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MH2814 - PYP 2021-2022 S1

Date No.

1. (a) case 1: A and B are selected

case 2: A and B not selected

no. of ways = $\binom{38}{2} = \frac{38!}{2!(36!)} = 703$

total no. of ways = 703 + 1 = 704 //

(b) $P(A) = 0.1$, $P(B) = 0.3$, $P(C) = 0.6$

let S be purchase from shop

$P(S|A) = 0.1$, $P(S|B) = 0.5$, $P(S|C) = 0.3$

$$P(B|S) = \frac{P(S|B)P(B)}{P(S|A)P(A) + P(S|B)P(B) + P(S|C)P(C)}$$

$$= \frac{0.5 \times 0.3}{(0.1 \times 0.1) + (0.5 \times 0.3) + (0.6 \times 0.3)} = \frac{15}{34} \text{ //}$$

(c) $\rho_{YZ} = \frac{\sigma_{YZ}}{\sigma_Y \sigma_Z}$

$$\sigma_X^2 = \sigma^2_{3Y+Z+5} = 3^2\sigma_Y^2 + 1^2\sigma_Z^2 + 2(3)(1)\sigma_{YZ}$$

$$8 = 9(2) + \sigma_Z^2 + 6(-2.5)$$

$$\sigma_Z^2 = 5$$

$$\sigma_Z = \sqrt{5}$$

$$\rho_{YZ} = \frac{-2.5}{\sqrt{5}\sqrt{2}} = -0.790569$$

$$= -0.791 \text{ (3s.f) //}$$

2.(a) $n=20$, $p=10\%$, $q=90\%$.

let X be no. of defective components

$$P(X\geq 2) = 1 - P(X=0) - P(X=1)$$

$$= 1 - \binom{20}{0}(0.10)^0(0.90)^{20} - \binom{20}{1}(0.10)^1(0.90)^{19}$$

$$= 0.60825$$

$$= 0.6083 \ (3s.f)$$

(b) $\lambda t = np = 35 \times 0.10 = 3.5$

$$P(5;3.5) = \frac{e^{-3.5}(3.5)^5}{5!} = 0.13216$$

$$= 0.1322 \ (3s.f)$$

(c) $\mu = np = 1000 \times 0.10 = 100$

$\sigma^2 = npq = 1000 \times 0.1 \times 0.9 = 90$

$\sigma = \sqrt{90}$

$\bar{x} \sim N(100, \sqrt{90})$

$$P(82 \leq X \leq 105) = P(81.5 < X < 105.5)$$

$$= P\left(\frac{81.5-100}{\sqrt{90}} < X < \frac{105.5-100}{\sqrt{90}}\right)$$

$$= P(-1.950071 < X < 0.579750)$$

$$= 0.69337$$

$$= 0.6934 \ (3s.f)$$

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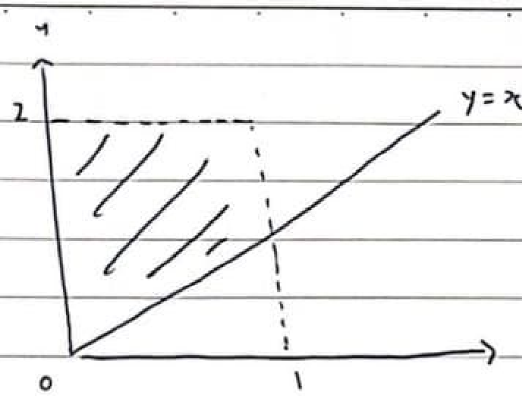
3. (a) $P(Y > X) = \int_0^1 \int_0^Y \frac{x^2}{2} + \frac{Y}{3} \, dx\, dy$

$= \int_0^1 \left[\frac{x^3}{6} + \frac{Y}{3} x \right]_0^Y dy$

$= \int_0^1 \frac{Y^3}{6} + \frac{Y^2}{3} \, dy$

$= \left[\frac{Y^4}{6 \times 4} + \frac{Y^3}{3 \times 3} \right]_0^1$

$= \frac{11}{72}$



(b) marginal distribution of x, $g(x)$

$g(x) = \int_0^2 \frac{x^2}{2} + \frac{y}{3} \, dy = \left[\frac{x^2}{2} y + \frac{y^2}{6} \right]_0^2 = x^2 + \frac{2}{3}$

marginal distribution of y, $h(y)$

$h(y) = \int_0^1 \frac{x^2}{2} + \frac{y}{3} \, dx = \left[\frac{x^3}{6} + \frac{y^2}{6} \right]_0^1 = \frac{1}{6} + \frac{1}{6} y^2$

(c) $f(x,y) = \int_0^1 \int_0^2 \frac{x^2}{2} + \frac{y}{3} \, dy\, dx = \int_0^1 \left[\frac{x^2}{2} y + \frac{y^2}{6} \right]_0^2 dx$

$= \int_0^1 x^2 + \frac{2}{3} \, dx$

$= \left[\frac{x^3}{3} + \frac{2}{3} x \right]_0^1$

$= 1$

$g(x)\, h(y) = \left(x^2 + \frac{2}{3} \right) \left(\frac{1}{6} + \frac{1}{6} y^2 \right) \neq f(x,y)$

$\therefore$ not independent.

(d) $E(x) = \int_0^1 x\, g(x)\, dx$

$= \int_0^1 x(x^2 + \frac{2}{3})\, dx$

$= \int_0^1 x^3 + \frac{2}{3}x\, dx$

$= \left[\frac{x^4}{4} + \frac{2}{3}\left(\frac{x^2}{2}\right)\right]_0^1$

$= \frac{4}{3}$

$E(y) = \int_0^2 y\, h(y)\, dy$

$= \int_0^2 y\left(\frac{1}{6} + \frac{1}{3}y\right) dy$

$= \int_0^2 \frac{1}{6}y + \frac{1}{3}y^2\, dy$

$= \left[\frac{1}{6}\left(\frac{y^2}{2}\right) + \frac{1}{3}\left(\frac{y^3}{3}\right)\right]_0^2$

$= 11/9$

(e) $\sigma_{xy} = E(x,y) - \mu_x \mu_y$

$E(x,y) = \int_0^2 \int_0^1 xy\left(\frac{x^2}{2} + \frac{y}{3}\right) dx\, dy$

$= \int_0^2 \int_0^1 \frac{x^3 y}{2} + \frac{x y^2}{3}\, dx\, dy$

$= \int_0^2 \left[\frac{y}{2}\left(\frac{x^4}{4}\right) + \frac{y^2}{3}\left(\frac{x^2}{2}\right)\right]_0^1 dy$

$= \int_0^2 \frac{1}{8}y + \frac{1}{6}y^2\, dy$

$= \left[\frac{1}{8}\left(\frac{y^2}{2}\right) + \frac{1}{6}\left(\frac{y^3}{3}\right)\right]_0^2$

$= \frac{25}{36}$

$\sigma_{xy} = \frac{25}{36} - \left(\frac{4}{3} \times \frac{11}{9}\right)$

$= -\frac{101}{108}$

4 (a) $\mu = 5000$, $\sigma = 300$

$n = 100$

$\bar{x} \sim N(5000, \frac{300^2}{100})$

$\bar{x} \sim N(5000, 900)$

(b) $\bar{x} \sim N(5000, 900)$

$P(\bar{x} < 4950) = P(Z < \frac{4950-5000}{\sqrt{900}})$

$= 0.04779$ //

(c) $1 - \alpha = 0.95$

$\alpha = 0.05$

$z_{0.05} = 1.645$

$\mu > \bar{x} - z_{0.05}(\frac{\sigma}{\sqrt{n}})$

$\mu > 5005 - 1.645(\sqrt{900})$

$\mu > 4955.65$

(d) $z_{0.025} = 1.960$

$45 < 1.960(\frac{300}{\sqrt{n}})$

$\sqrt{n} < \frac{196}{15}$

$n < 170.73$

$\therefore n = 171$ //

5. (a) $H_0 : \mu_A - \mu_B = 0$

(b) $H_1 : \mu_A - \mu_B > 0$

(c) $\alpha = 0.05$

(d) $n_A = 17, \ \bar{x}_A = 82, \ S_A = 6.1$
$n_B = 11, \ \bar{x}_B = 85, \ S_B = 5.5$

test stats : $T = \dfrac{(\bar{x}_A - \bar{x}_B) - (0,0)}{\sqrt{\frac{Sp^2}{17} + \frac{Sp^2}{11}}}$, $V = 17+11-2 = 26 \leq 30$

$$Sp^2 = \frac{(17-1)6.1^2 + (11-1)5.5^2}{17+11-2} = 34.53307692$$

$$t = \frac{82-85}{\sqrt{\frac{34.53}{17} + \frac{34.53}{11}}} = -1.319305595$$

p-value $= P(T > t) = P(T < -1.319) \approx 0.09928$

(e) p-value > 0.05

we do not reject Ho as there is not enough evidence to claim that mean mark of course A is smaller than of course B at 0.05 level of significance.

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6 (a) $B_1 = \frac{S_{xy}}{S_{xx}} = \frac{2588.5}{5495} = \frac{5177}{10990}$

$= 0.471064$

≈ 0.471 (3 s.f.)

$B_0 = \bar{y} - B_1 \bar{x} = 58.65 - 0.471(55.5)$

$= 32.5059$

$= 32.5$

$\therefore E(y) = 32.5 + 0.471x$

(b) $x = 40$

$y = 32.5 + 0.471(40)$

$= 51.348$

$S^2 = \frac{S_{yy} - \hat{B}_1 S_{xy}}{n-2} = \frac{5928.55 - 0.471064(2588.5)}{20-2}$

$= 261.622$

t-distribution, $v = 20 - 2 = 18$

a 95% cofidence interval

$E(y_0) \pm t_{0.025} (\sqrt{261.622}) \sqrt{\frac{1}{20} + \frac{(40-55.5)^2}{5495}}$

$= 51.348 \pm 2.101 \sqrt{261.622} \sqrt{0.09372}$

$\approx 51.348 \pm 10.4035$

$\approx (40.944, 61.7515)$

$\approx (40.9, 61.8)$

(c) $Y = 60$

$60 = 32.5 + 0.471x$

$x = 58.3864$

$58.3864 \pm t_{0.025} \left(\frac{\sqrt{261.622}}{\sqrt{5495}} \right)$

$\approx 58.3864 \pm 0.4584$

$\approx (57.9279, 58.844836)$

point estimator $= \frac{57.9279 + 58.8448}{2} = 58.38$

≈ 58.4 //