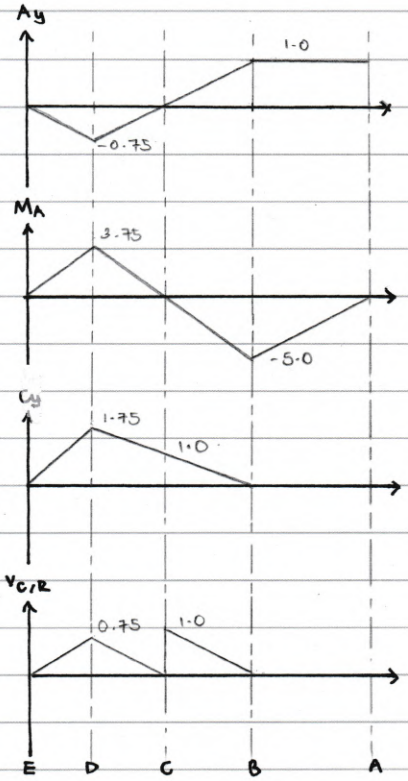
# CV2012 ~ STRUCTURAL ANALYSIS II
AY 20/21 S2

Done by: Kealeon Lee.

Q1. (a).




$M_{A,max(-ve)} = 11.25 \times 12 - 22.5 \times 12 - 22.5 \times 14 - 5 \times 30$

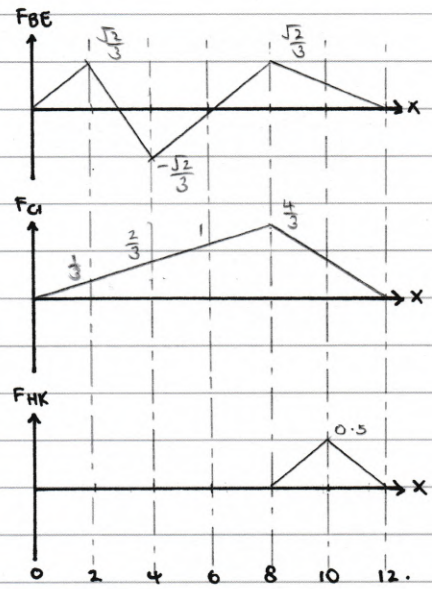
$= -600$ kNm.

$C_{y,max} = \frac{1}{2} \times 10 \times 1.75 \times (12 + 14) + 1.75 \times 30$

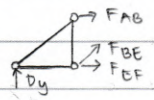
$= 280$ kN.

(b) To ease calculation, make table of $D_y$ values for different unit load configurations:

| x | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|---|
| $D_y$ (↑) | 1 | $\frac{2}{3}$ | $\frac{1}{3}$ | 0 | $-\frac{1}{3}$ | $\frac{1}{6}$ | 0 |



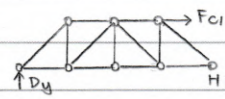

For $F_{BE}$:




when $x > 2$, unit load falls out of FBD, thus,

$F_{BE} \sin\theta = D_y \rightarrow F_{BE} = \frac{D_y}{\sin\theta} \rightarrow F_{BE} = \frac{2D_y}{\sqrt{2}}$

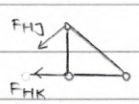
For $F_{CI}$:




similarly, when $x > 6$, take moment about H,

$\Sigma M_H = 0$, $D_y \times 8 = F_{CI} \times 2 \rightarrow F_{CI} = D_y \times 4$.

For $F_{HK}$:




$F_{BE,max} = \left(\frac{\sqrt{2}}{3} \times 15\right) + \left(\frac{\sqrt{2}}{3} \times \frac{3}{4} \times 20\right) = 14.14$ kN (T).

Q2. (a)(i) $M(x_1) = 30x - 10x^2$

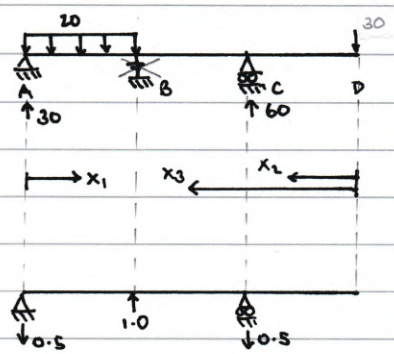
$M(x_2) = -30x$

$M(x_3) = -30x + 60(x-3) = 30x - 180$.

$m(x_1) = -0.5x$

$m(x_2) = 0$

$m(x_3) = -0.5(x-3) = -0.5x + 1.5$.




Take By as redundant and acts upwards,

$\Delta_B + f_{BB} B_y = -\frac{B_y}{K}$

$\Delta_B = \sum \int \frac{mM}{EI} dx = \frac{33.75}{1500} = 0.0225$

$f_{BB} = \sum \int \frac{m^2}{EI} dx = 0.003$.

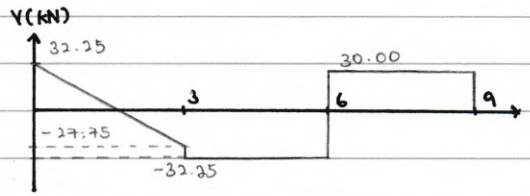
$0.0225 + 0.003 B_y = -\frac{B_y}{500} \rightarrow B_y = -4.5\,kN$

$= 4.5\,kN\,(\downarrow)$.

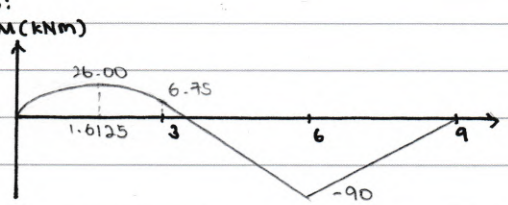
(ii) $\Sigma M_A = 0$, $C_y \times 6 = 20 \times 3 \times 1.5 + 4.5 \times 3 + 30 \times 9 \rightarrow C_y = 62.25\,kN\,(\uparrow)$.

$\Sigma F_y = 0$, $20 \times 3 + 30 + 4.5 = 62.25 + A_y \rightarrow A_y = 32.25kN\,(\uparrow)$.

SFD:




BMD:




(b) Take By as redundant and acts upwards,

$\Delta_B + f_{BB} B_y = -(\frac{B_y}{K} - 0.03)$.

From part (a)(i),

$0.0225 + 0.003x = -(\frac{B_y}{500} - 0.03) \rightarrow B_y = 1.5kN\,(\uparrow)$.

Q3 (a) $K_{BA} = \frac{3EI}{4}$, $K_{BE} = \frac{3(2EI)}{3}$, $K_{BC} = \frac{4EI}{4}$

$= 0.75EI$ $= 2EI$ $= EI \rightarrow \Sigma_B = 3.75EI$.

$DF_{BA} = 0.2$, $DF_{BE} = 0.533$, $DF_{BC} = 0.267$.

$K_{CB} = \frac{4EI}{4}$, $K_{CF} = \frac{3(2EI)}{3}$, $K_{CD} = \frac{4EI}{4}$

$= EI$ $= 2EI$ $= EI \rightarrow \Sigma_C = 4EI$

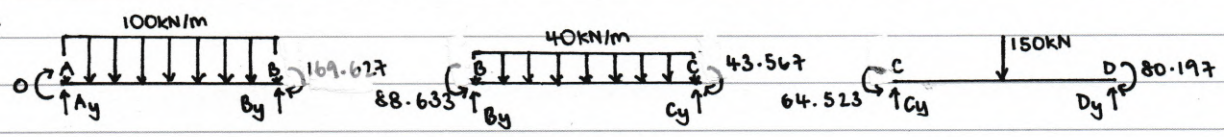
$DF_{CB} = 0.25$, $DF_{CF} = 0.5$, $DF_{CD} = 0.25$.

$(FEM)_{BA} = 200kNm$, $(FEM)_{BC} = -53.333kNm$, $(FEM)_{CB} = 53.333kNm$.

$(FEM)_{CD} = -75kNm$, $(FEM)_{DC} = 75kNm$.

| Joint | A | B | | | C | | | E | F | D |
|---|---|---|---|---|---|---|---|---|---|---|
| Member | AB | BA | BC | BE | CB | CF | CD | EB | FC | DC |
| DF | 1 | 0.2 | 0.267 | 0.533 | 0.25 | 0.5 | 0.25 | 1 | 1 | 0 |
| FEM | 0 | 200 | -53.333 | 0 | 53.333 | 0 | -75 | 0 | 0 | 75 |
| Dist. | | -29.333 | -39.111 | -78.222 | 5.417 | 10.834 | 5.417 | | | |
| CO. | | | 2.708 | | -19.556 | | | | | 2.708 |
| Dist. | | -0.542 | -0.722 | -1.444 | 4.889 | 9.778 | 4.889 | | | |
| CO. | | | 2.444 | | -0.361 | | | | | 2.444 |
| Dist. | | -0.489 | -0.652 | -1.304 | 0.090 | 0.181 | 0.090 | | | |
| CO. | | | 0.045 | | -0.326 | | | | | 0.045 |
| Dist. | | -0.009 | -0.012 | -0.024 | 0.081 | 0.163 | 0.081 | | | |
| Sum | 0 | 169.627 | -88.633 | -80.994 | 43.567 | 20.956 | -64.523 | 0 | 0 | 80.197 |

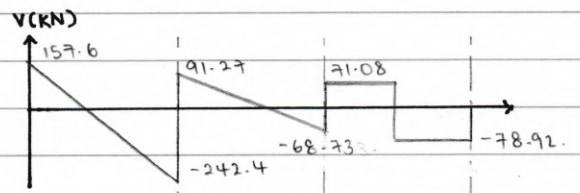
FBD:



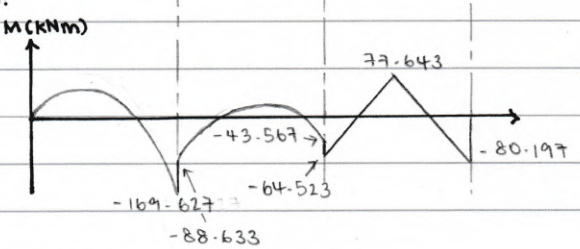

Using eqm: $A_y = 157.6kN$ $B_y = 91.27kN$ $C_y = 71.08kN$

$B_y = 242.4kN$ $C_y = 68.73kN$ $D_y = 78.92kN$

SFD:




BMD:

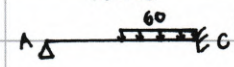


(b) For member ABC, develop new FEM:


$(FEM)_{CA} = \frac{100(8)^2}{8}$

$= 800\ kNm$

minus




$(FEM)_{CA} = \frac{9(60)(8)^2}{128}$

$= 270 kNm.$

$\rightarrow$ New $(FEM)_{CA} = 800 - 270 = 530 kNm.$

$K_{AC} = \frac{3EI}{8} = 0.375EI$, $K_{CD} = \frac{4EI}{4} = EI$, $K_{CF} = \frac{3(2EI)}{3} = 2EI \rightarrow \Sigma_C = 3.375EI$

$DF_{CA} = \frac{1}{9}$, $DF_{CD} = \frac{8}{27}$, $DF_{CF} = \frac{16}{27}$.

| Joint | A | C | | | D | F |
|---|---|---|---|---|---|---|
| Member | AC | CA | CF | CD | DC | FC |
| DF | 1 | $\frac{1}{9}$ | $\frac{16}{27}$ | $\frac{8}{27}$ | 0 | 1 |
| FEM | 0 | 530 | 0 | -75 | 75 | 0 |
| Dist. | | -50.556 | -269.630 | -134.815 | | |
| CO. | | | | 0 | -67.407 | |
| Dist. | | 0 | 0 | 0 | | |
| Sum. | 0 | 479.444 | -269.630 | -209.815 | 7.593 | 0 |

For ABC:

$\Sigma M_C = 0$, $8 \times A_y = 100 \times 4 \times 6 + 40 \times 4 \times 2 - 479.444 \rightarrow A_y = 280.069 kN (\uparrow)$

$M_{BA} = -(280.069 \times 4 - 400 \times 2) = -320.4 kNm.$

$M_{BC} = 320.4 kNm.$

Q4. (a). $(FEM)_{BA} = \frac{12(8)^2}{8} = 96 kNm$, $(FEM)_{BD} = -\frac{24(6)}{8} = -18 kNm$, $(FEM)_{DB} = 18 kNm.$

$M_{BA} = 3(2E)\left(\frac{1}{8}\right)[\theta_B - 0] + 96 = \frac{3EI}{4}\theta_B + 96$

$M_{BD} = 2(1.5E)\left(\frac{1}{6}\right)[2\theta_B + \theta_D - 3(0)] - 18 = EI\theta_B + \frac{EI}{2}\theta_D - 18$

$M_{DB} = 2(1.5E)\left(\frac{1}{6}\right)[2\theta_D + \theta_B - 3(0)] + 18 = EI\theta_D + \frac{EI}{2}\theta_B + 18$

$M_{DE} = -10$

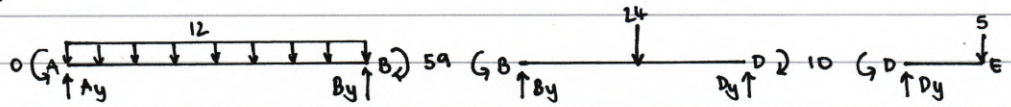
$M_{BA} + M_{BD} = 0$: $\frac{3EI}{4}\theta_B + 96 + EI\theta_B + \frac{EI}{2}\theta_D - 18 = 0 \rightarrow \frac{7EI}{4}\theta_B + \frac{EI}{2}\theta_D = -78 - (*)$

$M_{DB} + M_{DE} = 0$: $EI\theta_D + \frac{EI}{2}\theta_B + 18 - 10 = 0 \rightarrow \frac{EI}{2}\theta_D = -4 - \frac{EI}{4}\theta_B - (**)$

Sub $(**)$ into $(*)$: $\frac{7EI}{4}\theta_B - 4 - \frac{EI}{4}\theta_B = -78 \rightarrow \theta_B = -\frac{148}{3EI} \rightarrow \theta_D = \frac{50}{3EI}.$

$M_{AB} = 0 kNm$, $M_{BA} = 59 kNm$, $M_{BD} = -59 kNm$, $M_{DB} = 10 kNm$, $M_{DE} = -10 kNm.$

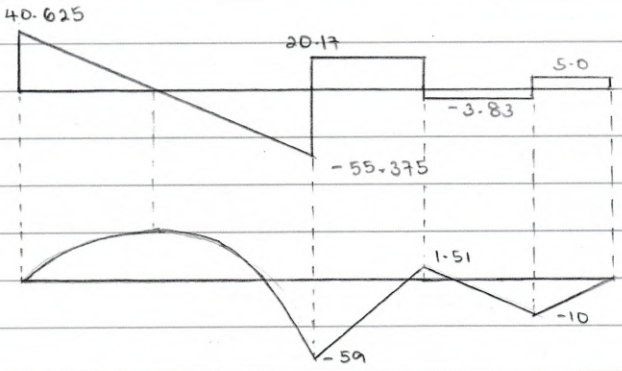
FBD:




Using eqm: $A_y = 40.625 kN$, $B_y = 55.375 kN$; $B_y = 20.17 kN$, $D_y = 3.83 kN$; $D_y = 5 kN.$

Q4. (a) [cont.]

SFD:




BMD:




(b) $\psi_{AB} = \psi_{BA} = \frac{15\times10^{-3}}{8} = 0.001875$, $\psi_{BD} = \psi_{DB} = -\frac{9\times10^{-3}}{6} = -0.0015$

$M_{BA} = 3(2E)(\frac{I}{8})[\theta_B - 0.001875) + 96 = 15000\theta_B + 67.875$

$M_{BD} = 2(1.5E)(\frac{I}{6})[2\theta_B + \theta_D - 3(-0.0015)] - 18 = 20000\theta_B + 10000\theta_D + 27$

$M_{DB} = 2(1.5E)(\frac{I}{6})[2\theta_D + \theta_B - 3(-0.0015)] + 18 = 20000\theta_D + 10000\theta_B + 63$

$M_{DE} = -10$

$M_{BA} + M_{BD} = 0$: $150000\theta_B + 67.875 + 200000\theta_B + 100000\theta_D + 27 = 0 \rightarrow 350000\theta_B + 100000\theta_D = -94.875$ - ①

$M_{DB} + M_{DE} = 0$: $200000\theta_D + 100000\theta_B + 63 - 10 = 0 \rightarrow 100000\theta_B + 200000\theta_D = -53$ - ②

↳ solving ① & ② gives $\theta_B = -0.0022792$ rad, $\theta_D = -0.0015104$ rad

$M_{AB} = 0$, $M_{BA} = 33.69$ kNm, $M_{BD} = -33.69$ kNm, $M_{DB} = 10$ kNm, $M_{DE} = -10$ kNm.