Jervis Teo

x, y, z

$\begin{pmatrix}1\\0\\0\end{pmatrix} \begin{pmatrix}0\\1\\0\end{pmatrix} \begin{pmatrix}0\\0\\1\end{pmatrix}$

$\vec{r} = \begin{pmatrix}x\\y\\z\end{pmatrix}$ origin to ~~tip of~~ tail of force

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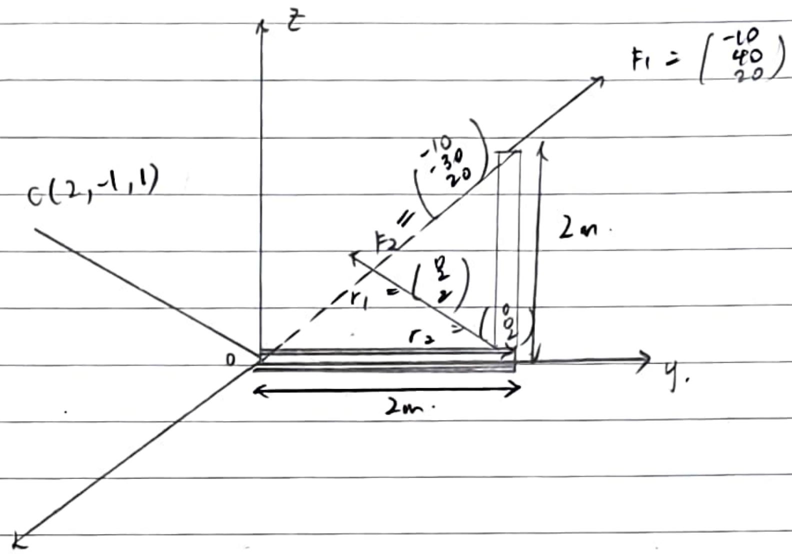
1(a)(i) $\vec{F_1} = \begin{pmatrix}-10\\40\\20\end{pmatrix}$, $\vec{F_2} = \begin{pmatrix}-10\\-30\\20\end{pmatrix}$

$|\vec{F_1}| = \sqrt{(-10)^2+(40)^2+(20)^2}$ $= \sqrt{2100}$

$|\vec{F_2}| = \sqrt{(-10)^2+(-30)^2+(20)^2}$ $= \sqrt{1400}$

$\vec{M} = \vec{r} \times \vec{F}$



$\vec{M}_{F_1} = r_1 \times \vec{F_1}$
$= \begin{pmatrix}0\\2\\2\end{pmatrix} \times \begin{pmatrix}-10\\40\\20\end{pmatrix}$
$= \begin{pmatrix}40-80\\-20-0\\0-(-20)\end{pmatrix}$
$= \begin{pmatrix}-40\\-20\\20\end{pmatrix}$

$\vec{M}_{F_2} = \vec{r_2} \times \vec{F_2}$
$= \begin{pmatrix}0\\2\\0\end{pmatrix} \times \begin{pmatrix}-10\\-30\\20\end{pmatrix}$
$= \begin{pmatrix}40\\0\\20\end{pmatrix}$

$|\vec{OC}| = \sqrt{(2)^2+(-1)^2+(-1)^2}$
$= \sqrt{4+1+1}$
$= \sqrt{6}$.

$\hat{\vec{OC}} = \frac{1}{\sqrt{6}}\begin{pmatrix}2\\-1\\1\end{pmatrix}$.

M_{F_1} about OC $= \vec{M}_{F_1} \cdot \hat{\vec{OC}}$
$= \begin{pmatrix}-40\\-20\\20\end{pmatrix} \cdot \begin{pmatrix}2\\-1\\1\end{pmatrix}\left(\frac{1}{\sqrt{6}}\right)$
$= (-80+20+20)\left(\frac{1}{\sqrt{6}}\right)$
$= \frac{-40}{\sqrt{6}}$ Nm.

M_{F_2} about OC $= \vec{M}_{F_2} \cdot \hat{\vec{OC}}$
$= \begin{pmatrix}40\\0\\20\end{pmatrix} \cdot \begin{pmatrix}2\\-1\\1\end{pmatrix}\left(\frac{1}{\sqrt{6}}\right)$
$= (80+20)\left(\frac{1}{\sqrt{6}}\right)$
$= \frac{100}{\sqrt{6}}$ Nm.

1b(i) projection vector of F1 along OC for it to be minimum.

projection vector = 0.

Since projection vector = 0 there is no direction.

$\vec{F_1} \cdot \vec{OC} = 0$.

$\begin{pmatrix} -10 \\ 40 \\ 20 \end{pmatrix} \cdot \begin{pmatrix} x \\ -1 \\ 1 \end{pmatrix} = 0$

$-10x - 40 + 20 = 0$

$-10x = 20$.

$x = -2$.

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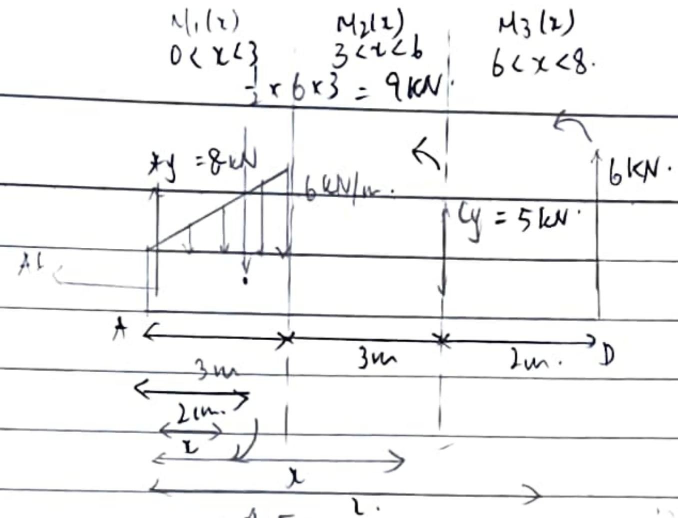
2a)



$+\uparrow \sum F_y = 0$

$A_y - 9 + C_y + 6 = 0$

$A_y + C_y - 3 = 0$

$A_y + C_y = 3$

when $C_y = 5kN$,

$A_y - 5 = 3$

$A_y = 8kN$

$+\circlearrowleft \sum M_A = 0$

$-(2 \times 9) + 6C_y + 6 \times 8 = 0$

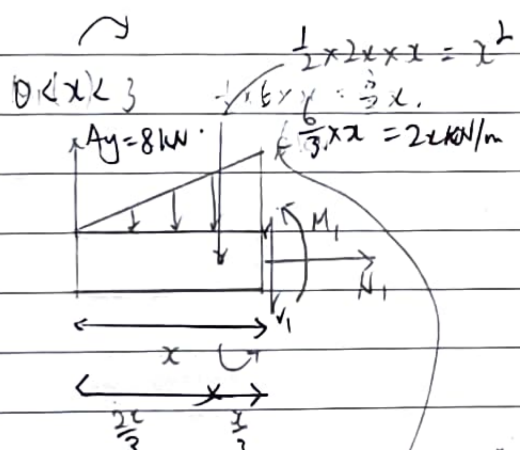
$-18 - 6C_y + 48 = 0$

$-6C_y + 30 = 0$

$-6C_y = -30$

$C_y = 5\ kN$

2b) $0 < x < 3$



$+\circlearrowleft \sum M_1 = 0$

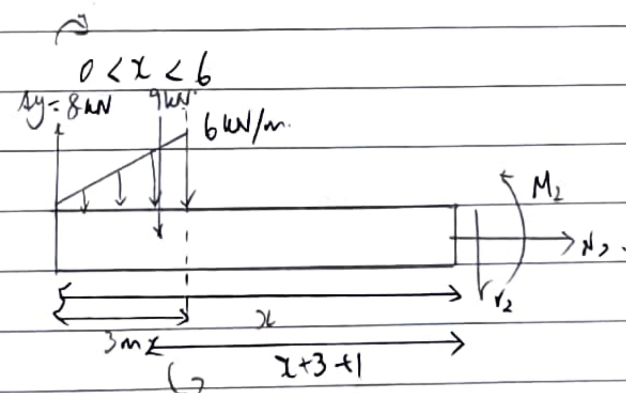
$M_1(x) + 8x + \left(\frac{x}{3}\right)(x^2) = 0$

$M_1(x) = 8x - \frac{x^3}{3}$

$= 8x - \frac{1}{3}x^3$

* using similar triangle:

$\frac{6}{3} = \frac{\quad}{x}$



$+\circlearrowleft \sum M_2 = 0$

$M_2(x) + 9(x-2) - 8x = 0$

$M_2(x) = 8x - 9(x-2)$

$= -x + 18$

$= 18 - x$

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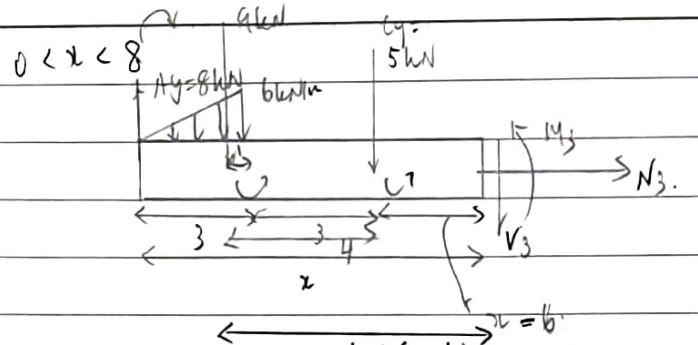
$0 < x < 8$



$4 + (x-6) = x - 2$

~~$+\circlearrowleft \sum M_3 = 0.$~~

~~$M_3(x) - 8x + 9(3 + x - 6) + 5(x-6) = 0.$~~

~~$M_3(x) = 8x - [9(3 + x - 6)] - 5(x-6)$~~

~~$= 8x - [-27 + 9x] - 5x + 30.$~~

~~$= 8x + 27 - 9x - 5x$~~

$+\circlearrowleft \sum M_3 = 0$

$M_3(x) - 8x + 9(x-2) + 5(x-6) = 0.$

$M_3(x) = 8x - 9(x-2) - 5(x-6)$

$= 8x - 9x + 18 - 5x + 30.$

$= -6x + 48$

when $x = 0$

$M_1(0) = 8(0) - \frac{(0)^3}{3}$

$= 0.$

when $x = 3$,

$M_1(3) = 8(3) - \frac{(3)^3}{3}$

$= 8(3) - \frac{27}{3}$

$= 24 - 9$

$= 15.$

when $x = 3$,

$M_2(3) = 18 - 3$

$= 15$

when $x = 6$,

$M_2(6) = 18 - 6$

$= 12.$

when $x = 6$,

$M_3(6) = -6(6) + 48$

$= -36 + 48$

$= 12.$

when $x = 8$,

$M_3(8) = -6(8) + 48$

$= 0.$

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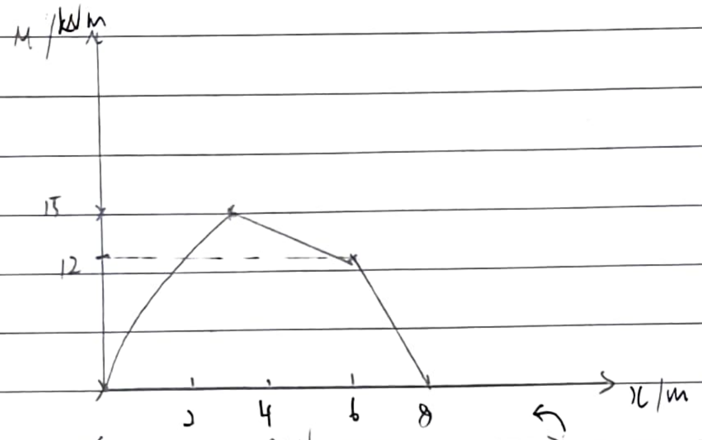
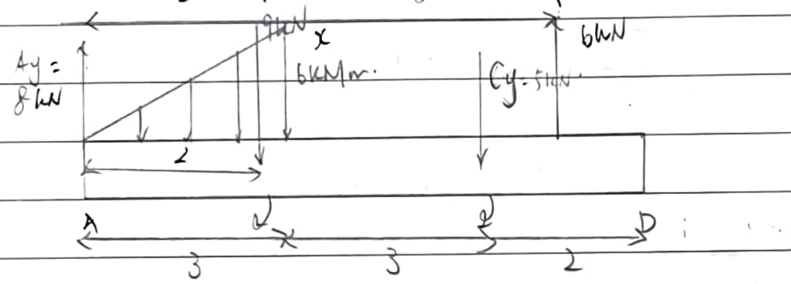


20)



$+\circlearrowright \Sigma M_A = 0$

$6x + C_y(6) - 9(2) = 0$

$6x = 6C_y + 18$

$x = C_y + 3$ → reaction force at C is minimum.

when $C_y = 0$,

$x = 3$.

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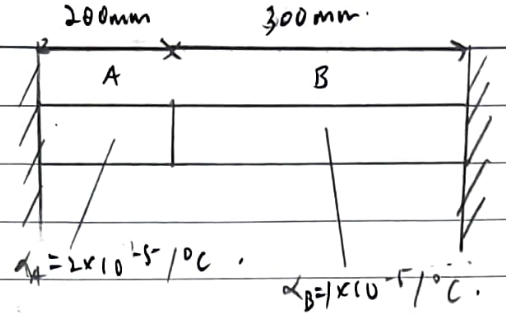
3a(i)



$$\delta_T = \alpha_A \Delta T L_A + \alpha_B \Delta T L_B$$
$$= (2\times10^{-5})(20)(200\times10^{-3}) + (1\times10^{-5})(20)(300\times10^{-3})$$
$$= 0.00008 + 0.00006$$
$$= 0.00014 \text{ m}$$

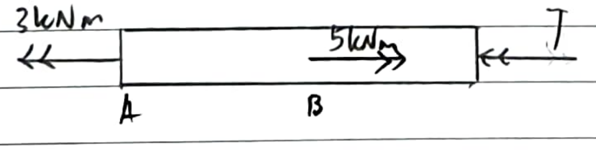
$$\frac{PL_A}{EA} + \frac{PL_B}{EA} - 0.00014 = 0.$$
$$P\left(\frac{L_A + L_B}{EA}\right) = 0.00014$$
$$P = 0.00014 \times \left(\frac{200\times10^{9} \times 4000\times10^{-6}}{(200\times10^{-3}) + (300\times10^{-3})}\right)$$
$$= 224\,000 \text{ N}$$
$$= 224 \text{ kN}$$

*. 3a(ii) This comment is valid when the bar is being heated up,

3b) Section AB $(\longrightarrow +ve)$

$$\Sigma T = 0$$
$$T - 3 = 0$$
$$T = 3 \text{ kNm} \quad (\longrightarrow)$$

Section BC



$$\Sigma T = 0$$
$$-T + 5 - 3 = 0$$
$$-T = -2$$
$$T = 2 \text{ kNm} \quad (\longleftarrow)$$

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Outer radius, $C_2 = \frac{110 \times 10^{-3}}{2}$

$= (55 \times 10^{-3})\,m$

$J = \frac{\pi}{2}\left[(C_2)^4 - (C_1)^4\right]$

$= \frac{\pi}{2}\left[(55 \times 10^{-3})^4 - (50 \times 10^{-3})^4\right]$

$= \frac{\pi}{2}\left[0.0000029\right]$

$= 0.000004556\,m^4$

$= 4.556 \times 10^{-6}\,m^4$

Inner radius, $C_1 = \frac{100 \times 10^{-3}}{2}$

$= (50 \times 10^{-3})\,m$

$\phi_A = \sum \frac{TL}{GJ} = \left(\frac{3 \times 10^3 \times 0.5}{90 \times 10^9 \times 4.556 \times 10^{-6}}\right) + \left(\frac{-2 \times 10^3 \times 1.0}{90 \times 10^9 \times 4.556 \times 10^{-6}}\right)$

$= -0.00122$ rad

$= 0.699^\circ$ (anticlockwise)

π rad $= 180^\circ$

1 rad $= \frac{180}{\pi}$

0.00122 rad $= \frac{180}{\pi} \times 0.00122$

$= 0.699^\circ$.

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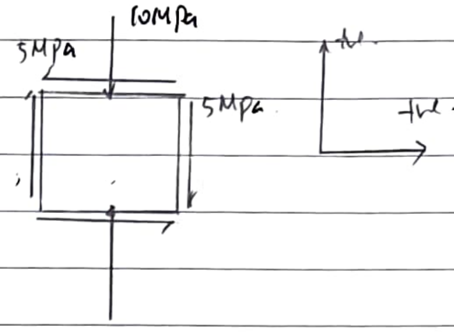
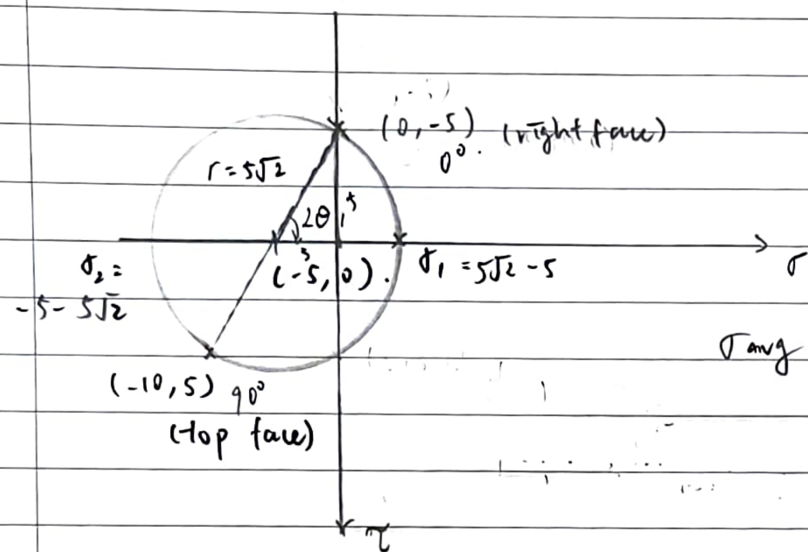
4a(i)





$\sigma_{avg} = \frac{-10+0}{2}$

$= -5$ MPa.

4a(ii) $r = \sqrt{5^2 + 5^2}$

$= \sqrt{25+25}$

$= 5\sqrt{2}$

$\sigma_1 = -5 + 5\sqrt{2}$

$= 5\sqrt{2} - 5$

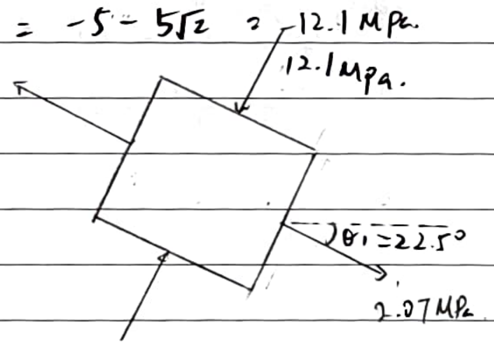
$= 2.07$ MPa

$\sigma_2 = -5 - 5\sqrt{2} = -12.1$ MPa.

$\tan 2\theta_1 = \frac{5}{5}$

$2\theta_1 = 45°$

$\theta_1 = 22.5°$



4a(iii) As the shear stress increases, the radius of the Mohr circle increases. The angle formed between the right face and the normal stress axis increases. Thus, the increase in shear stress would increase the likelihood of cracking.

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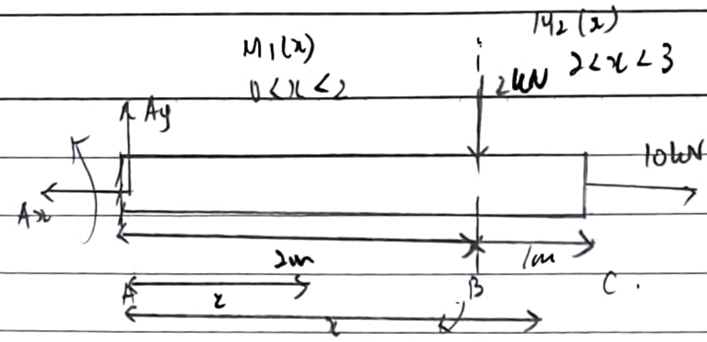
4b)



$+\uparrow \sum F_y = 0$

$A_y - 2 = 0$

$A_y = 2kN$

$+\circlearrowleft \sum M_A = 0$

$M_A - (2 \times 2) = 0$

$M_A = 4kNm$

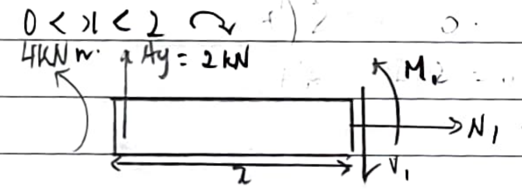
$\xrightarrow{+} \sum F_x = 0$

$-A_x + 10 = 0$

$-A_x = -10$

$A_x = 10 kN$

$0 < x < 2$



$+\circlearrowleft \sum M_1 = 0$

$M_1(x) - 2x + 4 = 0$

$M_1(x) = 2x - 4$

when $x = 0$

$M_1(0) = 2(0) - 4$

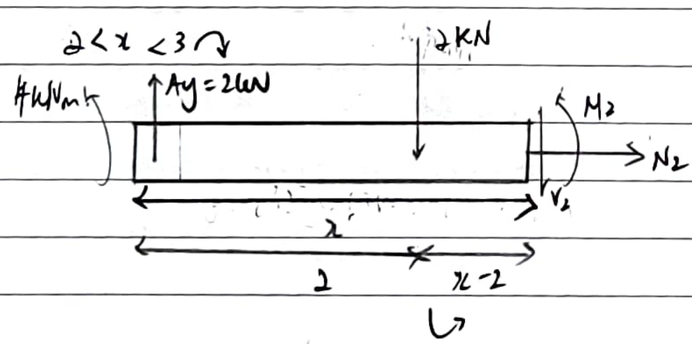
$= -4$

when $x = 2$

$M_1(2) = 2(2)$

$= 0$

$2 < x < 3$



$+\circlearrowleft \sum M_2 = 0$

$M_2(x) - 2x + 2(x-2) + 4 = 0$

$M_2(x) = 2x - 2(x-2) - 4$

$= 0$

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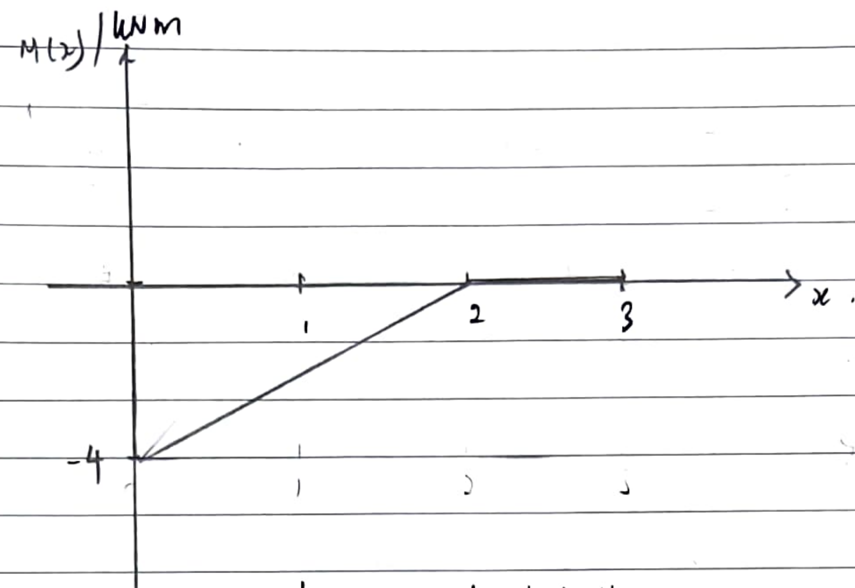
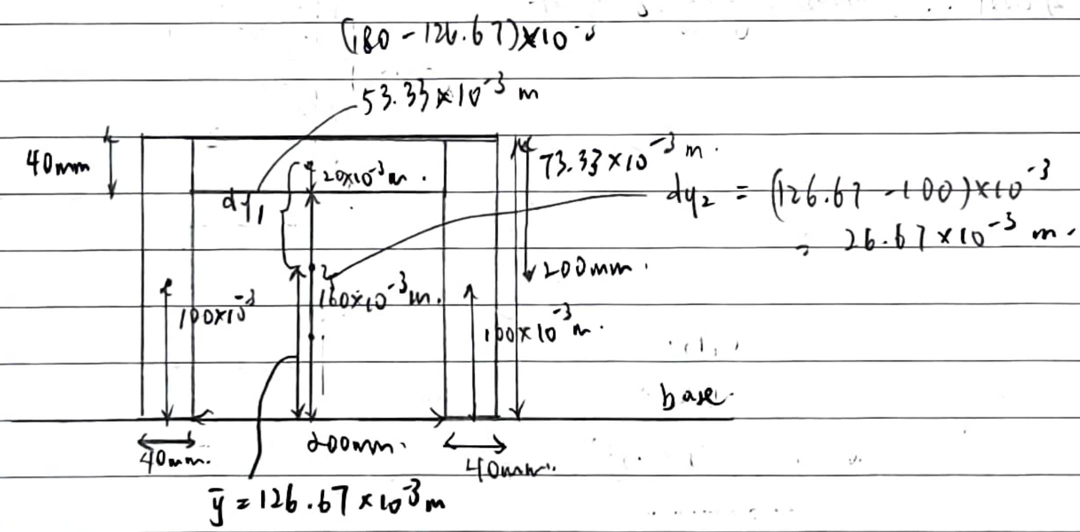


Max Bending moment = 4 kNm
= 4000 Nm.

Sagging negative / Hogging positive.
∴ Tensile positive



$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{[(40\times10^{-3})\times(200\times10^{-3})\times(100\times10^{-3})\times 2] + [(40\times10^{-3})(200\times10^{-3})\times(180\times10^{-3})]}{[(40\times10^{-3})\times(200\times10^{-3})\times 2] + (40\times10^{-3}\times200\times10^{-3})}$$

$$= \frac{0.0016 + 0.00144}{0.016 + 0.008} = \frac{0.00304}{0.024}$$

$$= 0.12667\ \text{m}.$$

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Normal stress, σ $= \dfrac{10\times10^{3}}{[(40\times10^{-3})\times(200\times10^{-3})\times2]+[40\times10^{-3}\times200\times10^{-3}]}$

$= 416670\ N/m^2$

$= 416670\ Pa$

$= 0.41667\ MPa.$

$$I = 2\Big[\big[\underbrace{\tfrac{1}{12}\times(40\times10^{-3})\times(200\times10^{-3})^3}_{\frac{1}{37500}}\big]+\big[\underbrace{(40\times10^{-3})\times(200\times10^{-3})\times(26.67\times10^{-3})^2}_{0.00000569}\big]\Big]$$

$$+\big[\underbrace{\tfrac{1}{12}\times(200\times10^{-3})\times(40\times10^{-3})^3}_{\frac{1}{937500}}\big]+\big[\underbrace{(200\times10^{-3})\times(40\times10^{-3})\times(53.33\times10^{-3})^2}_{0.000022752}\big]$$

$= 88.532\times10^{-6}\ m^4$

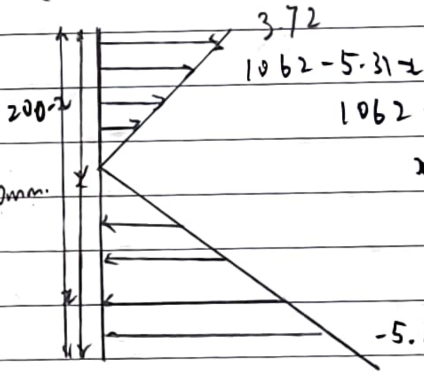
$\dfrac{200-x}{3.72} = \dfrac{x}{5.31}$

$(200-x)(5.31) = 3.72x$

$1062-5.31x = 3.72x$

$1062 = 9.03x$

$x = 118\ mm$ (3,1)



$\sigma = \dfrac{My}{I}$

$= \dfrac{4000\times(0.12667)}{88.532\times10^{-6}}$

$= 5723128.36$

$= 5.7231\ MPa$ ✗

$\sigma_{total} = 5.7231 + 0.41667$

$= 6.14\ MPa.$ (3,1) ✗

Tension

$\sigma_{Top} = \dfrac{My}{I} + (416670)$

$= \dfrac{(4000)(73.33\times10^{-3})}{88.532\times10^{-6}} + (416670)$

$= 3.72\ MPa$ (3,1)

But why minus, or is it +.

Compression

$\sigma_{bottom} = -\dfrac{My}{I} + (416670)$

$= -\dfrac{(4000)(126.67\times10^{-3})}{88.532\times10^{-6}} + (416670)$

$= -5.31\ MPa.$ (3,1)