sigma F_x = 0: \quad A_x + 1 = 0$

$A_x = -1$

Joint D



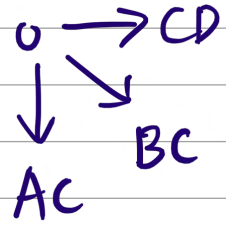
$\Sigma F_x = 0: \quad -BD = 0$

$BD = 0 \text{ (ZFM)}$

$\Sigma F_y = 0: \quad 1 - CD = 0$

$CD = 1$

Joint C



$\Sigma F_x = 0: CD + BC\left(\frac{3}{5}\right) = 0$

$\frac{3}{5} BC = -1$

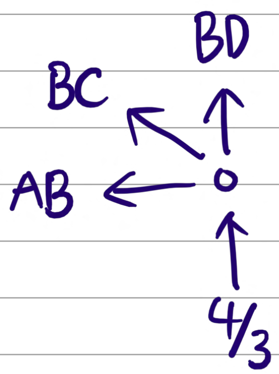
$BC = -5/3$

$\Sigma F_y = 0: -AC - BC\left(\frac{4}{5}\right) = 0$

$AC = \left(\frac{5}{3}\right)\left(\frac{4}{5}\right)$

$= \frac{4}{3}$

Joint B



$\Sigma F_x = 0: -AB - BC\left(\frac{3}{5}\right) = 0$

$AB = \left(\frac{5}{3}\right)\left(\frac{3}{5}\right)$

$= 1$

Member	N	n	L	NnL
AB	100	1	3	300
AC	$250/3$	$4/3$	4	$4000/9$
BC	$-500/3$	$-5/3$	5	$12500/9$
BD	$-100/3$	0	4	0
CD	0	1	3	0

$$\Delta_D = \sum \frac{NnL}{AE} = \frac{6400/3}{AE} \; (\rightarrow)$$

4bii

$$\frac{6400/3}{AE} \leq 10$$

$$\frac{(6400/3)(10^3)(10^3)}{(70 \times 10^3)\,10} \leq A$$

$$A \geq 3048 \text{ mm}^2$$