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CV4161 20/21 Date: No:

1a) 2DOF: u_3', v_3'

$E = 200 \times 10^6$ kN/m²

$L_1 = 2m$ $A_1 = 2 \times 10^{-3} m^2$

$L_2 = 2\sqrt{2} m$ $A_2 = 2\sqrt{2} \times 10^{-3} m^2$

$k_s = 400\,000$ kN/m

member 1 (1-3) ($\phi_1 = 0$, $c_1 = 1$, $s_1 = 0$)

$$\frac{A_1E}{L_1} = \frac{(2\times10^{-3})(200\times10^6)}{2} = 200\times10^3$$

$$K_1' = 1000 \times \left[\begin{array}{cc|cc} 200 & 0 & -200 & 0 \\ 0 & 0 & 0 & 0 \\ \hline -200 & 0 & 200 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

member 2 (2-3) ($\phi_2 = 135°$, $c_2 = -\frac{\sqrt{2}}{2}$, $s_2 = \frac{\sqrt{2}}{2}$)

$$\frac{A_2E}{L_2} = \frac{(2\sqrt{2}\times10^{-3}).(200\times10^6)}{2\sqrt{2}} = 200\times10^3 \quad s_2^2 = c_2^2 = \frac{1}{2} \quad c_2s_2 = -\frac{1}{2}$$

$$K_2' = 1000 \times \left[\begin{array}{cc|cc} 100 & -100 & -100 & 100 \\ -100 & 100 & 100 & -100 \\ \hline -100 & 100 & 100 & -100 \\ 100 & -100 & -100 & 100 \end{array}\right]$$

Assemble

$$\begin{bmatrix} R_{1x}' \\ R_{1y}' \\ R_{2x}' \\ R_{2y}' \\ 400 \\ -600, -k_s v_3' \end{bmatrix} = 10^3 \times \left[\begin{array}{cc|cc|cc} x & x & x & x & x & x \\ x & x & x & x & x & x \\ \hline x & x & x & x & x & x \\ x & x & x & x & x & x \\ x & x & x & x & 200+100 & 0-100 \\ x & x & x & x & 0-100 & 0+100 \end{array}\right] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ u_3' \\ v_3' \end{bmatrix}$$

$$\begin{bmatrix} R_{1x}' \\ R_{1y}' \\ R_{2x}' \\ R_{2y}' \\ 400 \\ -600 \end{bmatrix} = 10^3 \times \begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & 300 & -100 \\ & & & & -100 & 100+k_s \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ u_3' \\ v_3' \end{bmatrix}$$

$$\begin{bmatrix} 400 \\ -600 \end{bmatrix} = 10^3 \times \begin{bmatrix} 300 & -100 \\ -100 & 100+k_s \end{bmatrix} \begin{bmatrix} u_3' \\ v_3' \end{bmatrix}$$

$k_s = 400 \times 10^3$

$$\begin{bmatrix} 400 \\ -600 \end{bmatrix} = 10^3 \times \begin{bmatrix} 300 & -100 \\ -100 & 500 \end{bmatrix} \begin{matrix} u_3' \\ v_3' \end{matrix}$$

$u_3' = 0.001\text{M} = 1\text{mm}$

$v_3' = -0.001\text{M} = -1\text{mm}$

c) $-600 = 10^3 \times (-100\, u_3' + (100 + k_s)\, v_3')$

$-0.6 = -100\, u_3' + (100 + k_s)\, v_3'$

$u_3' = -2v_3'$

$400 = 10^3 \times (300\, u_3' - 100\, v_3')$

$0.4 = -600\, v_3' - 100\, v_3'$

$v_3' = \dfrac{0.4}{-700} = -5.714 \times 10^{-4}\text{m}$

$u_3' = 1.143 \times 10^{-3}\text{m}$

$-0.6 = -100 \times (1.143 \times 10^{-3}) + (100 + k_s) \cdot (-5.714 \times 10^{-4})$

$-0.6 = -0.1143 - 0.05714 - 5.714 \times 10^{-4}\, k_s$

$k_s = 750.02$

$k_s = 750\,000$ kN/m

2a) 2DOF = v_2' ; θ_2'

$E = 2\times10^8$ kN/m^2
$L_1 = L_2 = 1$m
$I_1 = I_2 = 4\times10^6$ mm^4
$= 4\times10^{-6}$ m^4

member 1

$$\frac{EI_1}{L_1^3} = \frac{2\times10^8 \cdot 4\times10^{-6}}{1^3} = 800 = \frac{EI}{L^2} = \frac{EI}{L}$$

$$K_1' = \left[\begin{array}{cc|cc} 9600 & 4800 & -9600 & \\ 4800 & 3200 & -4800 & \\ \hline -9600 & -4800 & & \\ 4800 & 1600 & & \end{array}\right]$$

member 2

$$\frac{EI_2}{L_2^3} = \frac{EI_2}{L_2^2} = \frac{EI_2}{L_2} = 800$$

$$K_{m2}' = \left[\begin{array}{cc|cc} 2400 & 2400 & -2400 & 0 \\ 2400 & 2400 & -2400 & 0 \\ \hline -2400 & -2400 & 2400 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Force factor

$$F_{e1}' = \begin{bmatrix} wL/2 \\ wL^2/12 \\ wL/2 \\ -wL^2/12 \end{bmatrix} = \begin{bmatrix} 24 \\ 4 \\ 24 \\ -4 \end{bmatrix} \qquad F_{e2}' = \begin{bmatrix} 11F/16 \\ 3FL/16 \\ 5F/16 \\ 0 \end{bmatrix} = \begin{bmatrix} 11 \\ 3 \\ 5 \\ 0 \end{bmatrix}$$

Assemble

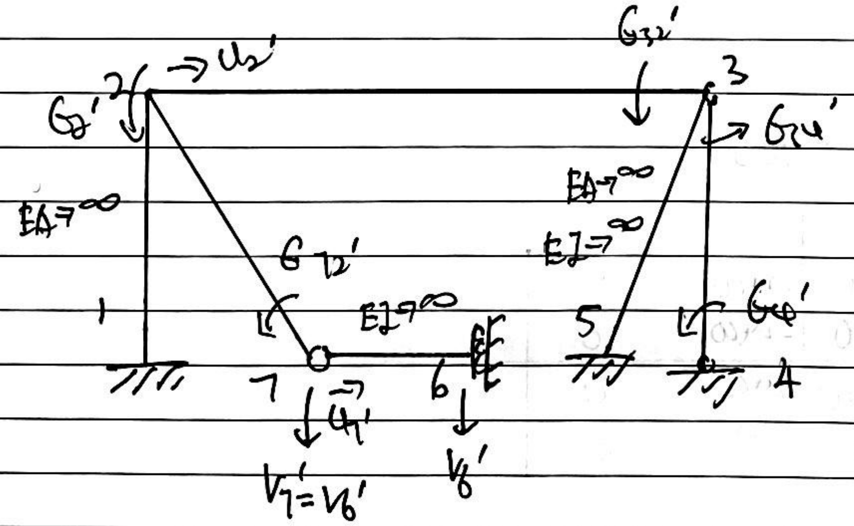
$$\begin{bmatrix} V_1' \\ M_1' \\ 0 \\ -12 \\ V_3' \\ M_3' \end{bmatrix} = \begin{bmatrix} & & & & & \\ & & & & & \\ & & 12000 & -2400 & & \\ & & -2400 & 5600 & & \\ & & & & & \\ & & & & & \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ V_2' \\ \theta_2' \\ 0 \\ \theta_2' \end{bmatrix} + \begin{bmatrix} 24 \\ 4 \\ 35 \\ -1 \\ 5 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ -12 \end{bmatrix} = \begin{bmatrix} 12000 & -2400 \\ -2400 & 5600 \end{bmatrix} \begin{bmatrix} V_2' \\ \theta_2' \end{bmatrix} + \begin{bmatrix} 35 \\ -1 \end{bmatrix}$$

$$12000\, V_2' - 2400\, \theta_2' = -35$$
$$-2400\, V_2' + 5600\, \theta_2' = -11$$

$V_2' = -3.62\text{mm}$
$\theta_2' = -3.52\text{mm}$

b)



Joint 1 & 5 (fixed joint) $\rightarrow$ No DOF
Joint 2 $\rightarrow$ 2 DOF $= u_2'$ & θ_2' (V_2' restricted by 1-2 because $EA \rightarrow \infty$)
Joint 3 $\rightarrow$ 2 DOF $= u_{34}'$, u_{35}' (member 3-5 has no axial deformation & can't bend, hence joint 3 does not have u_3', V_3', θ_{35}')
(Have u_{34}' and u_{35}' because internal pin)
Joint 4 $\rightarrow$ 1 DOF $= \theta_4'$ (pinned joint)
Joint 6 $\rightarrow$ 1 DOF (guided roller)
Joint 7 $\rightarrow$ 2 DOF $= u_7'$, θ_{72}' (V_7' is the same as V_6' because 6-7 has $EI \rightarrow \infty$; θ_{76}' restricted by 6-7 because $EI \rightarrow \infty$)

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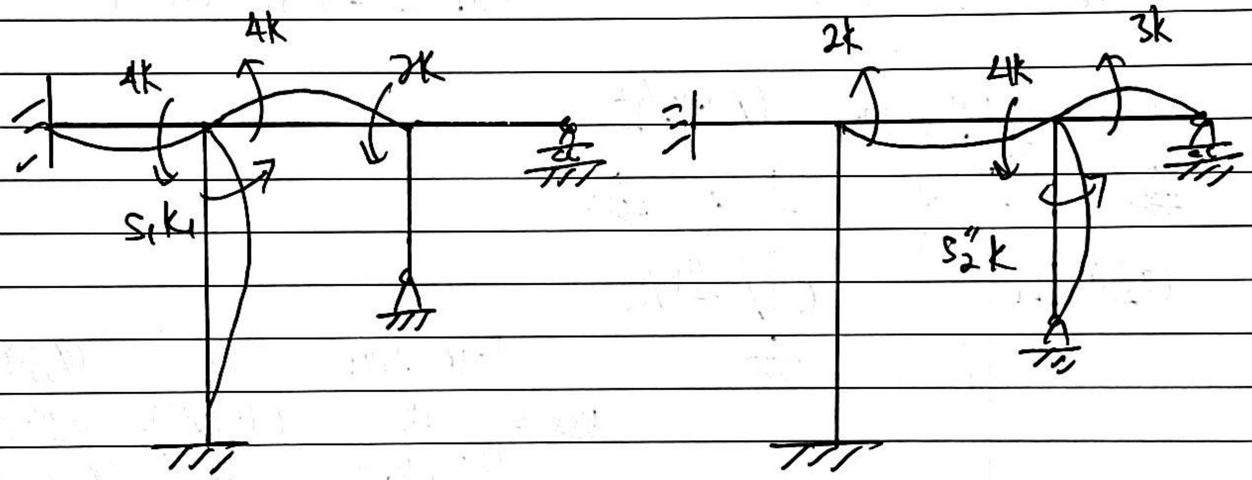
3a) For member 25, $k_1 = \frac{EI}{2L} = \frac{k}{2}$, $P_1 = P$, $\rho_1 = \frac{P}{\frac{\pi^2 EI}{(2L)^2}} = \frac{4P}{\frac{\pi^2 EI}{L^2}}$

For member 36, $k_2 = k$, $P_2 = 2P$, $\rho_2 = \frac{2P}{\frac{\pi^2 EI}{L^2}} \Rightarrow \rho_1 = 2\rho_2$

2DOF: θ_2' and θ_3'



(1) $\theta_2' = 1$, $\theta_3' = 0$ (2) $\theta_3' = 1$, $\theta_2' = 0$

$$K_{ff}' = \begin{bmatrix} 8k + s_1\left(\frac{k}{2}\right) & 2k \\ 2k & (7 + s_2'')k \end{bmatrix} \begin{matrix} \leftarrow \theta_2' \\ \leftarrow \theta_3' \end{matrix} \quad \text{where } k = \frac{EI}{L}$$

For the critical load, $|K_{ff}'| = 0 \Rightarrow \left(8 + \frac{s_1}{2}\right)(7 + s_2'') - 4 = 0$

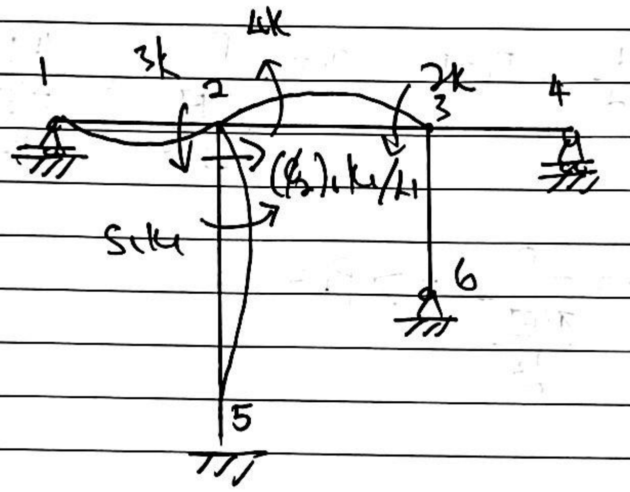
Let $f(s,c) = \left(8 + \frac{s_1}{2}\right)(7 + s_2'') - 4$

rho 1	s_1	rho 2	s_2''	$F(s,c)$
3	-5.0321	1.5	-4.2151	11.27225
3.2	-7.2973	1.6	-6.0321	0.21167
3.24	-7.8782	1.62	-6.4908	-1.9322

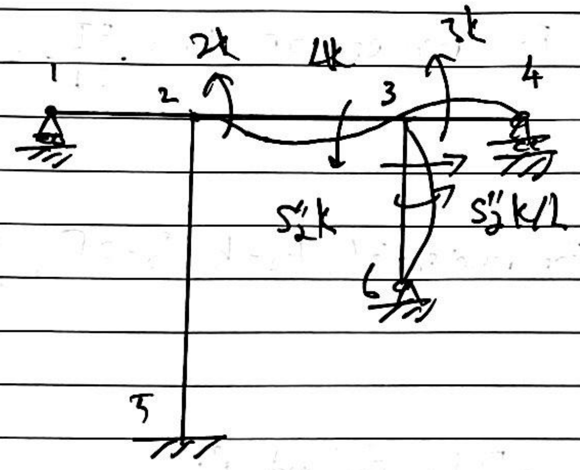
$P_{1,cr} = 3.2 + \frac{0.21167 - 0}{0.21167 - (-1.9322)}(3.24 - 3.2) = 3.204$

$P_{cr} = \rho_{1cr} P_{E1} = 3.204 \frac{\pi^2 EI}{(2L)^2} = 0.801 \frac{\pi^2 EI}{L^2}$

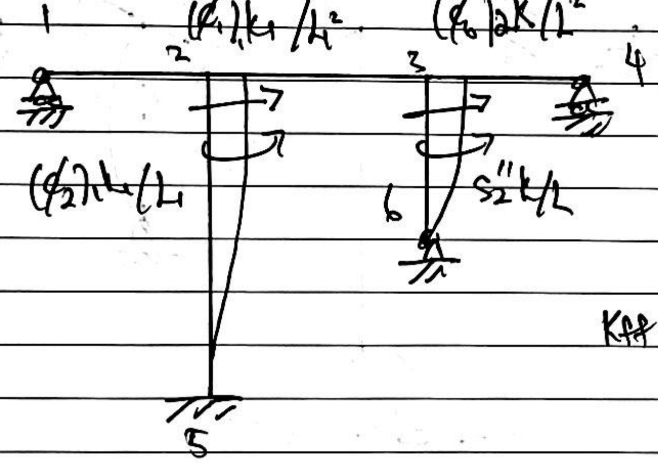
3b) 3DOF: θ_2', θ_3' and u_1' $(= u_2' = u_3' = u_4')$



(1) $\theta_2' = 1$, $\theta_3' = u_1' = 0$



(2) $\theta_3' = 1$, $\theta_2' = u_1' = 0$



$$K_{ff}' = \begin{array}{c|c} \begin{bmatrix} 7k + S_1 k_1 & 2k & (\phi_2)_1 k_1/L_1 \\ 2k & (7+S_2'')k & S_2'' k/L \\ (\phi_2)_1 k_1/L_1 & S_2'' k/L & (\phi_1)_1 k_1/L_1^2 + (\phi_6)_2 k/L^2 \end{bmatrix} & \begin{matrix} \theta_2' \\ \theta_3' \\ u_1' \end{matrix} \end{array}$$

(3) $u_1' = 1$, $\theta_2' = \theta_3' = 0$

For the critical load, $|K_{ff}'| = 0$

$$\left(7 + \frac{S_1}{2}\right)(7 + S_2'')\left[\frac{(\phi_1)_1}{8} + (\phi_6)_2\right] + S_2''(\phi_2)_1 - \frac{(\phi_2)_1^2 (7 + S_2'')}{16} - \left(7 + \frac{S_1}{2}\right)(S_2'')^2$$
$$-4\left[\frac{(\phi_1)_1}{8} + (\phi_6)_2\right] = 0$$

Let $f(S, C)$ be the LHS of the above equation.

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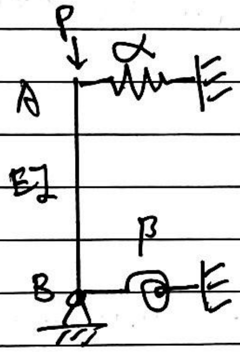
rho 1	S1	C1	(phi1)1	(phi2)1	rho 2	S"2	(phi6)2	f(S,C)
0.60	3.1462	0.7136	4.8403	5.3810	0.3	2.35	-0.6109	-52.048
0.52	3.2640	0.6749	5.8016	5.4669	0.26	2.4443	-0.1218	-9.074
0.48	3.3247	0.6571	6.2813	5.5094	0.24	2.4901	0.1217	12.917

$$(\rho_{cr})_1 = 0.52 + \frac{0-(-9.0739)}{12.9167-(-9.0739)}(0.48-0.52) = 0.5035$$

Critical load: $P_{cr} = \rho_{1cr} P_{E1} = 0.5035 \frac{\pi^2 EI}{(2L)^2} = \frac{0.1259\,\pi^2 EI}{L^2}$

4a)

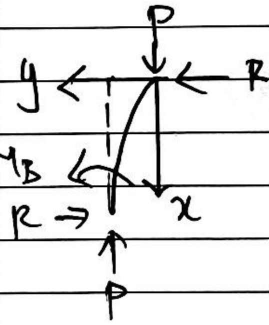


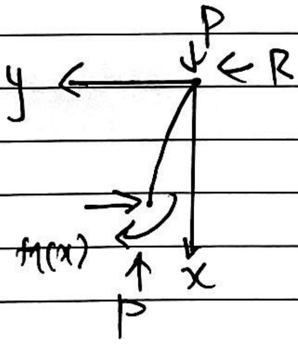
$$\alpha = \frac{12EI}{L^3}$$

$$\beta = \frac{3EI}{L}$$

Lower and upper bounds of critical load $\left(0, 2.05\frac{\pi^2 EI}{L^2}\right)$

b)





$$R = \frac{12EI}{L^3}v_B = \frac{12EI}{L^3}v(L), \quad M_B = \frac{3EI}{L}\theta(L)$$

$$M(x) + Pv(x) - Rx = 0 \Rightarrow v'' + \omega^2 v = \frac{R}{EI}x \quad \text{where } \omega^2 = \frac{P}{EI}$$

$$v = A\sin\omega x + B\cos\omega x + \frac{R}{P}x$$

$$v' = A\omega\cos\omega x - B\omega\sin\omega x + \frac{R}{P}, \quad v'' = -A\omega^2\sin\omega x - B\omega^2\cos\omega x$$

Boundary conditions:

$$v(0) = 0 \Rightarrow B = 0$$

$$EIv''(L) = -M_B \Rightarrow EI\left[-A\omega^2 \sin\omega L\right] = \frac{-3EI}{L}\theta(L).$$

$$= \frac{-3EI}{L}\left[A\omega\cos(\omega L) + \frac{R}{P}\right]$$

$$A = \frac{R}{P}\frac{3L}{\varphi(\varphi\sin\varphi - 3\cos\varphi)}, \quad \varphi = \omega L$$

$$M_B + RL = Pv_B \Rightarrow EI(A\omega^2\sin\varphi) + \frac{12EI}{L^2}v(L)\,L = Pv(L)$$

$$\Rightarrow \varphi^2 - 12 = \frac{36}{\varphi(\varphi - 3\cot\varphi)}$$

$$\varphi(\varphi^2 - 12)(\varphi - 3\cot\varphi) - 36 = 0$$

$$\varphi = \omega L = 3.3332 \Rightarrow P_{cr} = (3.3332)^2\frac{EI}{L^2} = 11.11\frac{EI}{L^2} = 1.1257\frac{\pi^2 EI}{L^2}$$