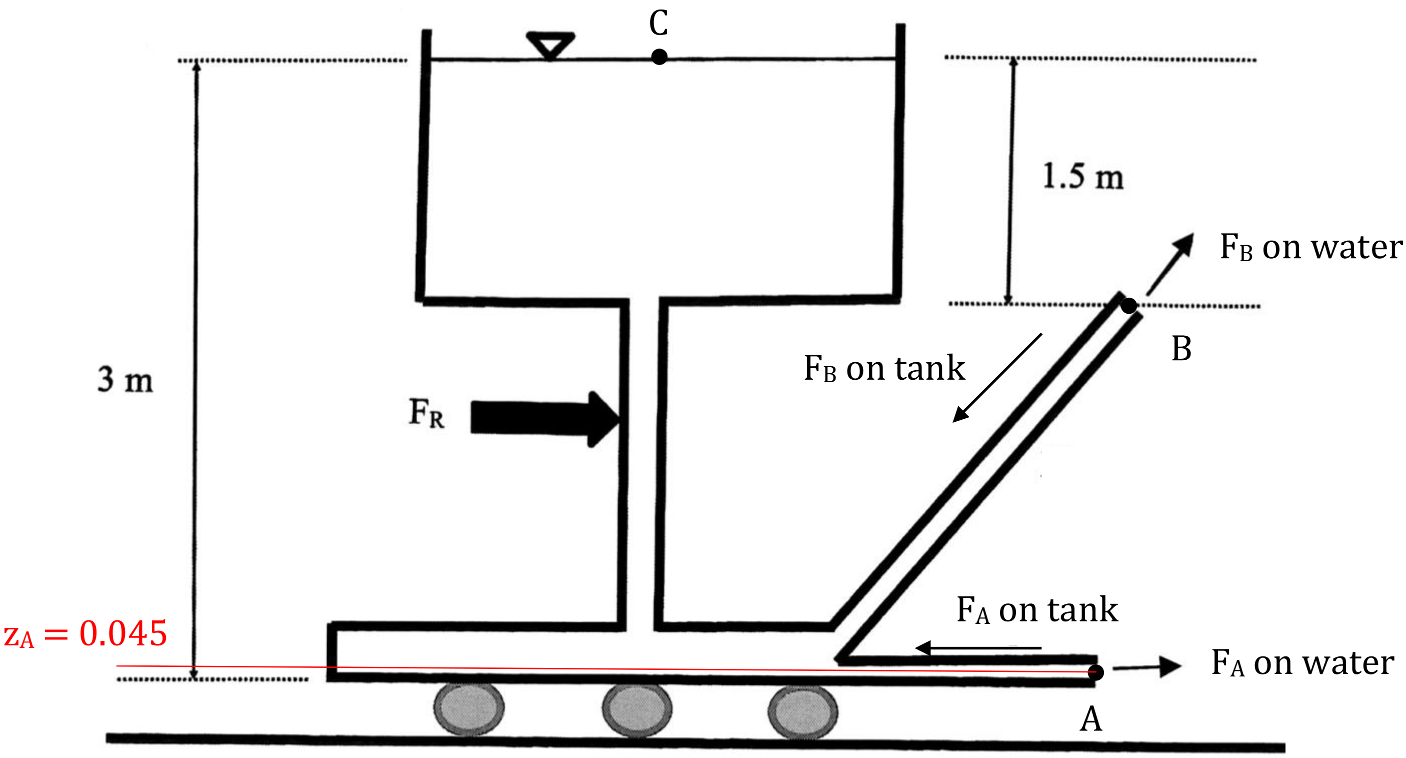
Name: Owen
Paper: CV1012 – Fluid Mechanics 2020–2021 Sem 2

1. (a)



Assuming $P_{atm} = 0$, $V_C = 0$, no head loss,
Bernoulli's equation C to A:

$z_C = \frac{{V_A}^2}{2g} + z_A$

$V_A = \sqrt{(3 - 0.045)(2)(9.81)}$

$= 7.6143$ m/s

Bernoulli's equation C to B:

$z_C = \frac{{V_B}^2}{2g} + z_B$

$V_B = \sqrt{(3 - 1.5)(2)(9.81)}$

$= 5.4249$ m/s

$F_A = M_{out} - M_{in}$

$= \rho Q_A V_A - 0$

$= 1000(7.6143(\pi/4)(0.09)^2)(7.6143)$

$= 368.83$ N

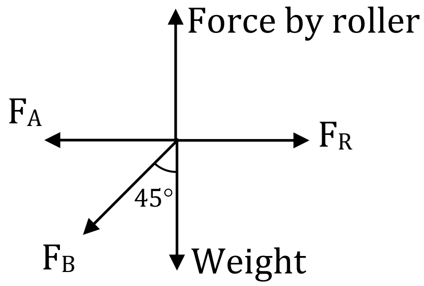
$F_B = \rho Q_B V_B - 0$

$= 1000(5.4249(\pi/4)(0.1)^2)(5.4249)$

$= 231.14$ N

1. (a)

FBD of tank:



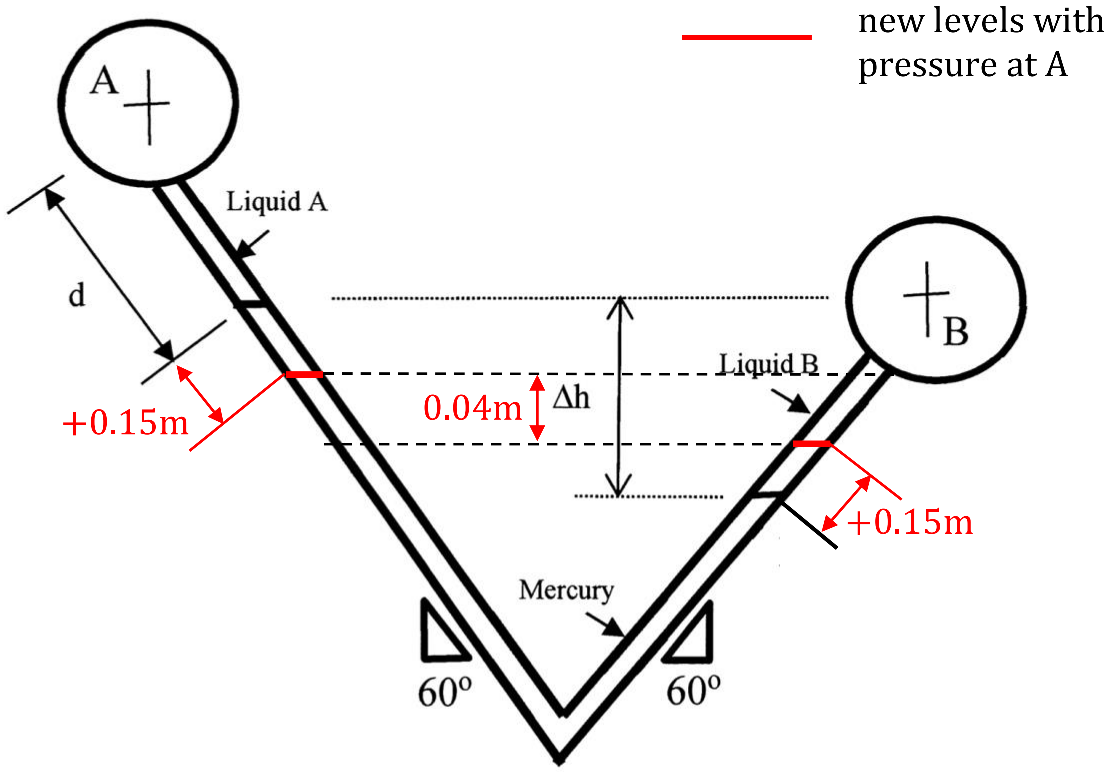
$\sum F_x = 0$ (taking right as positive)

$F_R - F_A - F_B\sin(45°) = 0$

$F_R = 368.83 + 231.14\sin(45°)$

$= \underline{\underline{532\ N}}$ (direction of F_R is correct as the value calculated is positive)

1. (b)



When d = 0.15m,

$P_A + (1000)(9.81)(0.15\sin 60°) + (13600)(9.81)(\Delta h) = P_B + (800)(9.81)(\Delta h)$

$(P_B - P_A) = 1274.4 + 125568\ \Delta h$

When d = 0.30m, Δh = 0.04m, if additional pressure P added to A,

$P_A + P + (1000)(9.81)(0.3\sin 60°) + (13600)(9.81)(0.04) = P_B + (800)(9.81)(0.04)$

$(P_B - P_A) - P = 2548.7 + 5336.64 - 313.92$

$= 7571.4\ \text{Pa}$

$\Delta h = 2(0.15)\sin 60° + 0.04$

$= \underline{0.3\ \text{m}}$

$(P_B - P_A) = 1274.4 + 125568\ (0.3)$

$= \underline{\underline{38.9\ \text{kPa}}}$

2. (a)

$$u = U\left(\frac{y}{d}\right)^{\frac{1}{n}}$$

When u = 10.50, y = 10,

$$10.50 = U\left(\frac{10}{d}\right)^{\frac{1}{n}}$$

$$n \ln(10.50) = \ln\left(10\left(\frac{U}{d}\right)\right)$$

$$\ln\left(\frac{U}{d}\right) = n\ln(10.50) - \ln(10)$$

When u = 7.42, y = 5,

$$7.42 = U\left(\frac{5}{d}\right)^{\frac{1}{n}}$$

$$\ln(7.42) = \frac{1}{n}\ln\left(5\left(\frac{U}{d}\right)\right)$$

$$\ln\left(\frac{U}{d}\right) = n\ln(7.42) - \ln(5)$$

$$n\ln(10.50) - \ln(10) = n\ln(7.42) - \ln(5)$$
$$n = 1.9964$$

$$\frac{du}{dy} = U\left(\frac{1}{n}\right)\left(\frac{y}{d}\right)^{\left(\frac{1}{n}-1\right)}\left(\frac{1}{d}\right)$$
$$= U\left(\frac{1}{n}\right)(y)^{\left(\frac{1}{n}-1\right)}\left(\frac{1}{d}\right)^{\left(\frac{1}{n}-1\right)}\left(\frac{1}{d}\right)$$
$$= \left(\frac{1}{n}\right)(y)^{\left(\frac{1}{n}-1\right)}\left(\frac{U}{d^{\frac{1}{n}}}\right)$$

$$\left(\frac{U}{d^{\frac{1}{n}}}\right) = \frac{10.50}{10^{\frac{1}{n}}}$$
$$= 3.3135$$

$\tau = \mu\,\frac{du}{dy}$

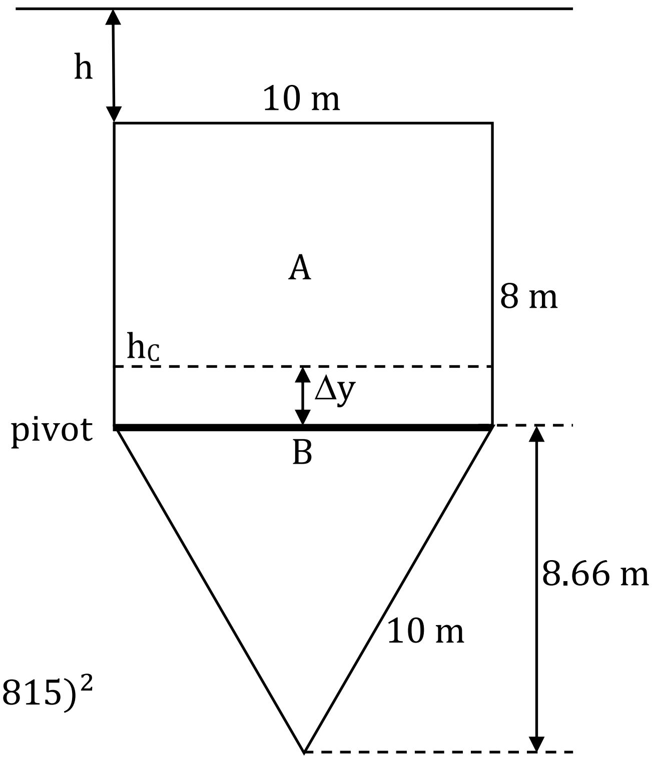
At y = 0.5,

$\underline{\underline{\tau = 0.00235\ \text{N/m}^2}}$

At y = 1.0,

$\underline{\underline{\tau = 0.00166\ \text{N/m}^2}}$

2. (b)



Centroid A (from pivot) = 4 m

Centroid B (from pivot) = $-\frac{1}{3}(10\sin60°)$

$= -2.8868$ m

Centroid $= \frac{4(10)(8)-2.8868(0.5)(10)(10\sin60°)}{(10)(8)+(0.5)(10)(10\sin60°)}$

= 1.5815 m (above pivot)

I_A about $h_C = \frac{1}{12}(10)(8)^3 + (8)(10)(4-1.5815)^2$

$= 894.60$ m^4

I_B about $h_C = \frac{1}{36}(10)(10\sin60°)^3 + (0.5)(10)(10\sin60°)(2.8868+1.5815)^2$

$= 1044.9$ m^4

Total $I_C = 894.60 + 1044.9$

$= 1939.5$ m^4

For no moment about pivot, Δy = 1.5815 m

$1.5815 = \frac{I_C}{Ay_C}$

$= \frac{1939.5}{[(8)(10)+(0.5)(10)(10\sin60°)]\,[h+8-1.5815]}$

h = 3.53 m

3. (a)

Laminar flow is characterized by fluid particles following smooth paths in layers, with each layer moving smoothly past the adjacent layers with little or no mixing.

Turbulent flow is fluid motion characterized by chaotic changes in pressure and flow velocity.

3. (b) (i)

$\text{Re} = \frac{VL}{\upsilon}$

For Re similarity,

$V_p L_p = V_m L_m$, if $\upsilon_p = \upsilon_m$

$\frac{V_p}{V_m} = \frac{L_m}{L_p}$

$V_r = (L_r)^{-1}$

$\text{Fr} = \frac{V}{\sqrt{gL}}$

For Fr similarity,

$\frac{V_p}{\sqrt{L_p}} = \frac{V_m}{\sqrt{L_m}}$, if $g_m = g_p$

$\frac{V_p}{V_m} = \sqrt{\frac{L_p}{L_m}}$

$V_r = (L_r)^{0.5}$

Hence, due to a conflict in the length scale used, it is generally not practical to satisfy both the Re and Fr similarity simultaneously if the same fluid is used.

3. (b) (ii)

For both Re and Fr similarity,

$$\frac{V_p}{V_m} = \frac{L_m}{L_p}\frac{\upsilon_p}{\upsilon_m} = \sqrt{\frac{L_p}{L_m}}, \text{ if } g_m = g_p$$

$$\frac{\upsilon_p}{\upsilon_m} = \frac{L_p}{L_m}\sqrt{\frac{L_p}{L_m}}$$

$$= \underline{\underline{\left(\frac{L_p}{L_m}\right)^{1.5}}}$$

3. (c)

$P = f(Q, D, \omega, H, \rho, g)$

Their dimesions are:

$P = [ML^{2}T^{-3}]$

$Q = [L^{3}T^{-1}]$

$D = [L]$

$\omega = [T^{-1}]$

$H = [L]$

$\rho = [ML^{-3}]$

$g = [LT^{-2}]$

Using D, ω, ρ as repeating variables, ∏ term for P:

$\prod_1 = P\ \rho^{a}\ \omega^{b}\ D^{c}$

Using MLT system:

$ML^{2}T^{-3}M^{a}L^{-3a}T^{-b}L^{c} = M^{0}L^{0}T^{0}$

$1 + a = 0$

$2 + c - 3a = 0$

$-3 - b = 0$

Solving: a = -1, c = -5, b = -3

$\prod_1 = \frac{P}{\rho\omega^{3}D^{5}}$ (shown)

3. (d) (i)

Fluid power = $\rho g Q h_p$

= (1000)(9.81)(0.05)(15)

= 7357.5 W

Shaft power = $\frac{7357.5}{0.75}$

= $\underline{\underline{9810 \text{ W}}}$

3. (d) (ii)

For similarity of flow coefficient:

$$\frac{Q_1}{\omega_1 D^3} = \frac{Q_2}{\omega_2 D^3}$$

$Q_2 = Q_1 \frac{\omega_2}{\omega_1}$

$= (0.05)\left(\frac{1800}{1200}\right)$

$= \underline{\underline{0.075 \text{ m}^3/\text{s}}}$

For similarity of head rise coefficient:

$$\frac{gH_1}{\omega_1{}^2 D^2} = \frac{gH_2}{\omega_2{}^2 D^2}$$

$H_2 = H_1 \left(\frac{\omega_2}{\omega_1}\right)^2$

$= (15)\left(\frac{1800}{1200}\right)^2$

$= \underline{\underline{33.75 \text{ m}}}$

4. (a)

1. Re < 2100, in laminar flow regime, f is a function of Re only & is independent of ε/D, the equation is f = 64/Re.
2. 2100 < Re <4000, in transition range, f is uncertain as flow may be laminar or turbulent.
3. Re > 4000 but not in wholly turbulent flow regime, f depends on both Re and ε/D.
4. Re very large and in the wholly turbulent flow regime, f depends on ε/D only, independent of Re.

4. (b) (i)

$$V = \frac{Q}{A}$$
$$= \frac{0.0000393}{(\pi/4)(0.05)^2}$$
$$= \underline{\underline{0.02\ m/s}}$$

$$Re = \frac{VD}{\upsilon}$$
$$= \frac{(0.02)(0.05)}{(1*10^{-6})}$$
$$= 1000 < 2100\ \underline{\underline{\text{(laminar flow regime)}}}$$

4. (b) (ii)

$$V_C = 2(0.02)$$
$$= \underline{\underline{0.04\ m/s}}$$

4. (c) (i)

Assuming $P_{atm} = 0$, $V = 0$ at top of reservoir,
Bernoulli's equation A to B:
$z_A = z_B + h_f$

$48 - 45 = (10.63+21.25)Q^2$

$Q = \sqrt{\frac{3}{(10.63+21.25)}}$

$= \underline{\underline{0.307 \text{ m}^3/\text{s}}}$

4. (c) (i)

Assuming $P_{atm} = 0$, $v = 0$ at top of reservoir,
$h_A = 48$ m, $h_B = 45$ m, $h_C = 35$ m
$h_f = h_1 - h_2$
$= KQ^2$

KQ^2 for pipe 3 $= 21.25Q^2 + 0.5\frac{Q^2}{A^2(2g)}$

$= 21.25Q^2 + \frac{0.5}{[(\pi/4)(0.6)^2]^2(2)(9.81)}Q^2$

$= 21.569Q^2$

$|Q| = \sqrt{\frac{|h_1 - h_2|}{K}}$

$Q_1 + Q_2 + Q_3 = 0$ (continuity, take into J as positive and out of J as negative)

$|Q_1| = \sqrt{\frac{|48 - h_J|}{10.63}}$

$|Q_2| = \sqrt{\frac{|45 - h_J|}{21.25}}$

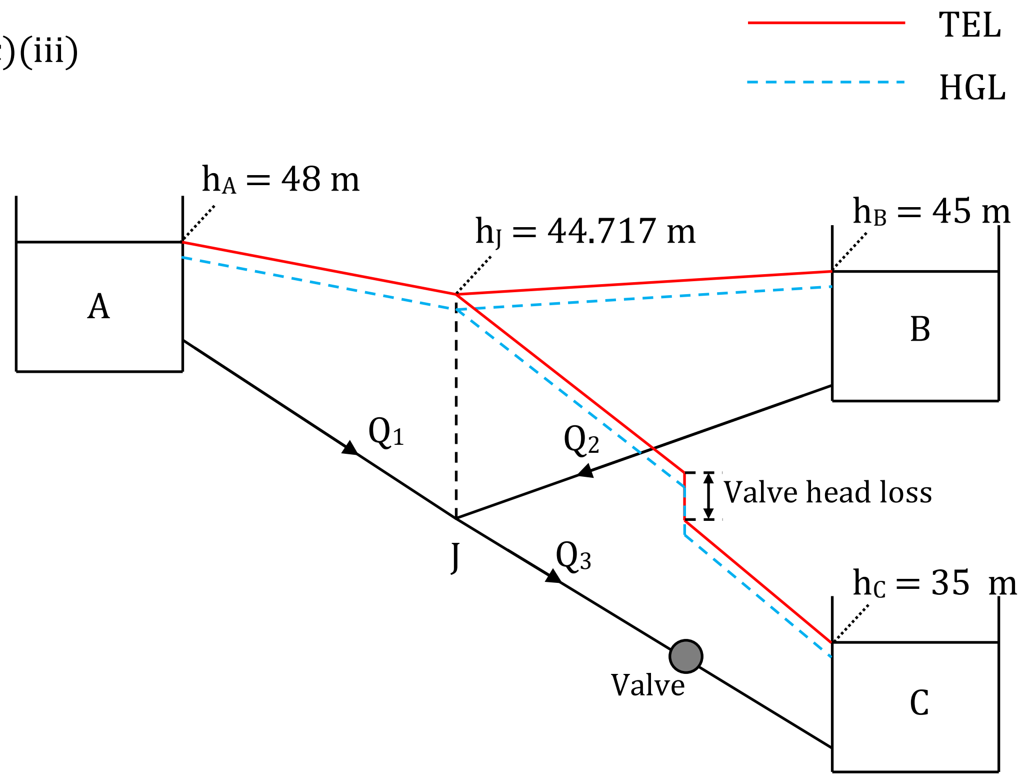
$|Q_3| = \sqrt{\frac{|35 - h_J|}{21.569}}$

Guess:
Q_1 into J, Q_2 out of J, Q_3 out of J
Using G.C., solve for $|Q_1| - |Q_2| - |Q_3| = 0$, $45 < h_J < 48$
No solution for h_J

Guess:
Q_1 into J, Q_2 into J, Q_3 out of J
Using G.C., solve for $|Q_1| + |Q_2| - |Q_3| = 0$, $35 < h_J < 45$
$h_J = 44.717$ m

Thus,
Q_1 = <u>0.556 m³/s into J</u>
Q_2 = <u>0.115 m³/s into J</u>
Q_3 = <u>0.671 m³/s out of J</u>

4. (c)(iii)



NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2020-2021

CV1012 - FLUID MECHANICS

April / May 2021 **Time Allowed: 2½ hours**

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **EIGHT (8)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. An **Appendix** of **ONE (1)** page containing useful data and formula is attached to the question paper.
5. This is a Closed-Book Examination.
6. All answers must be written in the answer book provided. Answer each question beginning on a **FRESH** page of the answer book.

---

1. (a) Figure Q1(a) shows a water tank setup supported on top of a rolling platform.

It is given that:

Diameter of Pipe A = 0.09 m
Diameter of Pipe B = 0.10 m
Vertical inclination of Pipe B = 45 degree

Compute the magnitude of the force F_R so that the platform is stationary (assume that the area of the tank is large, and that the energy loss is negligible). Is the direction of F_R indicated in the figure correct? Explain your reasoning.

(10 Marks)

Note: Question No. 1 continues on Page 2.

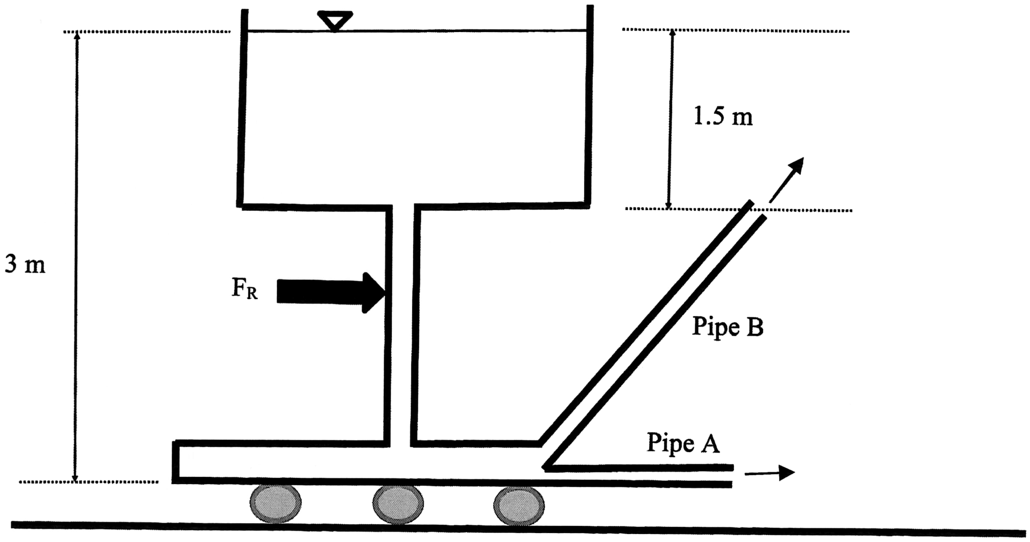


Figure Q1(a)

Note: Question No. 1 continues on Page 3.

(b) Figure Q1(b) shows an inclined manometer with 60 degree inclination from the horizontal axis.

- Based on the existing pressures at Points A and B, the interface between Liquid A and Mercury is at the same elevation as Point B and d = 0.15 m.

- If, however, an additional pressure is added to the pressure at Point A, d will increase from 0.15 m to 0.30 m, and new Δh = 0.04m.

Compute the pressure difference between Points A and B as well as the original value of Δh before the additional pressure is imposed.

It is given that:

Density of Mercury = 13,600 kg/m^3
Density of Liquid A = 1,000 kg/m^3
Density of Liquid B = 800 kg/m^3

(15 Marks)

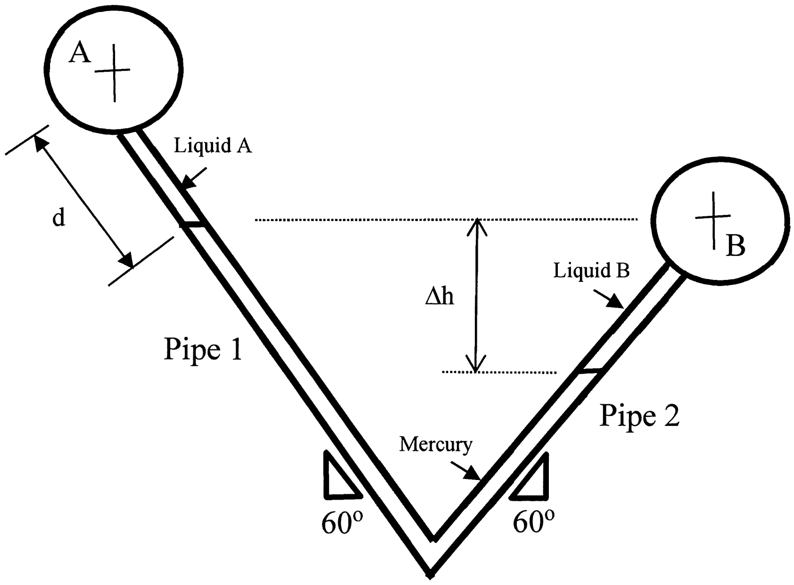


Figure Q1(b)

2. (a) The velocity profile of a laminar flow (dynamic viscosity = 10^{-3} kg/(m s)) next to a solid boundary is shown in Figure Q2(a). The velocity profile can be expressed as:

$$\frac{u}{U} = \left(\frac{y}{d}\right)^{1/n}$$

where the values of n, U and d are unknown constants.

It is given that:

u = 10.50 m/s at y = 10 m

u = 7.42 m/s at y = 5 m

Compute the magnitude of viscous shear stress at y = 0.5 and 1.0 m (hint: you can assume that the Newton's law of viscosity can be used).

(10 Marks)

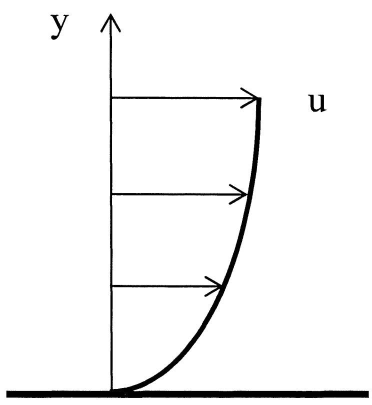


Figure Q2(a)

Note: Question No. 2 continues on Page 5.

(b) Figure Q2(b) shows the layout of a submerged gate opening with a rotational shaft in a water body. The cross-section of the gate consists of two areas. Area A is a rectangle of 8 m by 10 m above the shaft, and Area B is an equilateral triangle with a side length of 10 m below the shaft.

Compute the water depth, h, such that there is zero moment about the shaft.

(15 Marks)

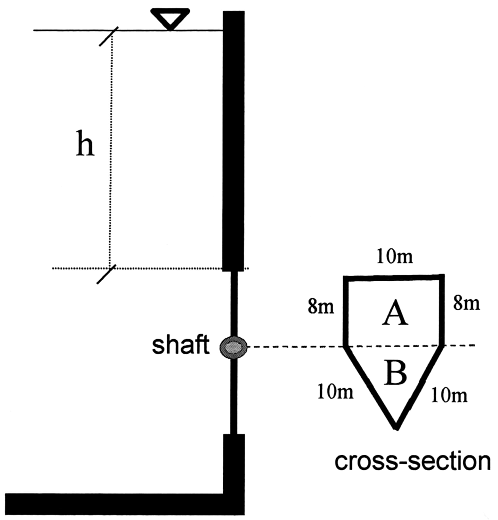


Figure Q2(b)

3. (a) Explain the meaning of the terms: (i) laminar flow, and (ii) turbulent flow.

(4 Marks)

(b) (i) Explain and show that it is generally not practical to satisfy both the Reynolds and Froude numbers similarity simultaneously if the same fluid is used in both prototype and model.

(ii) However, if the model needs to satisfy both the Reynolds and Froude numbers similarity simultaneously, what is the kinematic viscosity scale of the fluids, $\frac{\nu_p}{\nu_m}$?

(7 Marks)

(c) The power P (kg m^2/s^3) input to a pump is assumed to be a function of the discharge, Q (m^3/s), impeller diameter, D (m), impeller rotation speed, ω (rev/s), energy head H, fluid density, ρ, and acceleration due to gravity. Using dimensional analysis, show that the π-parameter for power is $\frac{P}{\rho \omega^3 D^5}$.

(4 Marks)

(d) A pump with an impeller diameter D = 0.3 m delivers water at a rate of Q = 0.05 m^3/s against a head H = 15 m, if the pump speed is 1200 rpm.

(i) Calculate the shaft power, P required if the pump efficiency is η = 75%.

(ii) Calculate the discharge and head if the pump speed is increased from 1200 rpm to 1800 rpm. You may assume the pump efficiency remains the same.

(10 Marks)

4. (a) Explain briefly 4 important characteristics of the Moody Diagram for pipe flow.

(4 Marks)

(b) Water (kinematic viscosity, ν = 1x10^{-6} m^2/s) flows in a pipeline (diameter D = 0.05 m) at a flow rate of Q = 3.93 x 10^{-5} (or 0.0000393) m^3/s.

(i) What is the average velocity in the pipeline and its flow regime?

(ii) What is the velocity at the centreline of the pipe?

(6 Marks)

(c) Figure Q4 shows 3 Reservoirs A, B and C are connected by 3 pipelines to a common junction **J**. The water levels at Reservoir A, B, and C are fixed at elevations of 48 m, 45 m and 35 m, respectively. The 3 pipelines have a constant diameter, D = 0.6 m, and their friction factor, *f* may be assumed to be the same at 0.02. The lengths of pipelines 1, 2 and 3 are 500 m, 1000 m and 1000 m, respectively. The computed K for the pipe characteristics are given in Table Q4.

If a <u>valve</u> is installed along pipe 3,

(i) Calculate the flow rate from Reservoirs A to B if the valve is completely closed.

(ii) Calculate the flow rates in the 3 pipes if the valve is completely opened with a minor loss coefficient, K_{valve} = 0.5.

<u>Note</u>: You must include the minor loss due to the <u>valve</u> in your computations. But you may ignore the entrance loss, exit loss and bend loss at **J**.

(iii) Sketch the total energy line (TEL) and hydraulic grade line (HGL) for Part (c)(ii).

(15 Marks)

Note: Question No. 4 continues on Page 8.

Table Q4

Pipeline	Pipe diameter D (m)	Length L (m)	Friction factor f	$K = \frac{8fL}{g\pi^2 D^5}$
1	0.6	500	0.02	10.63
2	0.6	1000	0.02	21.25
3	0.6	1000	0.02	21.25

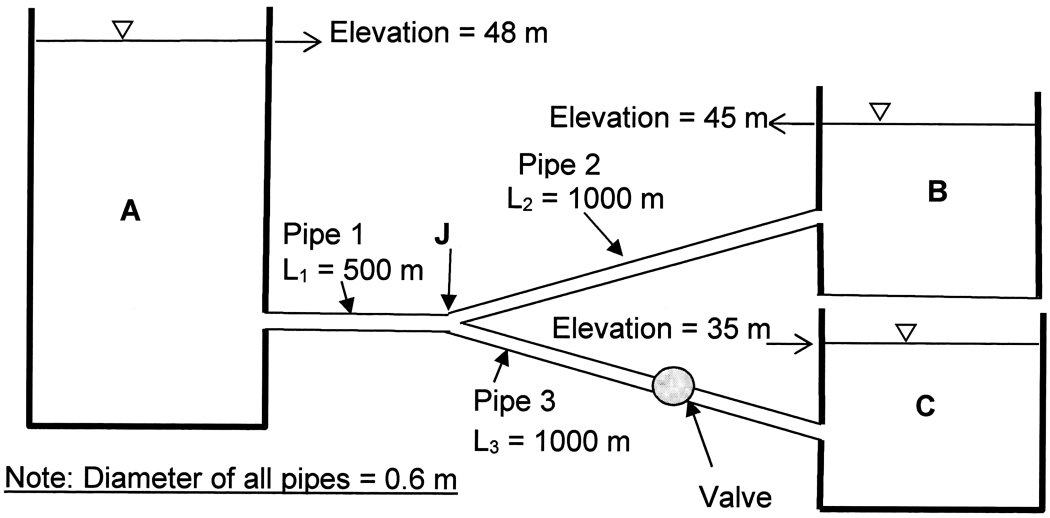


Figure Q4

END OF PAPER

Useful Formulas and Moody Diagram

$\gamma = \rho g$

$SG = \rho/\rho_{water}$

$\tau = \mu(du/dy)$

$\mu = \rho\nu$

$E_V = - dp/ (dV/V) = dp/ (d\rho/\rho)$

$p = p_o + \gamma h$

$p_{atm} \approx$ 1 bar = 100 kPa (abs) $\approx$ 760 mm Hg (abs)

$p_{gauge} = p_{abs} - p_{atm}$

$F_R = p_c A = \gamma h_c A$

$y_R = I_{xc}/(y_c A) + y_c$

$I_{xc} = ba^3/12$ (rectangle); $\pi R^4/4$ (circle); $0.11R^4$ (semicircle); $0.055R^4$ (quarter circle)

$I_{xc} = ba^3/36$ (triangle, b = base, a = height)

$F_B = \gamma V_{displaced}$

$BM = I_{yy}/V_{displaced}$

$\Sigma Q_{in} = \Sigma Q_{out}$

$p_1/(\rho g) + V_1^2/(2g) + z_1 = p_2/(\rho g) + V_2^2/(2g) + z_2$

$\Sigma F_x = \Sigma(\rho Q V_x)_{out} - \Sigma(\rho Q V_x)_{in}$

Darcy-Weisbach Eq: $h_f = \frac{f\ L\ V^2}{2\ g\ D} = \frac{8\,f\,L\,Q^2}{g\,\pi^2\,D^5}$

Poiseuille's Eq: $h_f = \frac{32\,\mu\,L\,V}{\rho\,g\,D^2} = \frac{128\,\mu\,L\,Q}{\rho\,g\,\pi\,D^4}$

For laminar flow, $f = \frac{64}{Re}$

Blasius Formula: $f = \frac{0.316}{Re^{0.25}}$,

$K_{expansion} = \left(1 - \frac{A_1}{A_2}\right)^2$, $K_{contraction}$ = 0.5, $K_{entrance}$ = 0.5, K_{exit} = 1.0.

Fluid power = $\rho\ g\ Q\ h_p$

Head rise coefficient = $\frac{gH}{\omega^2 D^2}$

Flow coefficient = $\frac{Q}{\omega D^3}$

Power coefficient = $\frac{P}{\rho\,\omega^3\,D^5}$

CV1012 FLUID MECHANICS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.