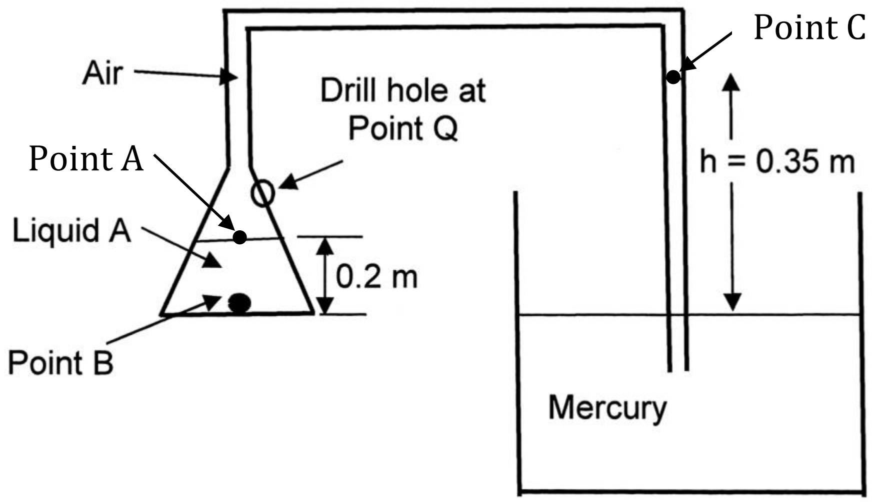
Name: Owen
Paper: CV1012 – Fluid Mechanics 2021–2022 Sem 1

1. (a) (i)



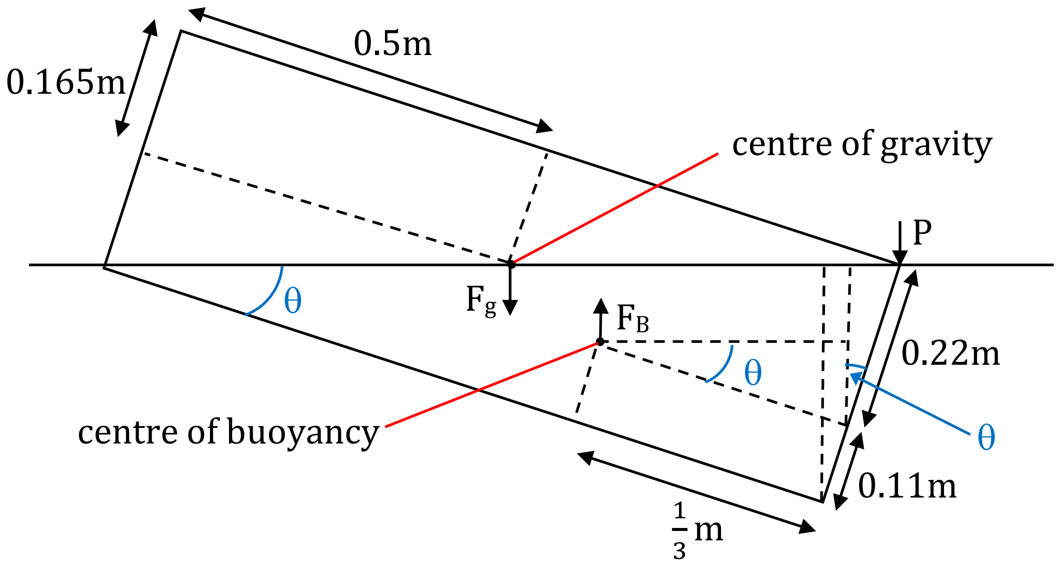
Assuming $P_{atm} = 0$, $\rho_{air} = 0$,

$P_C + \rho_{mercury}gh = 0$
$P_C = P_A$
$P_B = P_A + \rho_A g(0.2)$
$= -(13600)(9.81)(0.35) + (1250)(9.81)(0.2)$
$= -44243.1$ Pa
$= \underline{\underline{-0.442 \text{ bar}}}$

1. (a) (ii)

With hole, $P_C = P_A = P_{atm} = 0$
$P_B = \rho_A g(0.2)$
$= (1250)(9.81)(0.2)$
$= \underline{\underline{2452.5 \text{ Pa}}}$
$\rho_{mercury}gh = 0$
$h = \underline{\underline{0}}$

1. (b) (i)



1. (b) (ii)

Volume of block = (0.2)(1)(0.33)

$= 0.066\ m^3$

$\theta = \tan^{-1}\left(\frac{0.33}{1}\right)$

$= 18.263°$

$F_B = 0.5(0.066)(1000)(9.81)$

$= 323.73$ N

$F_g = (0.066)(\rho_s)(9.81)$

$= 0.64746\ \rho_s$

Taking pivot at P,

$\sum M = 0$

$F_g \sqrt{0.5^2 + 0.165^2} = F_B\left(\frac{1}{3}\cos(\theta) + 0.22\sin(\theta)\right)$

$0.64746\ \rho_s\sqrt{0.5^2 + 0.165^2} = 323.73\ (0.38549)$

$\rho_s = \underline{\underline{366\ kg/m^3}}$

Taking upwards as positive,

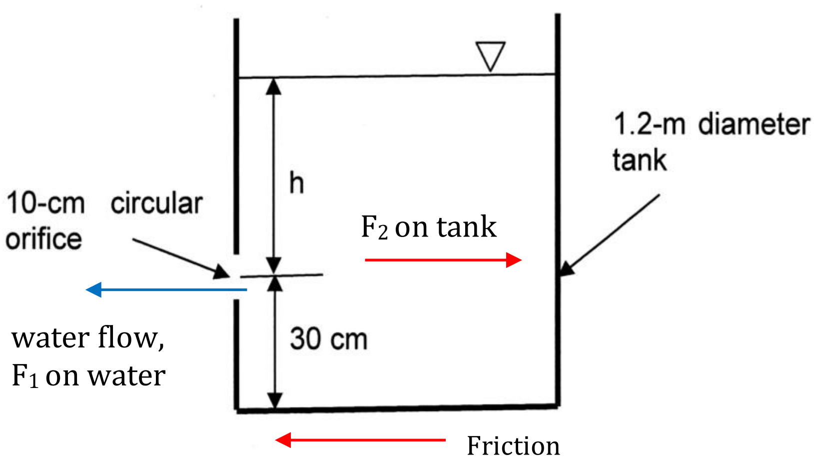
$\sum F_y = 0$

$F_B - F_g - P(0.2) = 0$

$323.73 - 0.64746(366.07) - P(0.2) = 0$

$P = \underline{\underline{434\ N/m}}$

2. (a) (i)



Assuming $P_{atm} = 0$, V = 0 at top of tank, no head loss,
Bernoulli's equation top of tank to orifice:

$h + 0.3 = \frac{V^2}{2g} + 0.3$

$h = \frac{V^2}{2g}$

$V^2 = 2gh$

$F_1 = M_{out} - M_{in}$

$= \rho QV - 0$

$= \rho V^2 A$

$= 2000gh(\pi/4)(0.1)^2 = F_2 =$ Friction, when tank just start to move.

Friction $= \mu N$

$= 0.01[240 + \rho g(\pi/4)(1.2)^2(h+0.3)]$

$2000gh(\pi/4)(0.1)^2 = 0.01[240 + \rho g(\pi/4)(1.2)^2(h+0.3)]$

h = 0.82705 m

Total height = 0.82705 + 0.3

= <u>1.127 m</u>

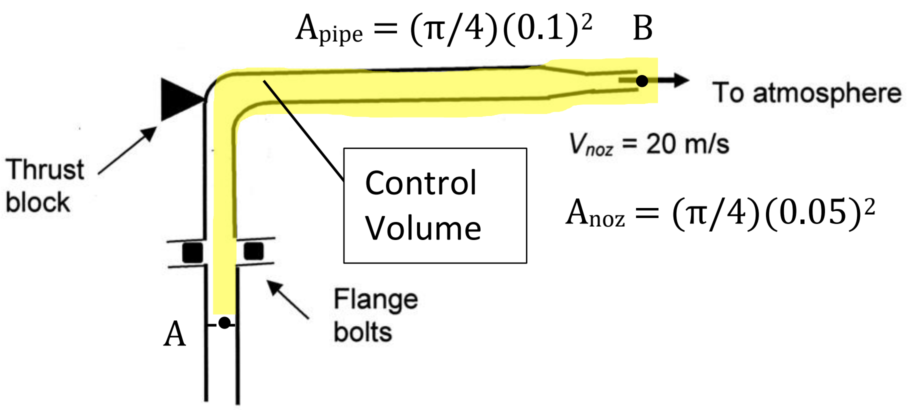
2. (a) (ii)

1. Water flows out horizontally in a 10cm diameter circle at constant speed.
2. Water flows out at a height of 30cm from the bottom.

2. (a) (iii)

Rightwards.

2. (b) (i)



$Q_B = 20(\pi/4)(0.05)^2$

$\quad = Q_A$ (continuity)

$V_A = \frac{20(\pi/4)(0.05)2}{(\pi/4)(0.1)^2}$

$\quad = 5$ m/s

Assuming $z_A = z_B$, $P_{atm} = 0$, no head loss,
Bernoulli's equation A to B:

$$\frac{P_A}{\rho g} + \frac{V_A{}^2}{2g} = \frac{V_B{}^2}{2g}$$

$$P_A = 1000g\left(\frac{20^2}{2g} - \frac{5^2}{2g}\right)$$

$$= \underline{\underline{187.5 \text{ kPa}}}$$

2. (b) (ii)

Momentum equation in x-direction (right as positive):

$F_x = M_{out} - M_{in}$

$\quad = \rho Q_B V_B - 0$

$\quad = 1000(20(\pi/4)(0.05)^2)(20)$

$\quad = \underline{\underline{785 \text{ N}}}$

Momentum equation in y-direction (up as positive):

$F_{net,y} = M_{out} - M_{in}$

$\quad = 0 - \rho Q_A V_A$

$\quad = -1000(20(\pi/4)(0.05)^2)(5)$

$\quad = -196.35$ N

$F_{net,y} = P_A A_{pipe} + F_y$

$F_y = -196.35 - 187500\pi(0.05)^2$

$\quad = \underline{\underline{-1669 \text{ N}}}$

3. (a)

1. Dimensional analysis simplifies problems by reducing the number of variables to work with, allowing analysis to be performed in an organized, meaningful, and systematic manner.
2. Dimensional analysis helps to solve problems that cannot be solved analytically without the use of experimental data.

3. (b) (i)

$Re = \frac{VL}{\upsilon}$

For Re similarity,

$V_p L_p = V_m L_m$, if $\upsilon_p = \upsilon_m$

$\frac{V_p}{V_m} = \frac{L_m}{L_p}$

$V_r = (L_r)^{-1}$

$Fr = \frac{V}{\sqrt{gL}}$

For Fr similarity,

$\frac{V_p}{\sqrt{L_p}} = \frac{V_m}{\sqrt{L_m}}$, if $g_m = g_p$

$\frac{V_p}{V_m} = \sqrt{\frac{L_p}{L_m}}$

$V_r = (L_r)^{0.5}$

Hence, due to a conflict in the length scale used, it is not practical to satisfy both the Re and Fr similarity simultaneously if the same fluid is used.

3. (b) (ii)

For both Re and Fr similarity,

$\frac{V_p}{V_m} = \frac{L_m}{L_p}\frac{\upsilon_p}{\upsilon_m} = \sqrt{\frac{L_p}{L_m}}$, if $g_m = g_p$

$\frac{\upsilon_p}{\upsilon_m} = \frac{L_p}{L_m}\sqrt{\frac{L_p}{L_m}}$

$\upsilon_r = \underline{\underline{(L_r)^{1.5}}}$

3. (c)

Assumptions:
1. Fully developed, steady, incompressible, Newtonian fluid flow.

For Re similarity,

$$\frac{Q_p}{Q_m} = \frac{V_p}{V_m}\left(\frac{L_p}{L_m}\right)^2$$

$$= \frac{L_p}{L_m}$$

$$Q_m = Q_p \frac{L_m}{L_p}$$

$$= 0.1\left(\frac{1}{10}\right)$$

$$= \underline{\underline{0.01\ m^3/s}}$$

For both Re and Fr similarity,

$$\upsilon_r = (L_r)^{1.5}$$

$$= (10)^{1.5}$$

$$= \underline{\underline{31.62}}$$

3. (d) (i)

$V^2 = \frac{Q^2}{A^2}$

$= \frac{(0.1)^2}{((\pi/4)(0.3)^2)^2}$

$= 2.0014\ m^2/s^2$

Entrance loss $= 0.5\,\frac{V^2}{2g}$

Exit loss $= 1.0\,\frac{V^2}{2g}$

$h_f = \frac{8fLQ^2}{g\pi^2D^5}$

$= \frac{8(0.02)(200)(0.1)^2}{(9.81)\pi^2(0.3)^5}$

$= 1.3601$ m

Assuming $P_{atm} = 0$, V = 0 at top of reservoir,

Bernoulli's equation A to B:

$z_A + h_p = z_B + h_f + 0.5\,\frac{V^2}{2g} + 1.0\,\frac{V^2}{2g}$

$h_p = z_B + h_f + 0.5\,\frac{V^2}{2g} + 1.0\,\frac{V^2}{2g} - z_A$

$= 130 + 1.3601 + 0.5\,\frac{2.0014}{2(9.81)} + 1.0\,\frac{2.0014}{2(9.81)} - 100$

$= 31.513$ m

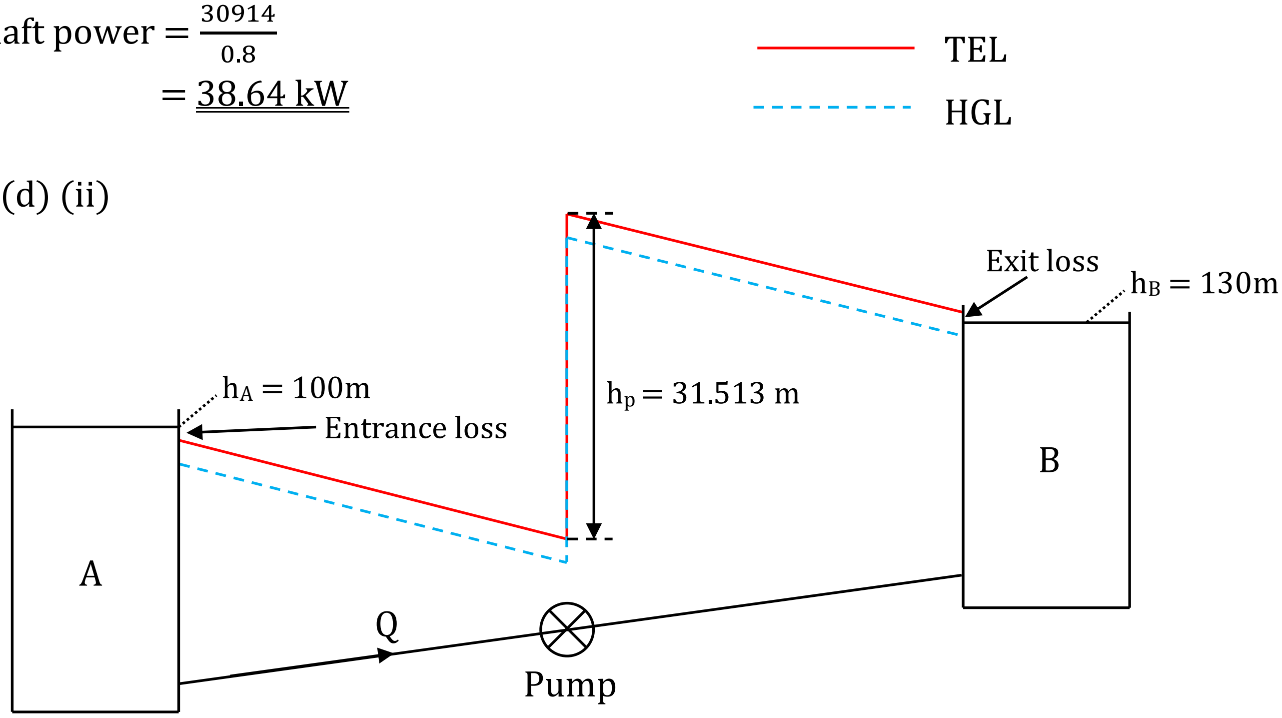
Fluid power $= (1000)(9.81)(0.1)(31.513)$

$= 30914$ W

Shaft power $= \frac{30914}{0.8}$

$=$ 38.64 kW

3. (d) (ii)



4. (a)

1. Re < 2100, in laminar flow regime, f is a function of Re only & is independent of ε/D, the equation is f = 64/Re.
2. 2100 < Re <4000, in transition range, f is uncertain as flow may be laminar or turbulent.
3. Re > 4000 but not in wholly turbulent flow regime, f depends on both Re and ε/D.
4. Re very large and in the wholly turbulent flow regime, f depends on ε/D only, independent of Re.

4. (b)

$P = f(Q, D, \omega, H, \rho, g)$

Their dimesions are:

$P = [ML^2T^{-3}]$

$Q = [L^3T^{-1}]$

$D = [L]$

$\omega = [T^{-1}]$

$H = [L]$

$\rho = [ML^{-3}]$

$g = [LT^{-2}]$

Using D, ω, ρ as repeating variables, ∏ term for P:

$\prod_1 = P\ \rho^a\ \omega^b\ D^c$

Using MLT system:

$ML^2T^{-3}M^aL^{-3a}T^{-b}L^c = M^0L^0T^0$

$1 + a = 0$

$2 + c - 3a = 0$

$-3 - b = 0$

Solving: a = -1, c = -5, b = -3

$$\prod_1 = \frac{P}{\rho\omega^3 D^5}$$

4. (c) (i)

Assuming $P_{atm} = 0$, $v = 0$ at top of reservoir,
$h_A = 60$ m, $h_B = 50$ m, $h_C = 30$ m, $h_D = 10$ m
Ignoring minor losses,
$h_f = h_1 - h_2$
$= KQ^2$
$|Q| = \sqrt{\frac{|h_1 - h_2|}{K}}$
$Q_1 + Q_2 + Q_3 + Q_4 = 0$ (continuity, take into J as positive and out of J as negative)

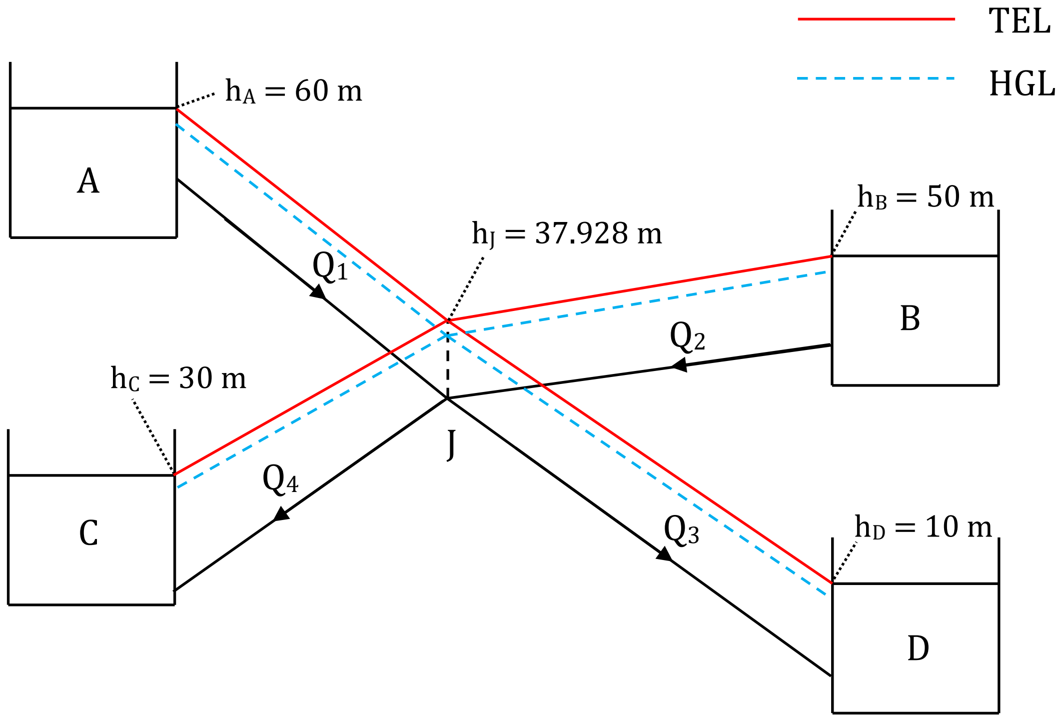
$|Q_1| = \sqrt{\frac{|60 - h_J|}{476}}$
$|Q_2| = \sqrt{\frac{|50 - h_J|}{272}}$
$|Q_3| = \sqrt{\frac{|30 - h_J|}{408}}$
$|Q_4| = \sqrt{\frac{|10 - h_J|}{340}}$

Guess:
Q_1 into J, Q_2 into J, Q_3 into J, Q_4 out of J
Using G.C., solve for $|Q_1| + |Q_2| + |Q_3| - |Q_4| = 0$, $10 < h_J < 30$
No solution for h_J

Guess:
Q_1 into J, Q_2 into J, Q_3 out of J, Q_4 out of J
Using G.C., solve for $|Q_1| + |Q_2| - |Q_3| - |Q_4| = 0$, $30 < h_J < 50$
$h_J = 37.928$ m

Thus,
$Q_1 = \underline{\underline{0.215 \text{ m}^3\text{/s into J}}}$
$Q_2 = \underline{\underline{0.211 \text{ m}^3\text{/s into J}}}$
$Q_3 = \underline{\underline{0.139 \text{ m}^3\text{/s out of J}}}$
$Q_4 = \underline{\underline{0.287 \text{ m}^3\text{/s out of J}}}$

4. (c) (ii)



NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2021-2022

CV1012 - FLUID MECHANICS

November / December 2021 **Time Allowed: 2½ hours**

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FIVE (5)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. An **Appendix** of **ONE (1)** page containing useful data and formula is attached to the paper.
5. This is a Closed-Book Examination.
6. All answers must be written in the answer book provided. Answer each question beginning on a **FRESH** page of the answer book.

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1. (a) The depth of Liquid A (with relative density = 1.25) in a closed vessel is 0.2 m as shown in Figure Q1(a). It is connected to a container filled with mercury (density = 13,600 kg/m^3) via an inverted U-tube manometer.

   (i) Calculate the pressure at Point B, p_B (at the bottom of the vessel) if the mercury level on the right-hand side of the U-tube, h = 0.35 m. Express your answer in gauge pressure in bars. (Given: 1 bar = 100 kPa).

   (ii) Calculate p_B and h if a hole is drilled on the vessel at Point Q.

(9 Marks)

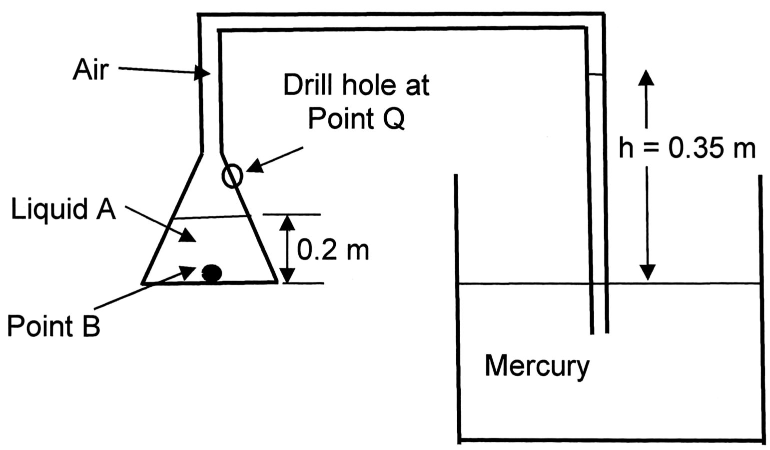

Figure Q1(a)

Note: Question No. 1 continues on page 2

(b) A rectangular block with length = 1 m, width = 0.2 m and depth = 0.33 m is floating on water (the densities of the block and water = ρ_s and 1,000 kg/m^3, respectively). When a uniform vertical force P, measured in Newtons per metre width of the block (N/m), is applied to its top right corner, the block has the configuration as shown in Figure Q1(b).

(i) With the aid of a diagram, clearly indicate the location of the centre of gravity and centre of buoyancy of the block.

(ii) Hence, compute the block density, ρ_s and vertical force, P in N/m.

(16 Marks)

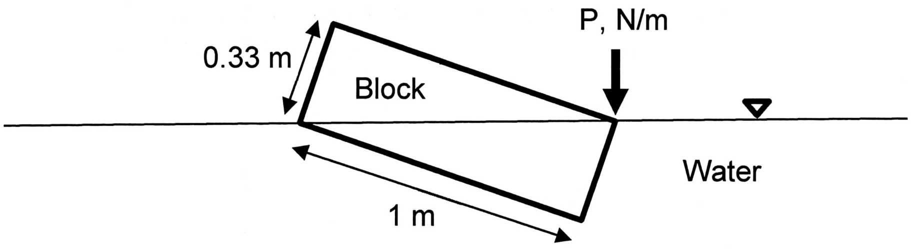

Figure Q1(b)

2. (a) Figure Q2(a) shows a 1.2-m diameter circular cylindrical tank that weighs 240 N when it is empty. The tank is resting on ice with a static friction coefficient, μ = 0.01. A 10-cm diameter circular orifice, which is located on one (left) side of the tank, is 30 cm from the bottom of the tank.

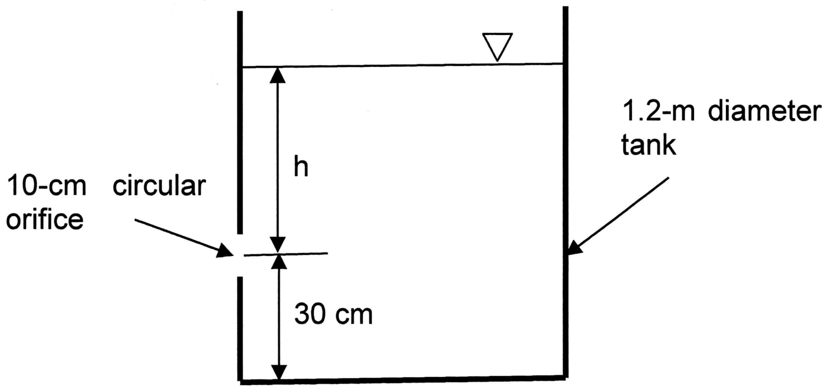

Figure Q2(a)

Note: Question No. 2 continues on page 3

(i) Calculate the **total depth** of water in the tank when it will just start to move as it is being filled with water.

(ii) State two assumptions needed for the calculation in part (i) to be correct.

(iii) If the actual depth of water in the tank exceeds the answer computed in part (i), what is the direction of its movement?

(14 Marks)

(b) Figure Q2(b) shows how water flows around a 90-degree **horizontal** pipe bend before discharging through a nozzle into the atmosphere. The diameters of the pipe and nozzle are 100 and 50 mm, respectively.

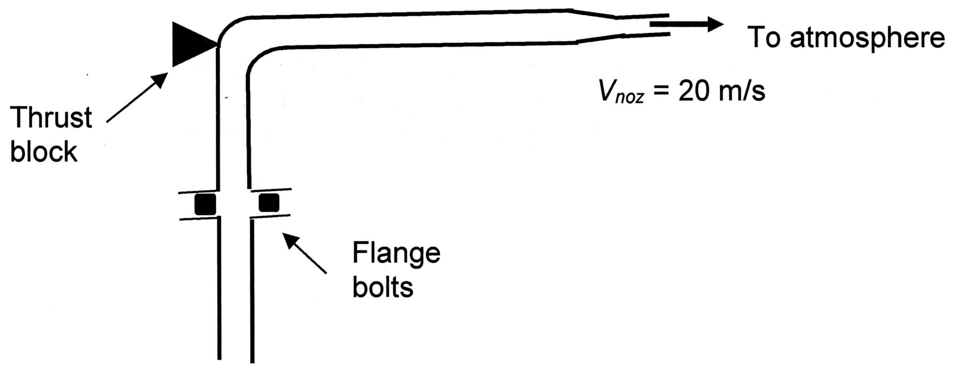

Figure Q2(b)

(i) Calculate the pressure at the start of the bend, p_1 if the velocity at the nozzle, V_{noz} = 20 m/s. Assume there is no energy loss in the flow through the pipe system.

(ii) If the hydrodynamic forces acting on the bend, which is induced by the flow, are designed to be resisted by the flange bolts and thrust block as shown in the figure, calculate the horizontal and vertical forces, F_x and F_y, respectively.

(11 Marks)

3. (a) Give 2 reasons why dimensional analysis is useful in analysing fluid mechanics problems.

(4 Marks)

(b) For practical reasons, a hydraulic scale model is typically designed either to follow the Reynolds number or Froude number criteria.

(i) Explain and show why it is not practical to fulfil the Reynolds and Froude numbers simultaneously in both prototype and model.

(ii) What is the scale ratio of the fluid kinematic viscosity if you want to satisfy both the Reynolds and Froude numbers modelling criteria?

(5 Marks)

(c) A dynamically similar 1:10 scale model is used to study pipe flow characteristics. If the prototype flow rate Q_p = 0.1 m^3/s, what is the flow rate, Q_m in the model? Determine the scale of the fluid kinematic viscosity to satisfy both Reynolds and Froude modelling criteria. State any assumptions made.

(5 Marks)

(d) Figure Q3 shows water is pumped from Reservoir A (Elevation, EL = 100 m) through a pipeline (diameter, D = 0.3 m, and length L = 200 m) to a hilltop Reservoir B (EL = 130 m).

(i) Calculate the shaft power of the pump required to supply a discharge Q = 0.1 m^3/s from reservoirs A to B. Include both the major and minor losses in your calculations. You may assume a pipe friction factor, f = 0.02, and a pump with an efficiency of 80%.

(ii) Sketch and label the Total Energy Line (TEL) and the Hydraulic Grade Line (HGL) for the flow system, including the minor losses components.

(11 Marks)

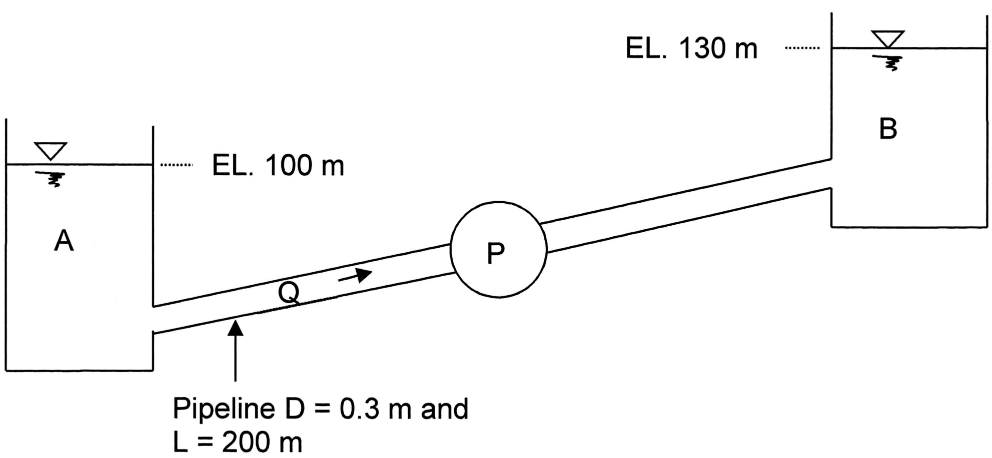

Figure Q3

4. (a) Briefly explain 4 important characteristics of the Moody Diagram for pipe flow.

(5 Marks)

(b) The pump power P, typically depends on 6 variables: discharge Q, impeller diameter D, impeller rotation speed ω, energy head H, fluid density ρ and gravitational acceleration g. Using dimensional analysis, derive the dimensionless π-term for the pump power P.

(5 Marks)

(c) Figure Q4 shows how 4 reservoirs, A, B, C and D are connected to a common junction J by pipes 1, 2, 3 and 4, respectively. The pipe characteristics are given in Table Q4.

Table Q4

Pipe	Water Elevation in reservoir (m)	Pipe diameter D (m)	Length, L (m)	Friction factor, f	$K = \frac{8fL}{g\pi^2 D^5}$ (s^2/m^5)
1	60	0.3	700	0.02	476
2	50	0.3	400	0.02	272
3	30	0.3	600	0.02	408
4	10	0.3	500	0.02	340

(i) Calculate the flow rates in the 4 pipes. Ignore the minor losses in your calculations. The kinematic viscosity of water, $\nu = 1 \times 10^{-6}$ m^2/s.

(ii) Sketch the Total Energy Line (TEL) and the Hydraulic Grade Line (HGL) for the pipeline system.

(15 Marks)

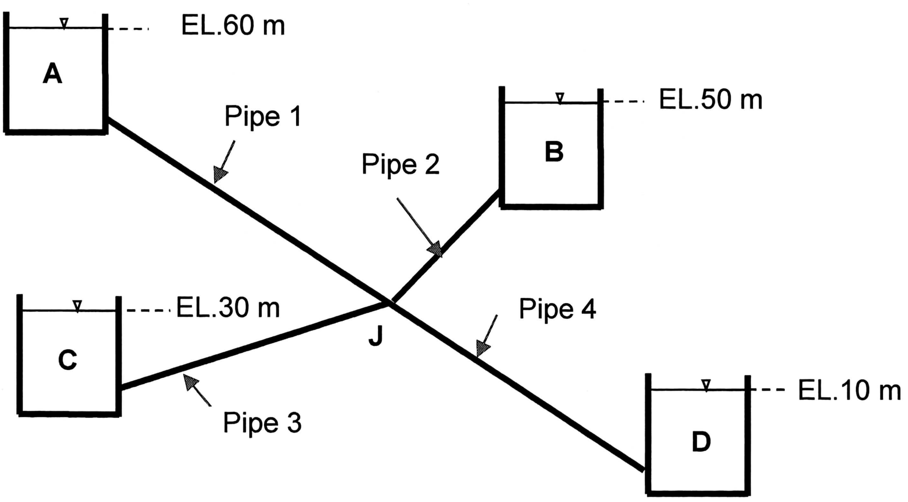

Figure Q4

END OF PAPER

Useful Formulas

$\gamma = \rho g$

$SG = \rho/\rho_{water}$

$\tau = \mu(du/dy)$

$\mu = \rho\nu$

$E_V = -\,dp/\,(dV/V) = dp/\,(d\rho/\rho)$

$p = p_o + \gamma h$

$p_{atm} \approx 1$ bar ≈ 760 mm Hg (abs)

$p_{gauge} = p_{abs} - p_{atm}$

$F_R = p_c A = \gamma h_c A$

$y_R = I_{xc}/(y_c A) + y_c$

$I_{xc} = ba^3/12$ (rectangle); $\pi R^4/4$ (circle); $0.11R^4$ (semicircle); $0.055R^4$ (quarter circle)

$I_{xc} = ba^3/36$ (triangle, b = base, a = height)

$F_B = \gamma V_{displaced}$

$BM = I_{yy}/V_{displaced}$

$\Sigma Q_{in} = \Sigma Q_{out}$

$p_1/(\rho g) + V_1^2/(2g) + z_1 = p_2/(\rho g) + V_2^2/(2g) + z_2$

$\Sigma F_x = \Sigma(\rho Q V_x)_{out} - \Sigma(\rho Q V_x)_{in}$

Darcy-Weisbach Eq: $h_f = \frac{f\ L\ V^2}{2\ g\ D} = \frac{8\,f\,L\,Q^2}{g\,\pi^2\,D^5}$

Poiseuille's Eq: $h_f = \frac{32\,\mu\,L\,V}{\rho\,g\,D^2} = \frac{128\,\mu\,L\,Q}{\rho\,g\,\pi\,D^4}$

For laminar flow, $f = \frac{64}{Re}$

Blasius Formula: $f = \frac{0.316}{Re^{0.25}}$, (3000 < Re < 100,000)

$K_{expansion} = \left(1 - \frac{A_1}{A_2}\right)^2$, $K_{contraction} = 0.5$, $K_{entrance} = 0.5$, $K_{exit} = 1.0$.

Fluid power = $\rho\ g\ Q\ h_p$

Head rise coefficient = $\frac{gH}{\omega^2 D^2}$

Flow coefficient = $\frac{Q}{\omega D^3}$

Power coefficient = $\frac{P}{\rho\,\omega^3\,D^5}$

CV1012 FLUID MECHANICS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.