NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2021-2022

CV1011 – MECHANICS OF MATERIALS

November / December 2021 **Time Allowed: 2.5 hours**

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **SIX (6)** pages.
2. Answer **ALL FOUR (4)** questions.
3. All questions carry equal marks.
4. An Appendix of **TWO (2)** pages is attached together with this paper.
5. This is a Closed-Book Examination.
6. All answers must be written in the answer book provided. Answer each question beginning on a **FRESH** page of the answer book.

---

1. A beam sits against the ground at point **B** and two walls at point **A** and point **C** as shown in **Figure Q1**. A uniformly distributed gravity force is applied on the beam. Assume wall and ground surfaces are smooth.

   (a) Draw the free-body diagram of the beam.

   (10 Marks)

   (b) Calculate the reactions at supports **A**, **B**, and **C**.

   (15 Marks)

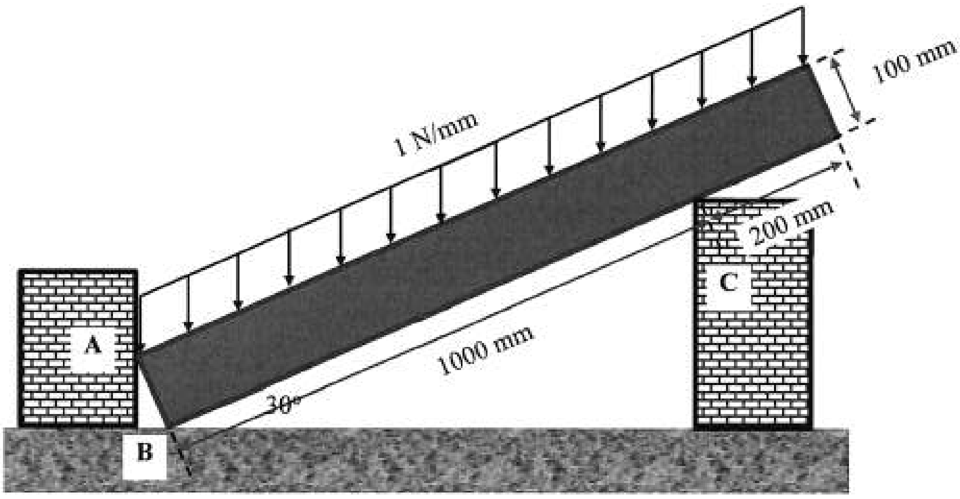


Figure Q1

2. (a) Determine the moment of inertia of the inverted T cross-sectional area with respect to the x' axis as shown in **Figure Q2**.

(5 Marks)

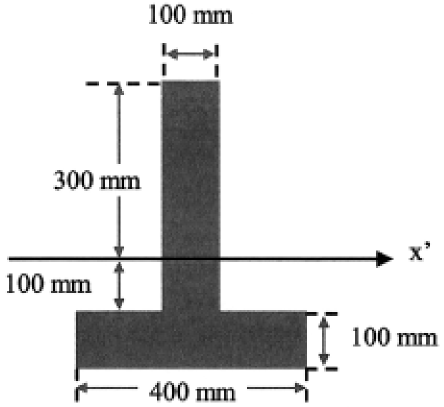


Figure Q2

(b) A rod has a diameter of 10 mm and a length of 100 mm. The rod is stretched from the two ends gradually with a displacement from 0 mm to 10 mm in 10 steps (that is, each stretching step is 1 mm). Assume the material is isotropic and behaves linear elastically with an elastic modulus of 1 GPa and a Poisson's ratio of 0.5.

(i) Determine the diameter of the rod when it is stretched by 10 mm.

(5 Marks)

(ii) Determine the engineering stress and engineering strain of the rod when it is stretched by 10 mm.

(5 Marks)

(iii) Determine the true strain of the rod when it is stretched by 10 mm. [Hint: Take the previous step as the baseline for the determination of strain in the next step.]

(5 Marks)

(iv) Compare and comment on the results in parts (b)(ii) and (b)(iii).

(5 Marks)

3. (a) **Figure Q3(a)** shows two bars, A (fixed at the left) and B (fixed at the right). They both have a uniform cross-sectional area of 3000 mm^2. Their Young's moduli and coefficients of thermal expansion are indicated in the figure. At room temperature, there is a gap between the two bars.

(i) From room temperature, suppose the two bars are heated up with the same temperature increase. Calculate the minimum temperature increase so that the gap is closed.

(5 Marks)

(ii) At room temperature, the gap is now filled with insulation material (i.e., no heat transfer between A and B) which can be considered incompressible. Bar A is then heated up uniformly by $10\,^{\circ}C$ but bar B remains to be at room temperature. Calculate the axial force induced (if any) in A and B. Express your answer in kN and indicate whether it is tensile or compressive.

(10 Marks)

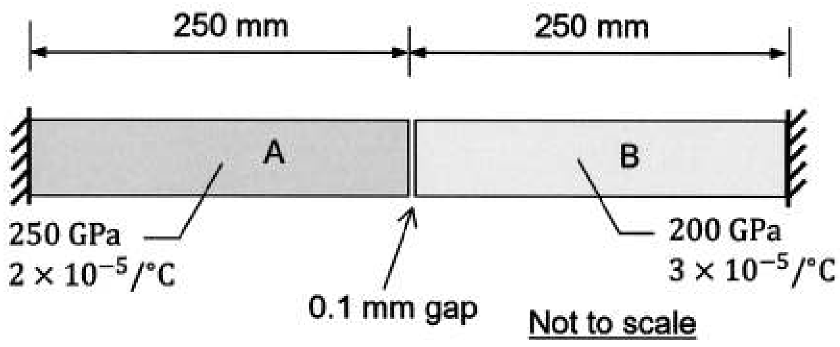


Figure Q3(a)

Note: Question No. 3 continues on Page 4.

(b) **Figure Q3(b)** shows a shaft with a uniform hollow cross-section. It is fixed at A and free at E. Part AD has a uniform shear modulus of 100 GPa. Part DE has a uniform shear modulus of 120 GPa. From the view point indicated near E, the shaft is subjected to an anti-clockwise couple of 4 kNm at E; and clockwise couples of 2 kNm at D and 5 kNm at B, respectively. Determine the smallest and largest shear stress in the cross-section at C, which is at 0.6 m from A.

(10 Marks)

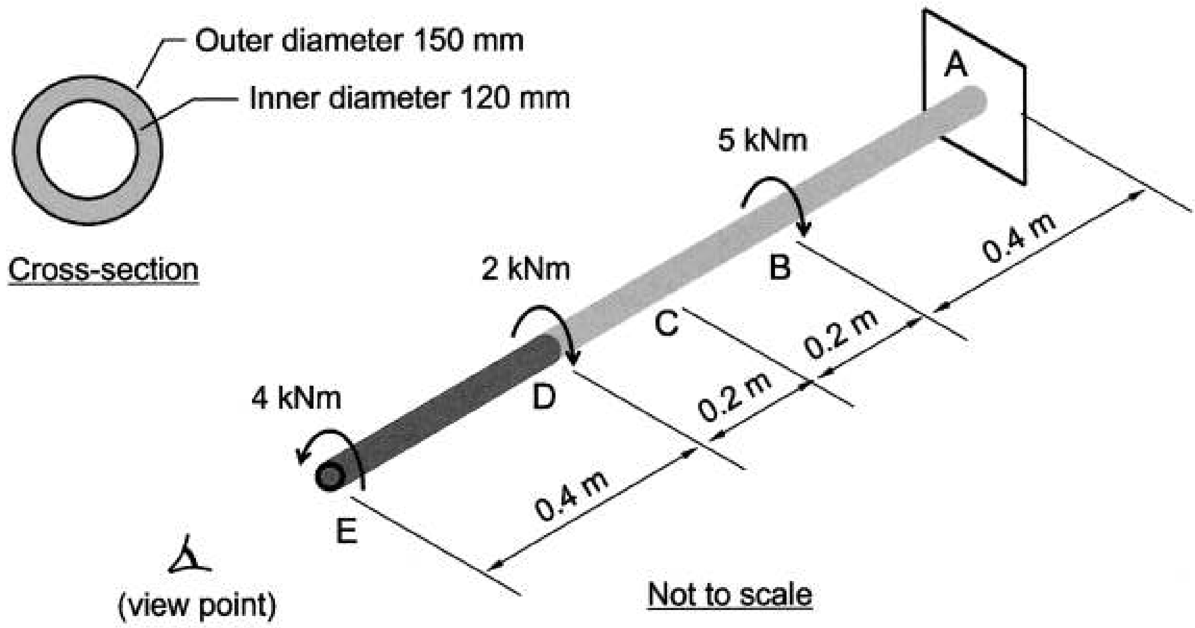
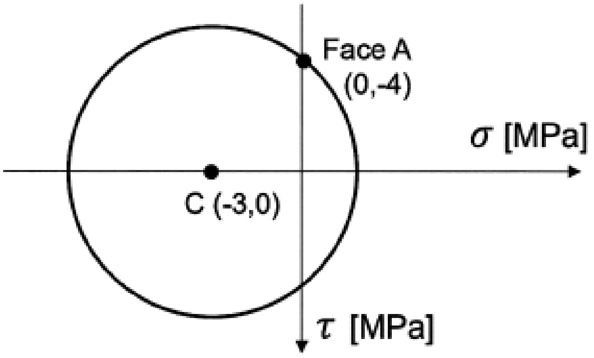


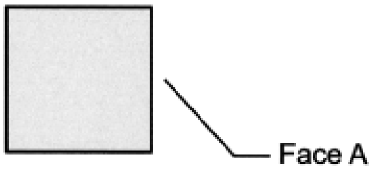
Figure Q3(b)

4. (a) **Figure Q4(a)** shows the Mohr circle (left) of a stress block (right) under an in-plane stress state. The centre of the circle is at C with coordinates $(-3,0)$. The point $(0,-4)$ corresponds to Face A as shown in the stress block on the right. Determine the angle (in degree, anti-clockwise positive) where the stress block should be rotated so that the magnitude of normal stress on Face A is largest. Draw the stress block oriented at such angle, and indicate the magnitude and direction of the stresses on all faces.

(8 Marks)



Mohr circle
(sign convention as in lecture notes)

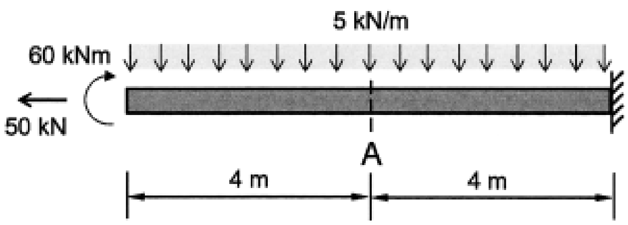


Stress block
(stresses not shown)

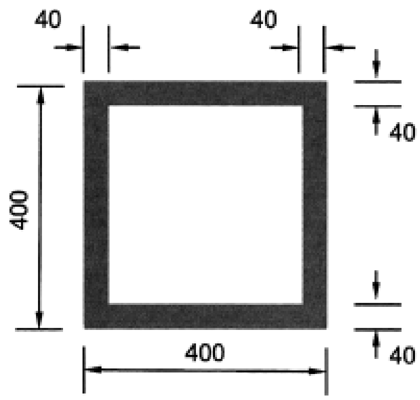
Figure Q4(a)

(b) **Figure Q4(b)** (left) shows a cantilever beam subjected to a uniformly distributed load along its length; and an axial force and moment couple at the left end. The cantilever has a uniform cross-section as shown on the right of the figure. For the cross-section at A, draw the distribution of normal stress. Indicate the extreme values in MPa and their locations. Ignore self-weight and stress concentration effects.

(12 Marks)



Elevation



Beam section
(dimension in mm)

Figure Q4(b)

Note: Question No. 4 continues on Page 6.

(c) A cantilever beam has homogenous material properties and uniform cross-section along its length. **Figure Q4(c)** (left) shows its cross-section, which comprises ten squares of equal size. Except at the support, the cantilever is unrestrained in both the x and y direction. At the free end it is subjected to a compressive axial load. In order to increase the critical (i.e., lowest) buckling load using the same amount of material, an engineer proposed to modify the section to be the one shown on the right of the figure. Is this modification effective? Explain your answer briefly.

(5 Marks)

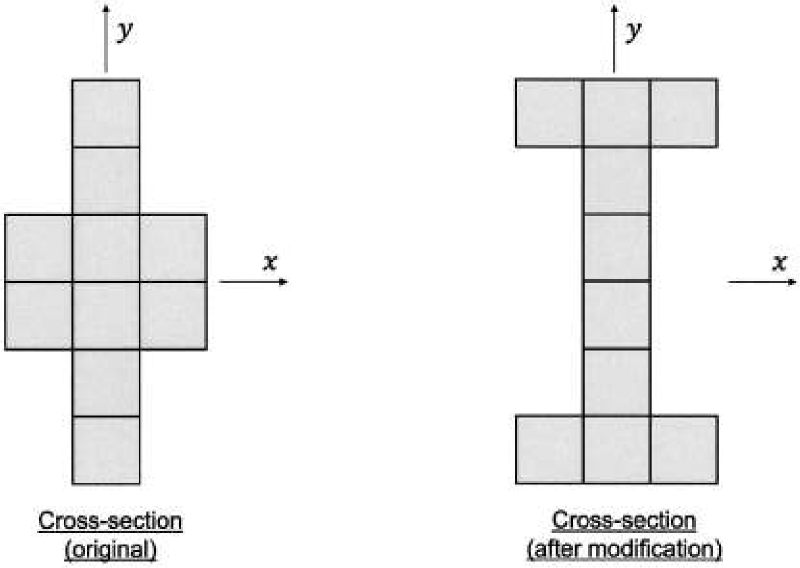


Figure Q4(c)

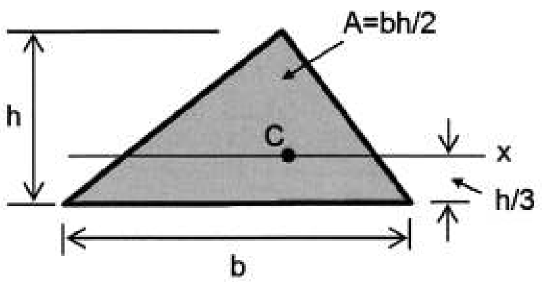
END OF PAPER

1. Equilibrium

Particle $\sum F_x = 0$, $\sum F_y = 0$, $\sum F_z = 0$

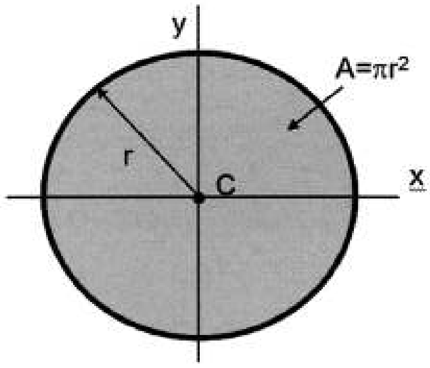
Rigid Body – Two dimensions $\sum F_x = 0$, $\sum F_y = 0$, $\sum M_o = 0$

2. Geometric properties of area elements



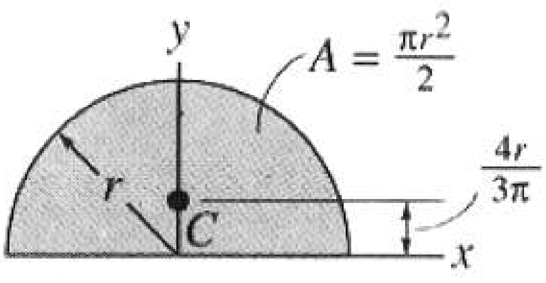
Triangular

$$I_x = \frac{1}{36} b h^3$$



Circular area

$$I_x = \frac{1}{4}\pi r^4$$
$$I_y = \frac{1}{4}\pi r^4$$



Semicircular area

$$I_x = \frac{1}{8}\pi r^4$$
$$I_y = \frac{1}{8}\pi r^4$$

Parallel-Axis Theorem $I = \bar{I} + Ad^2$

3. Axial load

Normal Stress $\sigma = \frac{P}{A}$

Displacement $\delta = \int_0^L \frac{P dx}{EA}$, $\delta = \sum \frac{PL}{EA}$, $\delta_T = \alpha \Delta T L$

4. Torsion

Shear Stress in Circular Shaft $\tau = \frac{T\rho}{J}$

where $J = \frac{\pi}{2} c^4$ solid cross section; $J = \frac{\pi}{2}(c_o^4 - c_i^4)$ tubular cross section

Angle of Twist $\phi = \sum \frac{TL}{GJ}$

5. Bending

Normal Stress $\sigma = \frac{My}{I}$

Unsymmetric Bending $\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$, $\tan\alpha = \frac{I_z}{I_y}\tan\theta$

6. Shear

Average Direct Shear Stress $\tau_{ave} = \frac{V}{A}$

Transverse Shear Stress $\tau = \frac{VQ}{It}$

Shear Flow $q = \tau \cdot t = \frac{VQ}{I}$

7. Stress Transformation Equations

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Principal Stress,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Maximum In-Plane Shear Stress,

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\ , \quad \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

8. Buckling

Critical Axial Load, $P_{cr} = \frac{\pi^2 EI}{(KL)^2}$

CV1011 MECHANICS OF MATERIALS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.

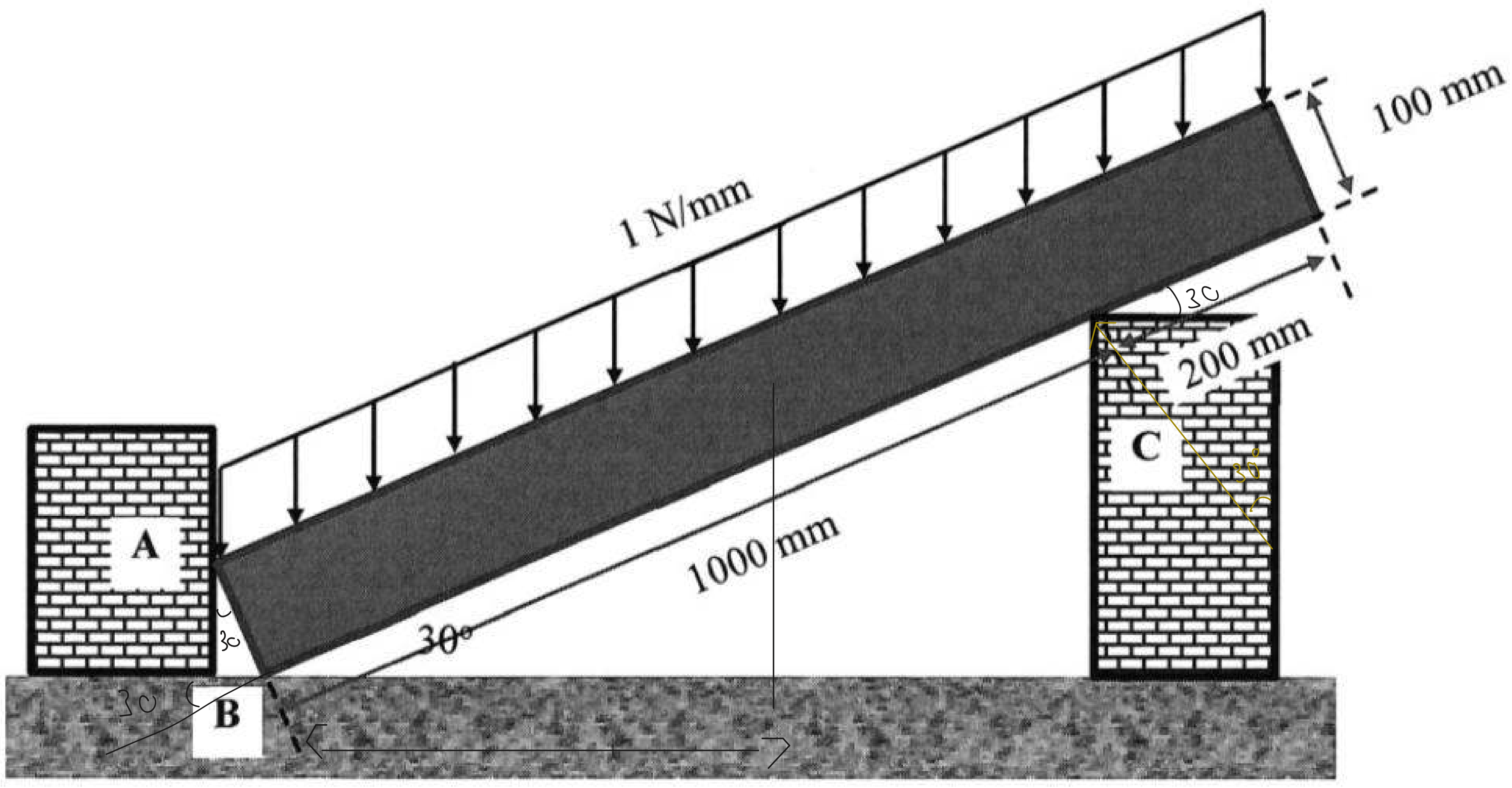
A beam sits against the ground at point **B** and two walls at point **A** and point **C** as shown in **Figure Q1**. A uniformly distributed gravity force is applied on the beam. Assume wall and ground surfaces are smooth.

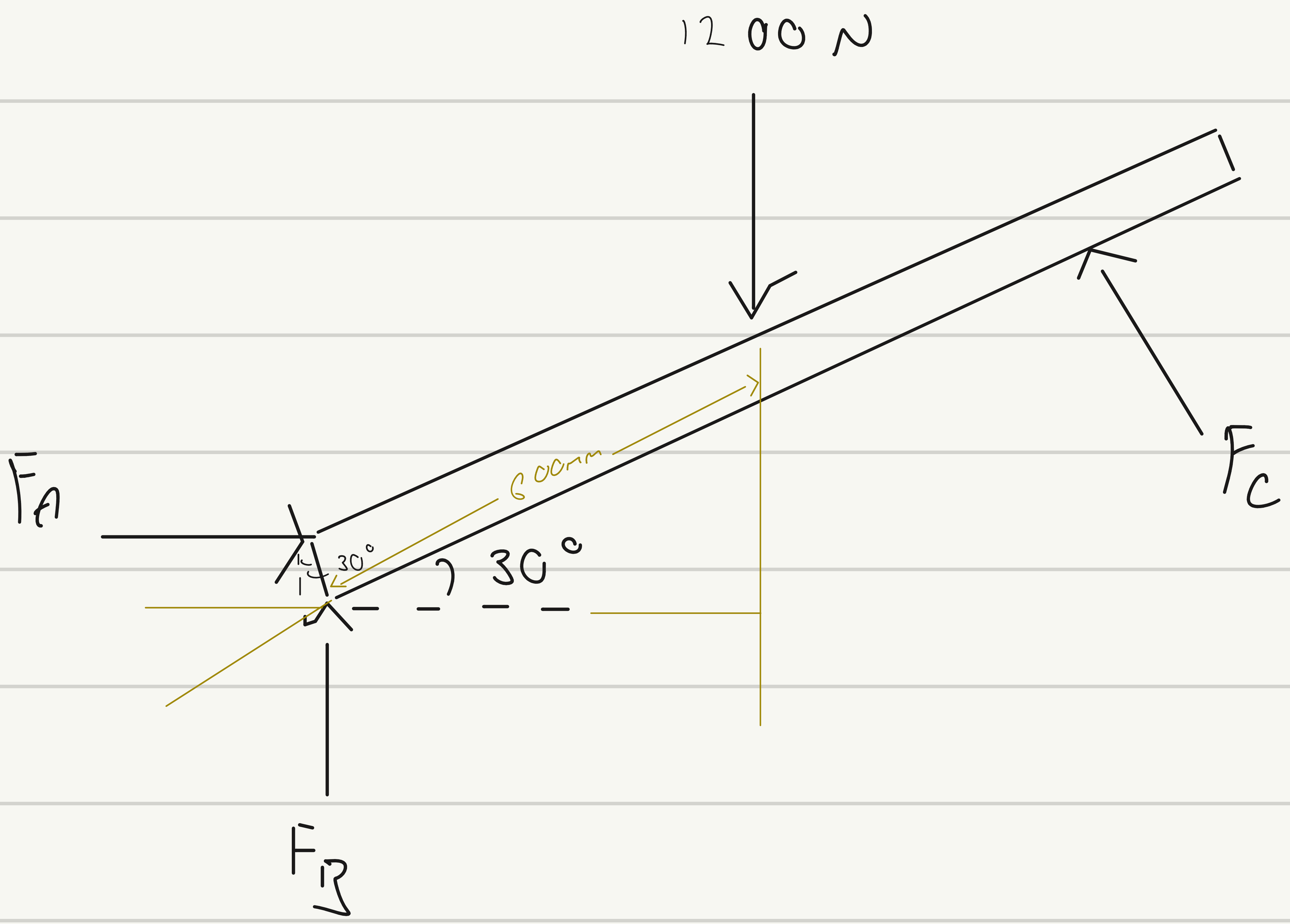
(a) Draw the free-body diagram of the beam.

(10 Marks)

(b) Calculate the reactions at supports **A**, **B**, and **C**.

(15 Marks)





$\Sigma M_B = 0$

$\circlearrowleft +$, $-F_A\, 100 \cos 30° - 1200(600)\cos 30° + F_c(1000) = 0$

$-86.6 F_A - 623538.29 + 1000 F_c = 0$ ①

$\Sigma F_y = 0$,

$+\uparrow$, $F_B - 1200 + F_c \cos 30° = 0$

$\Sigma F_x = 0$

$\overset{+}{\rightarrow}$, $F_A - F_c \sin 30 = 0$

$F_A = F_c \sin 30°$ ②

sub ① into ②

$-86.6(F_c \sin 30) - 623538.29 + 1000 F_c = 0$

$956.7 F_c = 623538.29$

$F_c = 651.759\ N$

2. (a) Determine the moment of inertia of the inverted T cross-sectional area with respect to the x' axis as shown in **Figure Q2**.

(5 Marks)

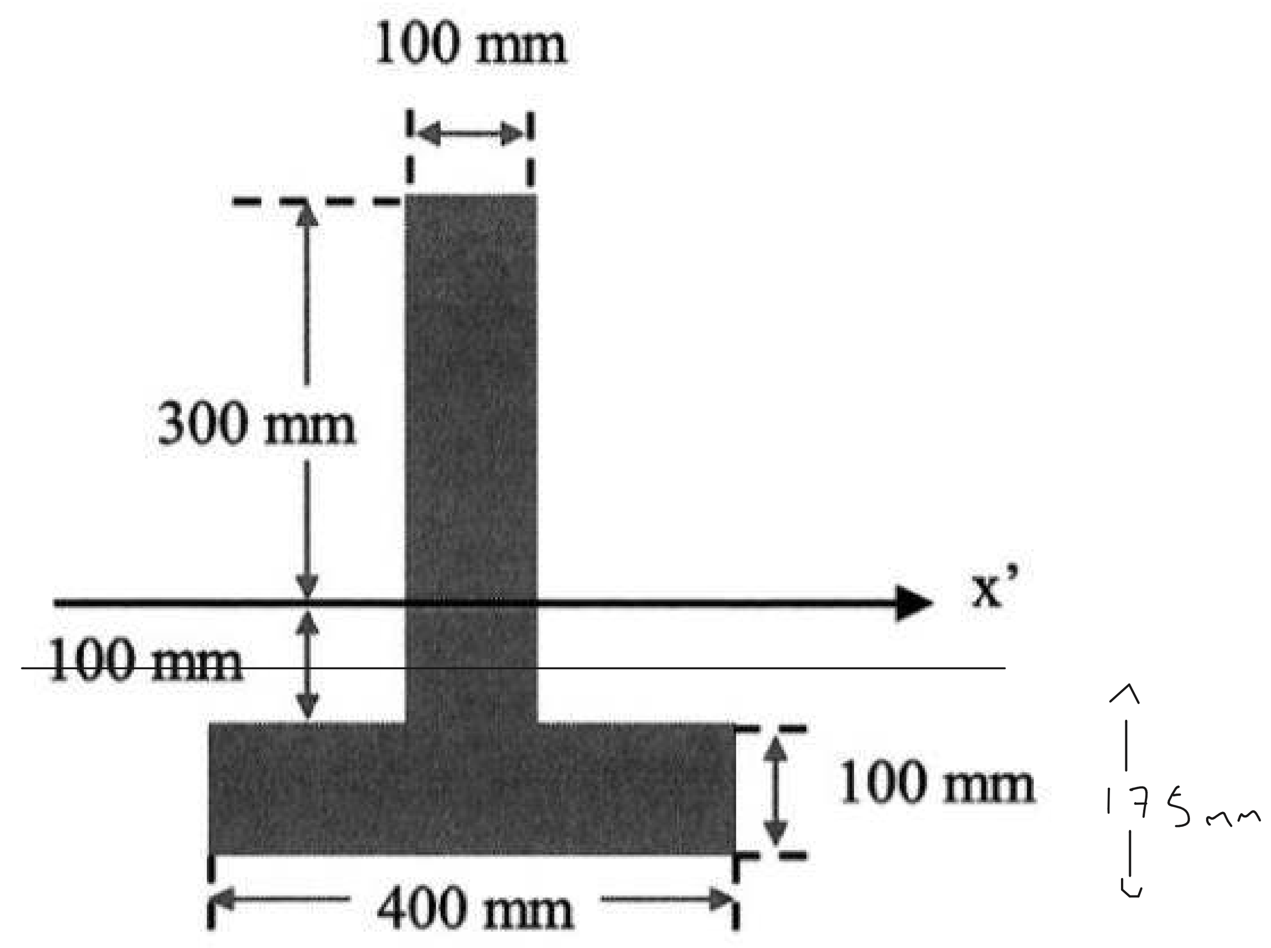


Figure Q2

$$I = \frac{1}{12}(100)(400)^3 + 100(400)(100)^2 +$$
$$\frac{1}{12}(400)(100)^3 + 100(400)(150)^2$$
$$= 1.86667 \times 10^9 \text{ mm}^4$$

(b) A rod has a diameter of 10 mm and a length of 100 mm. The rod is stretched from the two ends gradually with a displacement from 0 mm to 10 mm in 10 steps (that is, each stretching step is 1 mm). Assume the material is isotropic and behaves linear elastically with an elastic modulus of 1 GPa and a Poisson's ratio of 0.5.

(i) Determine the diameter of the rod when it is stretched by 10 mm.

(5 Marks)

(ii) Determine the engineering stress and engineering strain of the rod when it is stretched by 10 mm.

(5 Marks)

(iii) Determine the true strain of the rod when it is stretched by 10 mm. [Hint: Take the previous step as the baseline for the determination of strain in the next step.]

(5 Marks)

(iv) Compare and comment on the results in parts (b)(ii) and (b)(iii).

(5 Marks)

(i) $V = -\frac{\varepsilon_y}{\varepsilon_x}$

$0.5 = \frac{\Delta d \div 10}{10 \div 100}$

$= \frac{\Delta d}{10} \times \frac{10}{1}$

$0.5 = \Delta d$

diameter of rod,

$10 - 0.5 = 9.5mm$

(ii) $\delta = \frac{PL}{EA}$

$10\times10^{-3} = \frac{P(100\times10^{-3})}{10^9 \frac{\pi}{4}(5\times10^{-3})^2}$

$P = 1963.495$ Pa

$\sigma = \frac{P}{A}$

$= \frac{1963.495}{\frac{\pi}{4}(5\times10^{-3})^2}$

$= 100\times10^6$ Pa

$\varepsilon = \frac{\sigma}{E}$

$= \frac{100\times10^6}{10^9}$

$= 0.1$ Pa

$\sigma_{True} = \sigma_{Eng}(\varepsilon_{Eng} + 1)$

$\varepsilon_{True} = ln\left(\frac{L_i}{L_o}\right) = ln(\varepsilon_{Eng} + 1)$

(iii) $ln\left(\frac{l_f}{l_i}\right)$

$= ln\left(\frac{110}{100}\right)$

$= 0.095$

$ln(1 + \varepsilon)$

$= ln(1 + 0.1)$

$= 0.0953$

True stress, $\sigma_{Eng}(\varepsilon_{Eng} + 1)$

$= 100\times10^6(0.1 + 1)$

$= 110\times10^6$

3. (a) **Figure Q3(a)** shows two bars, A (fixed at the left) and B (fixed at the right). They both have a uniform cross-sectional area of 3000 mm^2. Their Young's moduli and coefficients of thermal expansion are indicated in the figure. At room temperature, there is a gap between the two bars.

(i) From room temperature, suppose the two bars are heated up with the same temperature increase. Calculate the minimum temperature increase so that the gap is closed.

(5 Marks)

(ii) At room temperature, the gap is now filled with insulation material (i.e., no heat transfer between A and B) which can be considered incompressible. Bar A is then heated up uniformly by $10\,°C$ but bar B remains to be at room temperature. Calculate the axial force induced (if any) in A and B. Express your answer in kN and indicate whether it is tensile or compressive.

(10 Marks)

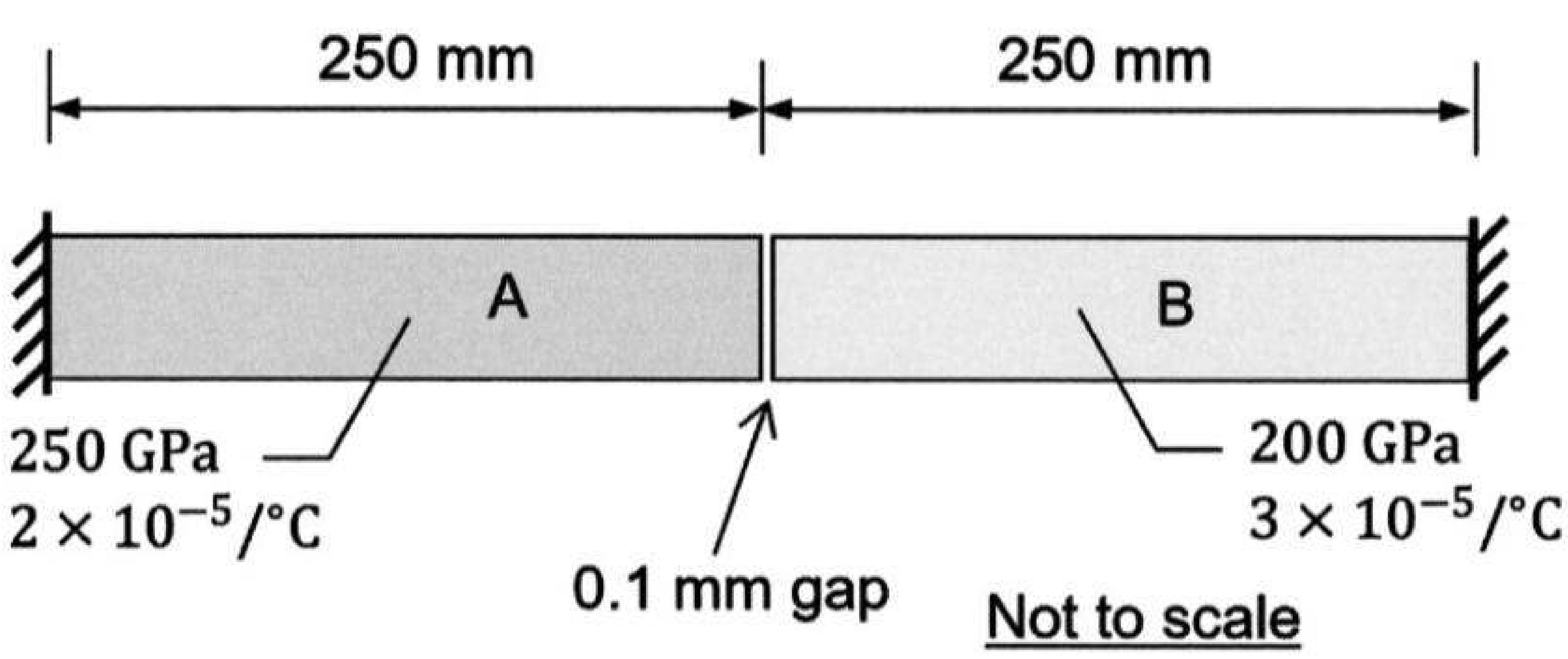


Figure Q3(a)

(ai) $\delta_T = \alpha L \Delta T$

$2\times10^{-5}(250\times10^{-3})\Delta T + 3\times10^{-5}(250\times10^{-3})\Delta T = 0.1\times10^{-3}$

$0.005\ \Delta T + 0.0075\ \Delta T = 0.1$

$0.0125\ \Delta T = 0.1$

$\Delta T = 8°C$

(ii) $\delta = 2\times10^{-5}(250\times10^{-3})(10)$

$= 0.005\,m$

$\delta = \frac{PL}{EA}$

$0.005 = \frac{P(250\times10^{-3})}{250\times10^{9}(3000(10^{-3})^2)}$

$P = 15\times10^{6}\ N$

Axial force induced on A & B

(b) **Figure Q3(b)** shows a shaft with a uniform hollow cross-section. It is fixed at A and free at E. Part AD has a uniform shear modulus of 100 GPa. Part DE has a uniform shear modulus of 120 GPa. From the view point indicated near E, the shaft is subjected to an anti-clockwise couple of 4 kNm at E; and clockwise couples of 2 kNm at D and 5 kNm at B, respectively. Determine the <u>smallest and largest shear stress in the cross-section at C</u>, which is at 0.6 m from A.

(10 Marks)

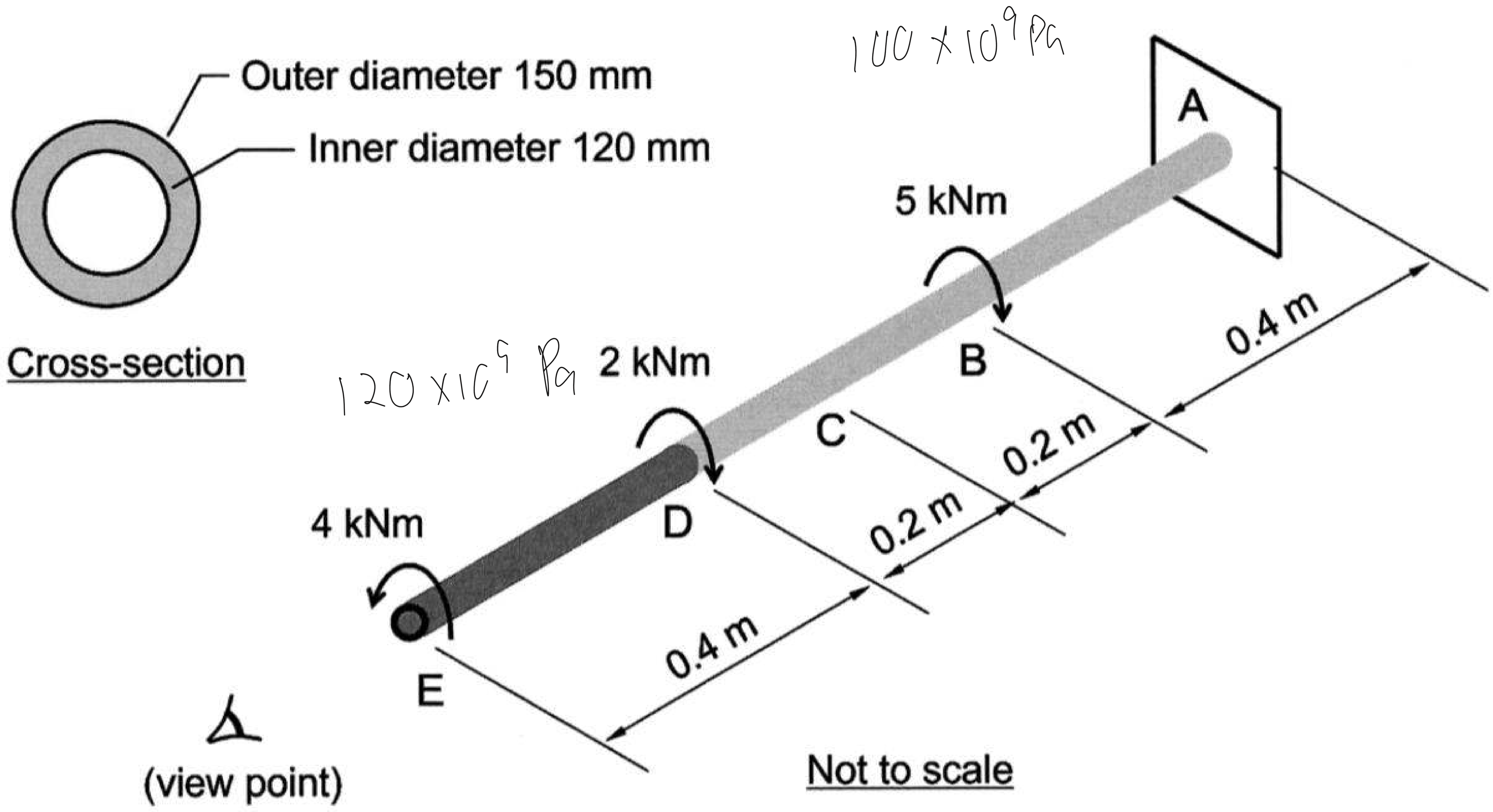
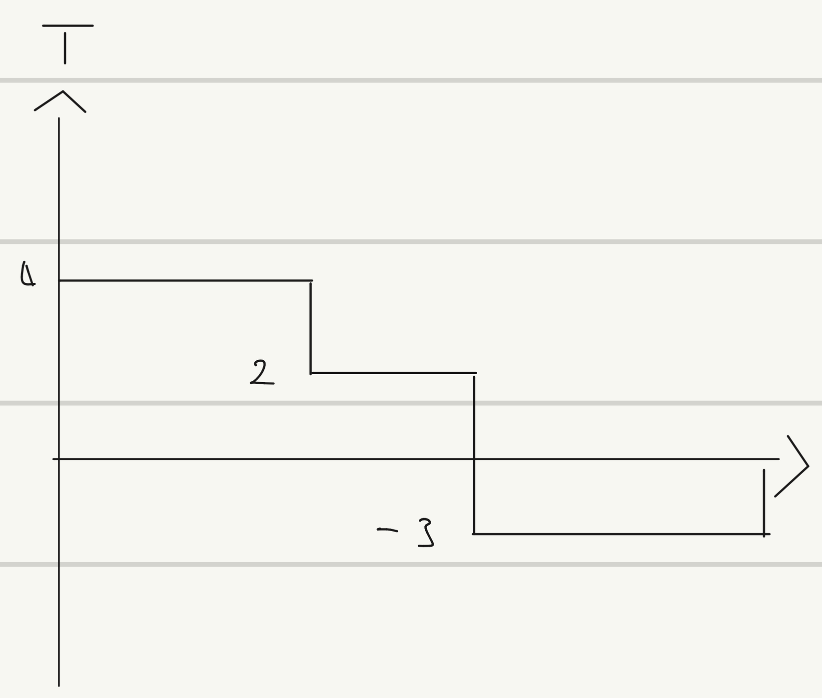


Figure Q3(b)

$J = \frac{\pi}{2}(75^4 - 60^4)$

$= 29.3434 \times 10^6 \ mm^4$

$= 29.343 \times 10^{-6} \ m^4$



$\tau = \frac{T\rho}{J}$

$\tau_{largest} = \frac{2\times10^3(75\times10^{-3})}{29.343\times10^{-6}}$

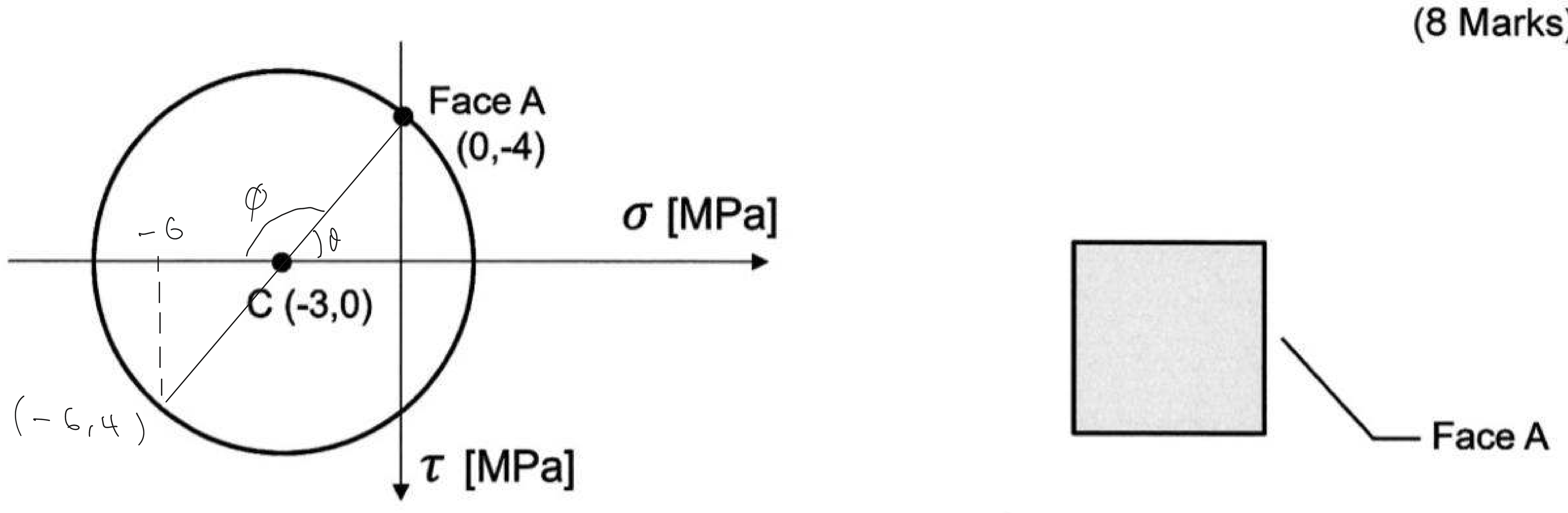
$= 5.1195 \times 10^6 \ Pa$

$\tau_{smallest} = \frac{2\times10^3(60\times10^{-3})}{29.343\times10^{-6}}$

$= 4.0895 \times 10^6 \ Pa$

4. (a) **Figure Q4(a)** shows the Mohr circle (left) of a stress block (right) under an in-plane stress state. The centre of the circle is at C with coordinates $(-3,0)$. The point $(0,-4)$ corresponds to Face A as shown in the stress block on the right. Determine the <u>angle</u> (in degree, anti-clockwise positive) where the stress block should be rotated so that the <u>magnitude of normal stress</u> on Face A is <u>largest</u>. Draw the stress block oriented at such angle, and indicate the magnitude and direction of the stresses on <u>all faces</u>.

(8 Marks)



Mohr circle
(sign convention as in lecture notes)

Stress block
(stresses not shown)

Figure Q4(a)

$\tan\theta = \frac{4}{3}$

$\theta = 53.13°$

$r = (3^2+4^2)^{1/2}$

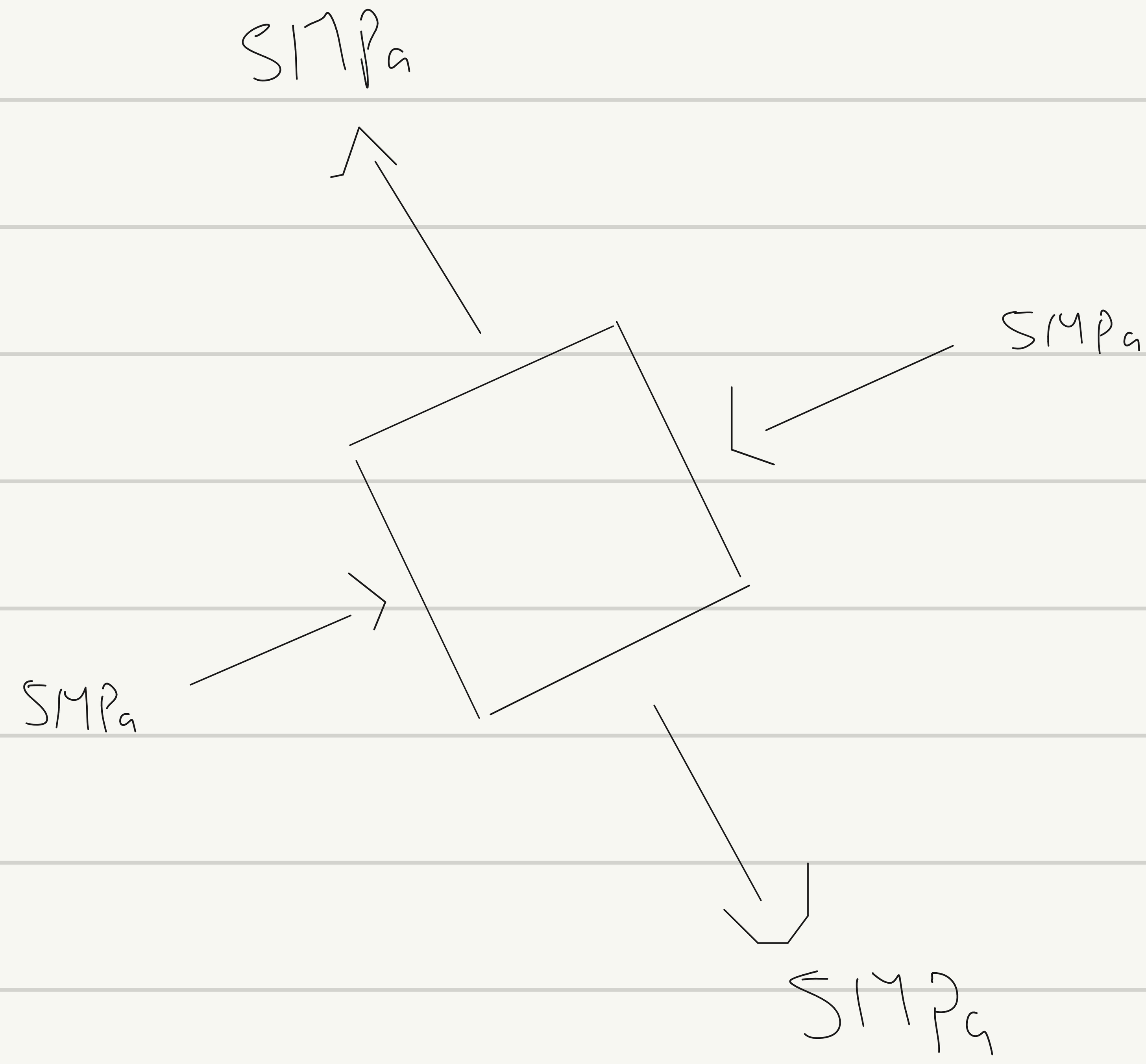
$= 5\ MPa$

$\phi = 180° - 53.13°$

$= 126.869$

$\delta = 126.869 \div 2$

$= 63.4°$ ↺



(b) **Figure Q4(b)** (left) shows a cantilever beam subjected to a uniformly distributed load along its length; and an axial force and moment couple at the left end. The cantilever has a uniform cross-section as shown on the right of the figure. For the <u>cross-section at A</u>, draw the distribution of normal stress. Indicate the extreme values in MPa and their locations. Ignore self-weight and stress concentration effects.

(12 Marks)

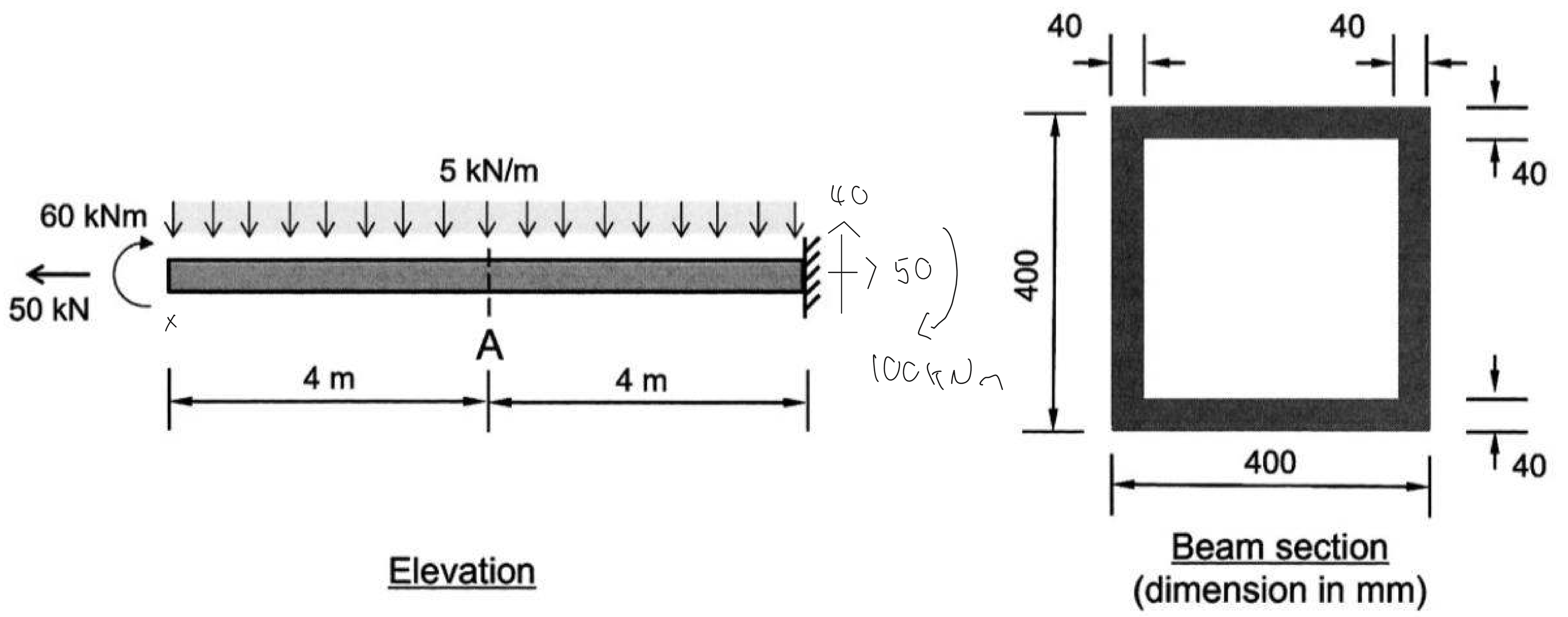


Figure Q4(b)

$\sigma = \frac{V}{A} + \frac{My}{I}$

$\sum M_x = 0$

$\circlearrowleft +, \; -60 + 5(8)(4) + M = 0$

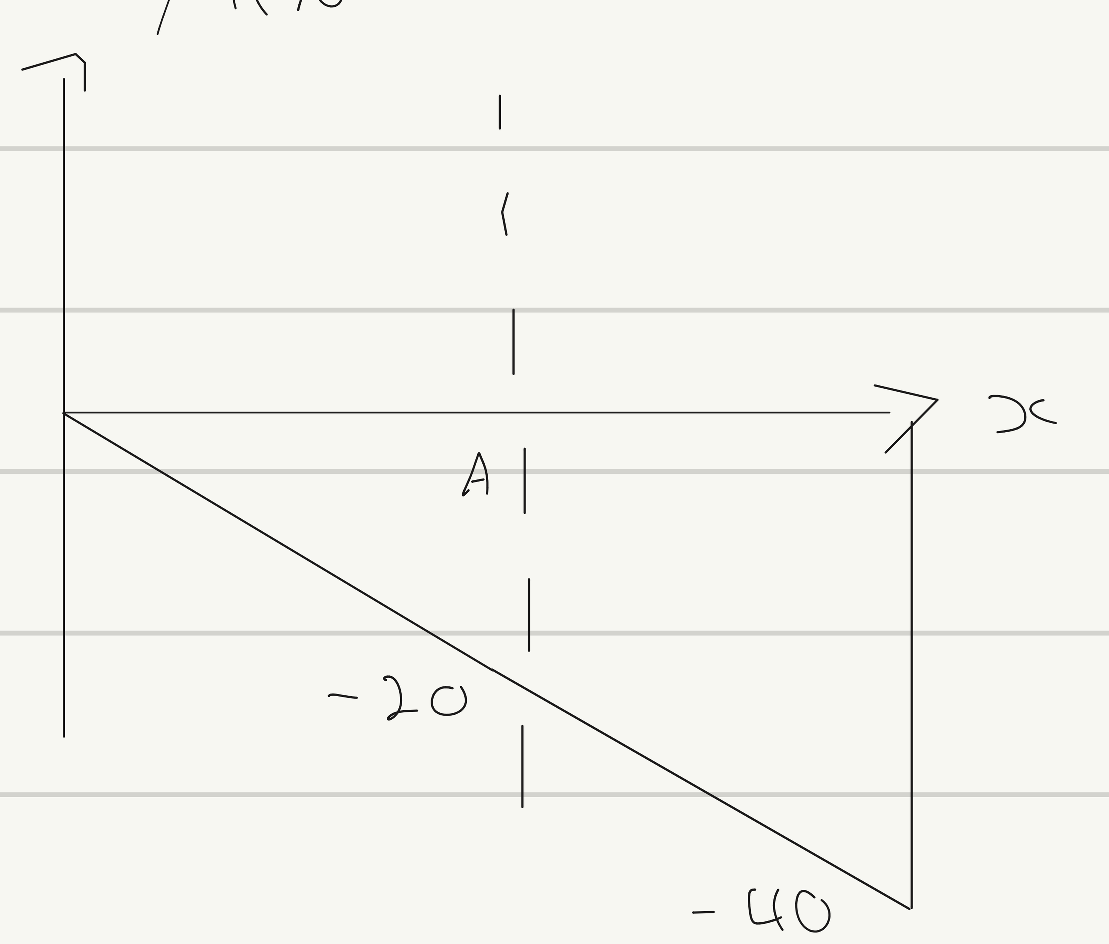
$M = -100 \text{ kNm } (\circlearrowright)$

$\sum F_y = 0$

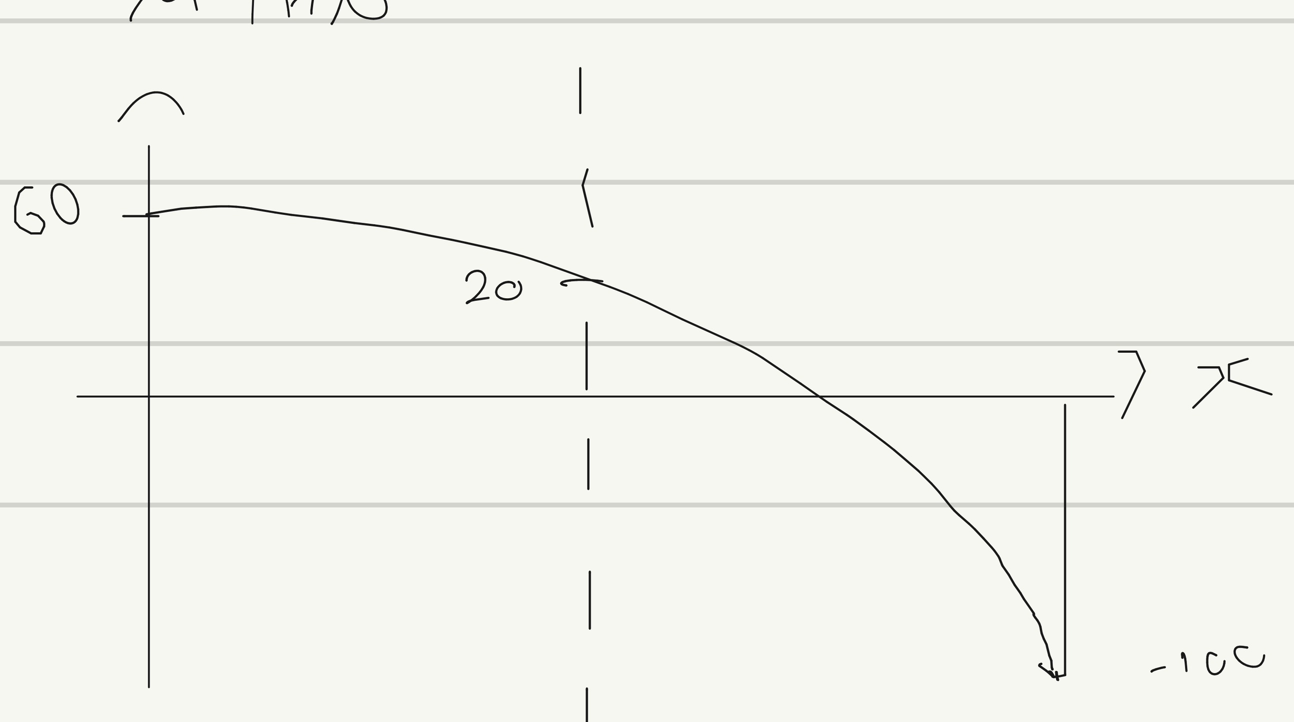
$\uparrow +, \; -5(8) + F_y = 0$

$F_y = 40 \text{ kN}$

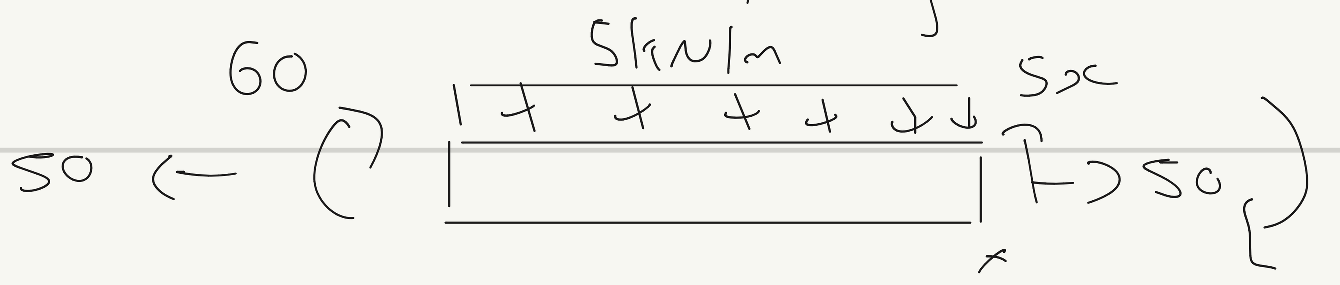
V / kN



M / kN



$0 < x < 4$, taking reference from x



$\sum F_y = 0$

$\uparrow +, \; -5x + V = 0$

$V = 5x$

$\sum M_x = 0$

$\circlearrowleft +, \; -60 + 5x\left(\frac{x}{2}\right) + M = 0$

$M = 60 - \frac{5}{2}x^2$

$I = \frac{1}{12}(400)(400)^3 - \frac{1}{12}(320)(320)^3$

$= 1.2595 \times 10^9 \text{ mm}^4$

$= 1.2595 \times 10^{-3} \text{ m}^4$

$A = 400^2 - 320^2$

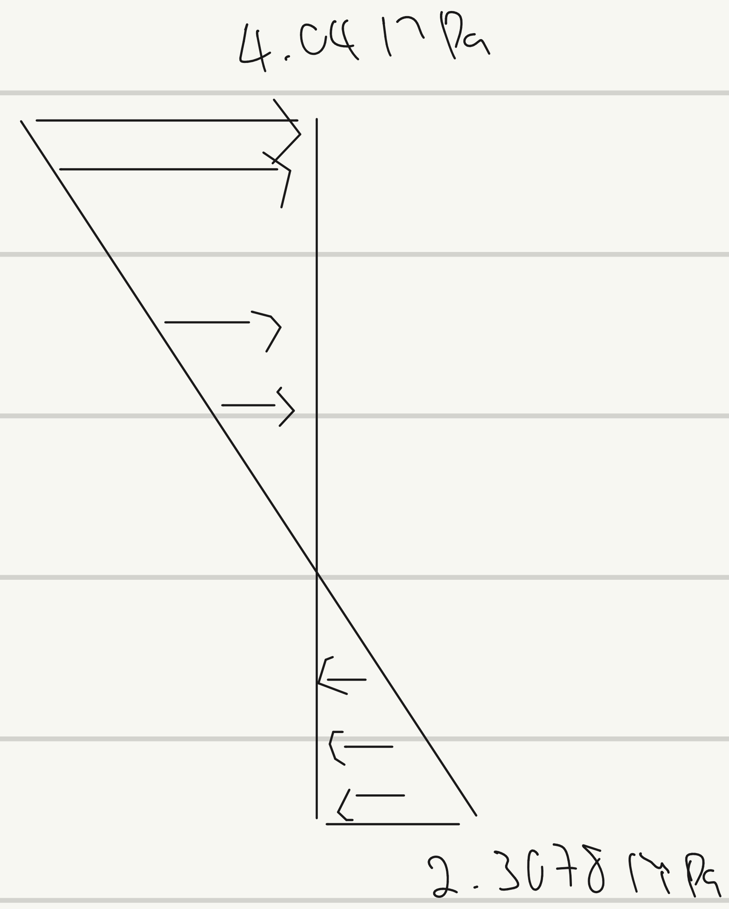
$= 57600 \text{ mm}^2$

$= 0.0576 \text{ m}^2$

$\sigma_{max} = \frac{50 \times 10^3}{0.0576} + \frac{20 \times 10^3 (200 \times 10^{-3})}{1.2595 \times 10^{-3}}$

$= 404 \times 10^6$

$\sigma_{min} = \frac{50 \times 10^3}{0.0576} - \frac{20 \times 10^3 (200 \times 10^{-3})}{1.2595 \times 10^{-3}} = -2.307 \times 10^6$

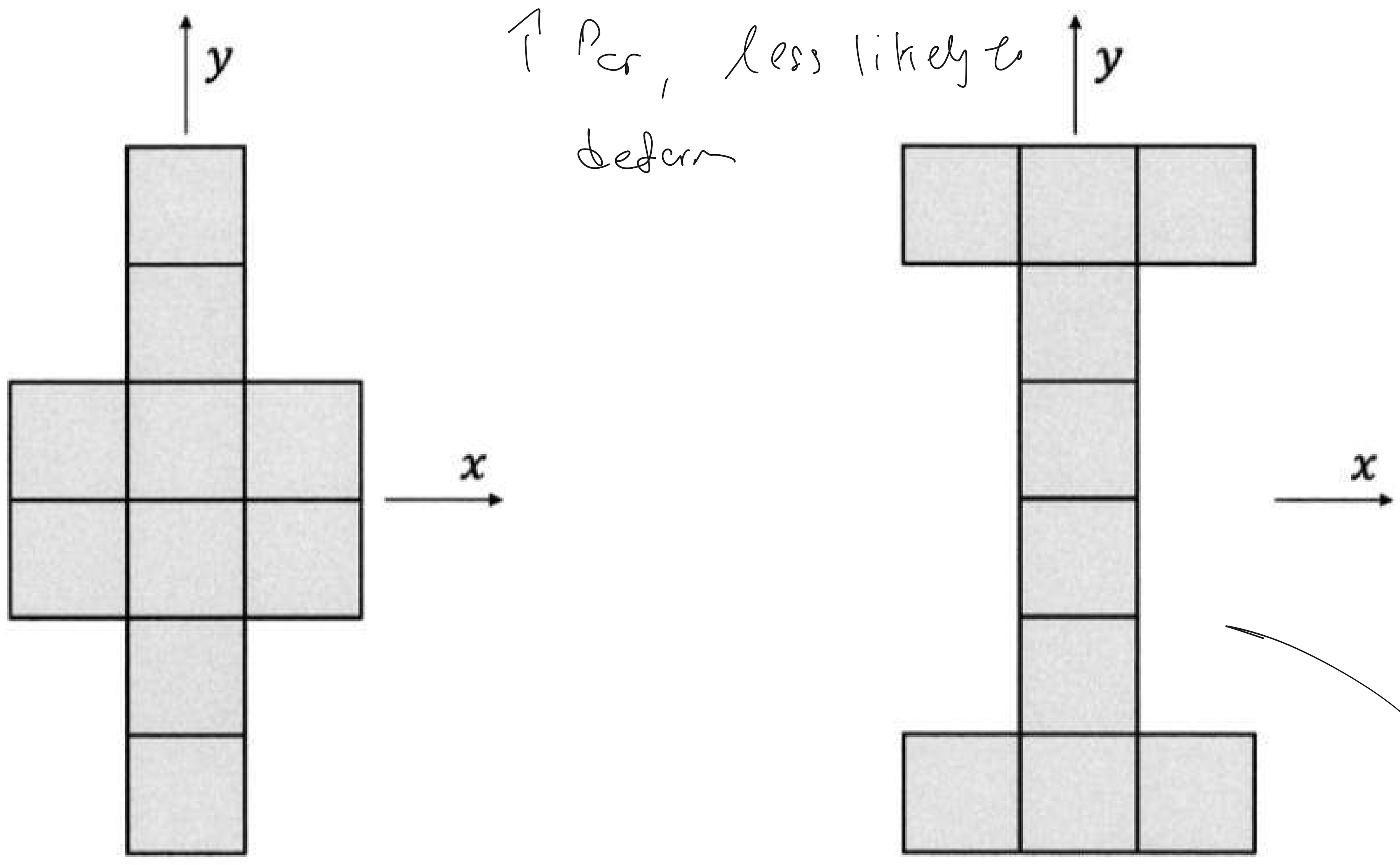


(c) A cantilever beam has homogenous material properties and uniform cross-section along its length. **Figure Q4(c)** (left) shows its cross-section, which comprises ten squares of equal size. Except at the support, the cantilever is unrestrained in both the x and y direction. At the free end it is subjected to a compressive axial load. In order to increase the critical (i.e., lowest) buckling load using the same amount of material, an engineer proposed to modify the section to be the one shown on the right of the figure. Is this modification effective? Explain your answer briefly.

greatest load that will not cause lateral deflection

(5 Marks)

↑ P_{cr}, less likely to deform



Cross-section (original)

Cross-section (after modification)

Figure Q4(c)

Critical Axial Load $P_{cr} = \frac{\pi^2 E I}{(KL)^2}$

I increase

P_{cr} increase