Hiew ming Yan

CV4116 AY 19-20

1 a) (i) $N = 21+57+72+63+42+24+12+9+0$
$= 300$

(ii) $H_{rms} = \sqrt{\frac{1}{N}\Sigma H_i^2}$
$= \sqrt{\frac{1}{300}(56.808)}$
$= 0.435$

Range of wave height (m)	mid-pt of wave height, H_i	no. of waves
0 - 0.12	0.06	21
0.12 - 0.24	0.18	57
0.24 - 0.36	0.30	72
0.36 - 0.48	0.42	63
0.48 - 0.60	0.54	42
0.60 - 0.72	0.66	24
0.72 - 0.84	0.78	12
0.84 - 0.96	0.9	9
0.96 - 1.08	1.02	0

H_{33} - average wave height of largest 1/3 waves
$= \frac{1}{100}[9\times0.9 + 12\times0.78 + 24\times0.66 + 42\times0.54 + 13\times0.42]$
$= 0.614$

(top 10%) → top 30 wave
$H_{10} = \frac{1}{10}[9\times0.9 + 12\times0.78 + 9\times0.66]$
$= 0.78$

H_i^2
0.0036
0.0324
0.09
0.1764
0.292
0.4356
0.6084
0.81
1.0404

$\Sigma H_i^2 = 0.0036\cdot21 + 0.0324\cdot57 + 0.09\cdot72 + 0.1764\cdot63 + 0.292\cdot42 + 0.4356\cdot24 + 0.6084\cdot12 + 0.81\cdot9 + 1.0404\cdot0$
$= 56.808$

b) $U = 7.5\,ms^{-1}$
$H_s = 0.614$
From fig II-2-25 $T = 8hrs$ (duration needed)
From fig II-2-26 $T = 3.35$

c) $E(f) = \frac{A}{f^5} e^{\frac{-1}{f^4}}$

$E_j(\Delta f_j) = \frac{a_j^2}{2} - S_j$

$S_{m0} = \int_0^\infty E(f)\,df = \frac{A}{4}\int_0^\infty \frac{4}{f^5} e^{-\frac{1}{f^4}}\,df$

$= \int_0^\infty \frac{A}{f^5} e^{\frac{-1}{f^4}}\,df = \frac{A}{4}\left[e^{\frac{-1}{f^4}}\right]_0^\infty$

$\frac{H_s^2}{16} = \frac{-A}{4}$

$H_s = 4\sqrt{S_{m0}}$

$H_s = 0.614$ [from (a) (H_{33})]

$A = -0.0943$

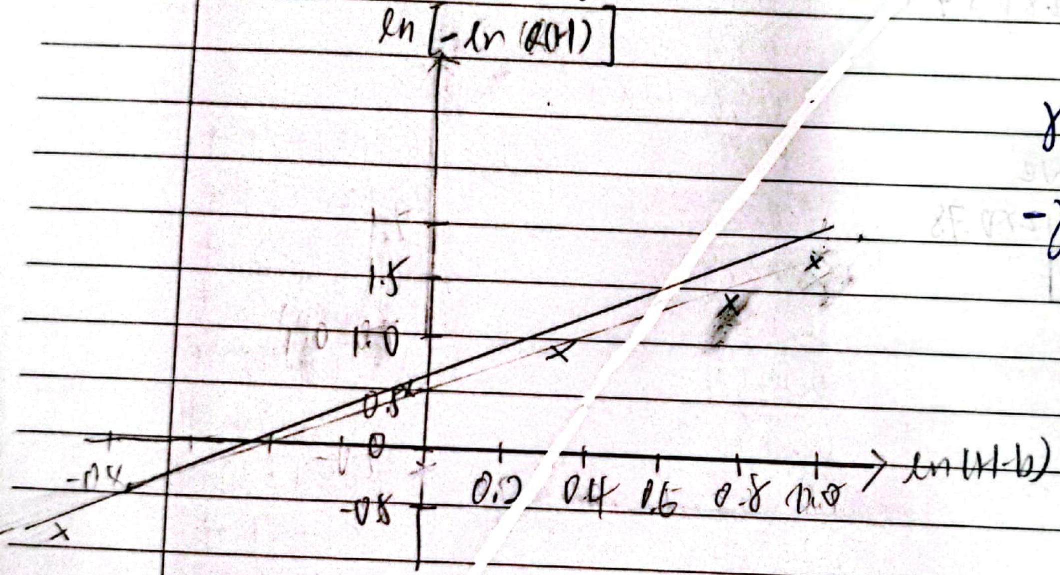
$[A] = m^2$

Besform

3a(i) $N = 520$

$N+1 = 521$

$H \geq H_{obs}$ (m)	$Q(H)$	$\ln [-\ln Q(H)]$	$H-b$	$\ln(H-b)$
0	1.998	-0.214	-0.08	-
0.5	0.576	-0.595	0.42	-0.87
1.0	0.192	0.501	0.92	-0.083
1.5	0.069	0.983	1.42	0.351
2.0	0.027	1.284	1.92	0.652
2.5	0.008	1.574	2.42	0.884
3.0	0.0	-	-	-



$\gamma \approx 1.5$ (gradient)

$-\gamma \ln a = 0.79$

$a = 0.59$

(ii) ~~$P_e = 1 - (1 - \frac{1}{T_r})^L$~~

~~$\frac{1}{30} = e^{-\left(\frac{H - 0.08}{1.44}\right)^{1.5}}$~~

$\left(\frac{1/52}{30}\right) = e^{-\left(\frac{H-0.08}{0.59}\right)^{1.5}}$

$H = 2.8$

(iii) $P_e = 1 - (1 - \frac{1}{T_r})^L$

$= 1 - (1 - \frac{1}{30})^{50}$

$= 0.816$

$= 81.6\%$

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b) ~~d=5~~ $d = 5m$ $\rho_s g = 2.6 \times 10^4$ From fig VI-5-21
$H_s = 1.2m$ $\rho_w g = 10^4$ $K_D = 4.0$
$\cot\alpha = 2$
$n = 2$

rubble mound breakwater → $H_{design} = H_{1/10} = 1.27 H_s = 1.52m$

$$\frac{H_{design}}{d} = \frac{1.52}{5} = 0.30 < 0.78$$

(Non breaking wave)

$$W = \frac{\rho_s g H^3}{K_D \left(\frac{\rho_s}{\rho_w} - 1\right)^3 \cot\alpha}$$
$= 2.8 kN$

4) $H_{s0} = 0.8$
$T = 6$
$\theta_0 = 45°$

a) From Fig II-3-6, $\frac{d}{gT^2} = 0.014$

$K_S K_R = 0.85$

$H_s = K_S K_R H_0$
$= (0.85)(0.8)$
$= 0.68$

$$\frac{H_s}{d} = \frac{0.68}{5} = 0.136 < 0.78$$

Fig II-1-7, $\frac{d}{L_0} = 0.65$
$L \approx 36.5 m$

b) non-wave-topping

$H_{design} = (1.8)(0.68)$
$= 1.224$

$\eta^* = 0.75(1+\cos 0)(1.0)(1.224)$
$= 1.836$

$\alpha_1 = 0.6 + 0.5\left[\frac{4\pi h_s/L}{\sinh(4\pi h_s/L)}\right]^2$
$= 0.802$

$\beta = 0$ (normal incident wave)

$\alpha_3 = 1 - \frac{h_w - h_c}{h_s}\left[1 - \frac{1}{\cosh(2\pi h_s/L)}\right]$

$h_s = 5m$
$L = 365m$ (from a)

$h_w - h_c = 3.5$ (the part immersed in water)
$h_s = 5$
$\alpha_3 = 0.802$

$\alpha_2 = \min\left[\frac{h_b - d}{3h_b}\left[\frac{H_{design}}{d}\right]^2, \frac{2d}{H_{design}}\right]$

$H_b = 5$
$d = 3.5$
$H_{design} = 1.224$

$\alpha_2 = \alpha_*$

$= \min[0.0122, 5.72]$
$= 0.0122$

$p_1 = 0.5(1+\cos\beta)[\alpha_1 + \alpha_* \cos^2\beta]\rho_w g H_{design}$
$= 10 \text{ kN/m}^2$

$p_2 = 0 \ (\eta^* \le h_c)$

$p_3 = \alpha_3 p_1$
$= (0.802)(10)$
$= 8.02 \text{ kN/m}^2$

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4c) $P_u = 0.5(1+\cos\beta)\,\alpha_1\alpha_3\,\rho_w g\,H_{design}$

pressure @ SWL → z=0

$$p = \rho g \eta \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} - \rho g z$$

$P = \rho g \eta$

$= \rho g H/2$

~~$= \left(\frac{10^4}{2}\right)(3.5)$~~

$= \frac{\rho_w g H_{design}}{2}$

$= \left(\frac{10^4}{2}\right)(3.5)$

~~$r_f = \frac{P - P_u}{P}$~~

~~$= \frac{[0.5(1+\cos\beta)\alpha_1\alpha_3 - 0.5]\,\rho_w g H_{design}}{\rho_w g H_{design}/2}$~~

~~$= (1+\cos\beta)\alpha_1\alpha_3$~~

~~From (a) (b)~~

~~$r_f = (2)(0.802)(0.802)$~~

~~$= 1.286$~~

~~$r_f = \frac{P_u}{P}$~~

~~$= \frac{0.5(1+\cos\beta)\alpha_1\alpha_3\,\rho_w g H_{design}}{0.5\,\rho_w g H_{design}}$~~

$r_f = \frac{P - P_u}{P}$

$= \frac{[0.5(1+\cos\beta)\alpha_1\alpha_3 - 0.5]\,\rho_w g H_{design}}{\rho_w g H_{design}/2}$

$= 1.286 - 1$

$= 0.286$

The uplift force acting on the caisson might imposed an overturning moment to the caisson which might cause the caisson to be less stable.

1.

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(a)

(i) Total number of waves in the record

= 21 + 57 + 72 + 63 + 42 + 24 + 12 + 9 + 0

= 300

(ii) $$H_{rms} = \sqrt{\frac{21\times0.06^2 + 57\times0.18^2 + 72\times0.30^2 + 63\times0.42^2 + 42\times0.54^2 + 24\times0.66^2 + 12\times0.78^2 + 9\times0.9^2}{300}}$$

$$= 0.435 \text{ m}$$

To calculate H_{33}, we need $300 \times \frac{1}{3} = 100$ largest wave heights from the record, which are 9 waves in range 0.84 – 0.96, 12 waves in range 0.72 – 0.84, 24 waves in range 0.60 – 0.72, 42 waves in range 0.48 – 0.60 and 13 waves in range 0.36 – 0.48.

Using mid-value in the range to approximate:

$$H_{33} = \frac{1}{100}\left(9\times0.9 + 12\times0.78 + 24\times0.66 + 42\times0.54 + 13\times0.42\right)$$

$$= 0.614 \text{ m}$$

To calculate H_{10}, we need $300 \times 10\% = 30$ largest wave heights from the record, which are 9 waves in range 0.84 – 0.96, 12 waves in range 0.72 – 0.84 and 9 waves in range 0.60 – 0.72

$$H_{10} = \frac{1}{30}\left(9\times0.9 + 12\times0.78 + 9\times0.66\right) = 0.78 \text{ m}$$

(b) $U = 7.5$ m/s, duration-limited

$H_s = H_{33} = 0.614$ m

From CEM Fig II-2-25 ⇒ D = 8 hours

From CEM Fig II-2-26 ⇒ T = 3.2 s

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(c) $H_s = H_{m_0} = 4\sqrt{s_{m_0}}$

$$s_{m_0} = \int_0^\infty E(f)\,df$$

$$s_{m_0} = \int_0^\infty \frac{A}{f^5} e^{-\frac{1}{f^4}}\,df$$

$$u = e^{-1/f^4} = e^{-f^{-4}}$$
$$du = 4f^{-5} e^{-f^{-4}}\,df$$
$$f \to 0 \Rightarrow u \to 0$$
$$f \to \infty \Rightarrow u \to 1$$

$$s_{m_0} = \int_0^1 \frac{A}{4}\,du$$

$$s_{m_0} = \frac{A}{4} u \Big|_0^1 = \frac{A}{4}$$

$$\Rightarrow H_s = 4\sqrt{\frac{A}{4}}$$

$$\Rightarrow 0.614 = 2 \times \sqrt{A}$$

$\therefore A = 0.0942$

Unit dimension of $E(f)$ = $[m^2 s]$

$\Rightarrow$ Unit dimension of A = $[m^2 s^{-4}]$

2.

(a) $T = 10s$

$$L_o = \frac{gT^2}{2\pi} = \frac{9.81 \times 10^2}{2\pi} = 156\,m$$

Deep water: $d/L > \frac{1}{2} \Rightarrow d_{min} = \frac{1}{2} \times 156 = 78\,m$

$$C_{go} = \frac{gT}{4\pi} = \frac{9.81 \times 10}{4\pi} = 7.81\ m/s$$

Shallow water: $d/L < \frac{1}{20}$

$L = T\sqrt{gd}$

$$\Rightarrow \frac{d}{T\sqrt{gd}} < \frac{1}{20}$$

$$\Rightarrow d_{max} = \frac{1}{20^2} T^2 g = 2.45\,m$$

$$C_g = \sqrt{gd} = \sqrt{9.81 \times 2.45} = 4.90\ m/s$$

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∴ Range of transitional water depth is from 2.45m to 78m
Range of wave group speed is from 4.90 m/s to 7.81 m/s

(b)

(i) Kinematic Boundary Condition

(ii) $$\left(\frac{2\pi}{T}\right)^2 = g\left(\frac{2\pi}{L}\right)\tanh\left(\frac{2\pi d}{L}\right)$$

$$\left(\frac{2\pi}{10}\right)^2 = 9.81\left(\frac{2\pi}{L}\right)\tanh\left(2\pi \times 0.1\right)$$

$\Rightarrow L = 86.9$ m

$\Rightarrow d = 0.1L = 8.69$ m

$H_b = 0.78d = 0.78 \times 8.69 = 6.78$ m

From Fig II-1-8 : $h/L_o = \frac{8.69}{156} = 0.056 \Rightarrow \frac{H_b}{H_o} = 1.2$

$\Rightarrow H_o = \frac{6.78}{1.2} = 5.65$ m

(c) $H = 0.6$ m

$$p = \rho g \eta \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} - \rho g z$$

$$P_{max} = 10^4 \times \frac{0.6}{2} \times \frac{1}{\cosh(2\pi \times 0.1)} - 10^4(-8.69)$$

$= 89.4$ kN/m²

$$P_{min} = 10^4 \times \frac{-0.6}{2} \times \frac{1}{\cosh(2\pi \times 0.1)} - 10^4(-8.69)$$

$= 84.4$ kN/m²

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To determine the oblique angle, we need at least 2 pressure gauges set at 2 different water depths. From the measurements of pressure at seabed, we can determine the wave height at each location, namely H_1 and H_2.

$$H_1 = K_{R_1} K_{S_1} H_0$$
$$H_2 = K_{R_2} K_{S_2} H_0$$

$$\Rightarrow \frac{K_{R_1} K_{S_1}}{K_{R_2} K_{S_2}} = \frac{H_1}{H_2} \Rightarrow (K_{R_1} K_{S_1}) = (K_{R_2} K_{S_2}) \frac{H_1}{H_2} \quad (1)$$

Iterate for θ_0 with eqn (1) and CEM Fig II-3-6.

E.g.

Try $\theta_0 = 45°$ with d_1, d_2 } $\Rightarrow$ read $K_{R_1} K_{S_1}$ and $K_{R_2} K_{S_2}$ from the chart

- Check whether the readings satisfy eqn (1)

If not, try another θ_0.

In this case, one would need 2 additional gauges at 2 different water depths. From 3 gauges, one can find 3 values for θ_0 (using 3 combinations of 2 gauges) then average the value for the oblique angle needed.

Author's note: There is no exactly correct answer for Q2 (c). You may suggest another number and it will be marked as correct as long as you can provide reasonable arguments.

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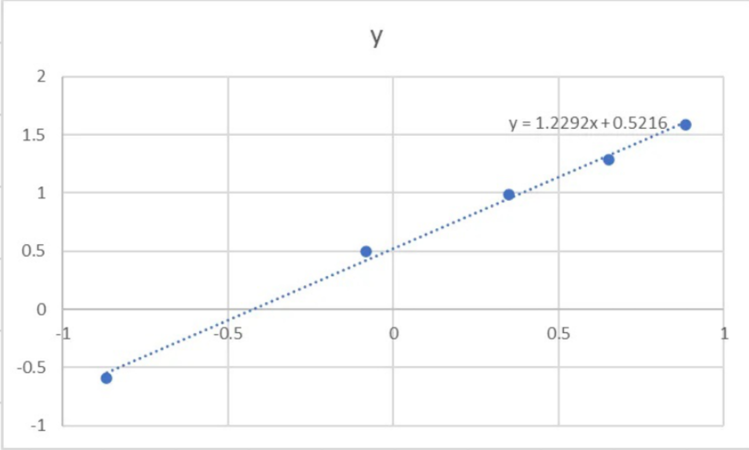
3.

(a) $N = 52 \times 10 = 520$ $\qquad b = 0\ 08$

(i)

H	#obs >H	Q	x	y
0.5	300	0.575816	-0.8675	-0.59427
1	100	0.191939	-0.08338	0.501127
1.5	36	0.069098	0.350657	0.982914
2	14	0.026871	0.652325	1.28556
2.5	4	0.007678	0.883768	1.582982
3	0	0		



$$Q = e^{-\left(\frac{H-b}{a}\right)^{\gamma}}$$

$$\ln[-\ln(Q)] = \gamma \ln(H-b) - \gamma \ln a$$

$$y \quad = \gamma x \quad - \gamma \ln a$$

From the graph: $\gamma = 1.2292$

$\gamma a = -0.5216 \Rightarrow a = -0.4243$

$$Q = e^{-\left(\frac{H-0.08}{-0.4243}\right)^{1.2292}}$$

(ii) Sampling interval $= \frac{1}{52}$ yr (weekly)

for $T_r = 30$ years $\qquad Q(H_s > H) = \frac{1/52}{30} = 0.000641$

$$\ln(-\ln(0.000641)) = 1.2292 \ln(H_r - 0.08) + 0.5216$$

$\Rightarrow H_r = 3.4$ m

(iii) Encounter probability over 50 years period

$$= 1 - \left(1 - \frac{1}{T_r}\right)^L = 1 - \left(1 - \frac{1}{30}\right)^{50} = 0.816$$

(b) $d = 5m$ $\gamma_s = 26 kN/m^3$ Hoang Thu Minh

$H_s = 1.2 m$ $\gamma_w = 10 kN/m^3$

slope 1V:2H

2 units thick

Design wave height $H_{1/10} = 1.27 H_s = 1.27 \times 1.2 = 1.52 m$

Check for breaking $\frac{H}{d} = \frac{1.52}{5} < 0.78 \rightarrow$ non-breaking

From CEM Table VI-5-22: $K_D = 4.0$

(rough angular, non-breaking, no damage)

Hudson's formula:

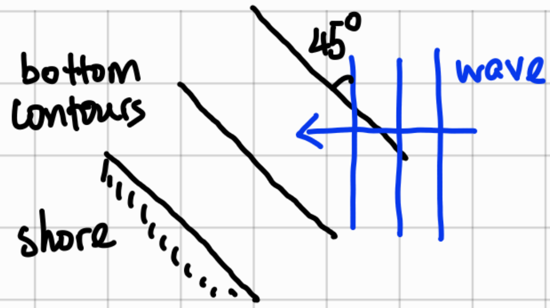
$$M = \frac{\rho_s H^3}{K_D\left(\frac{\rho_s}{\rho_w} - 1\right)^3 \cot\alpha}$$

$$= \frac{\frac{26 \times 10^3}{9.81} \times 1.52^3}{4.0\left(\frac{26}{10} - 1\right)^3 \times 2}$$

$$= 284 \text{ kg}$$

4.

$H_{so} = 0.8m$
$T = 6s$



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(a) $d = 5m$

From CEM Fig II-3-6 : $\frac{d}{gT^2} = \frac{5}{9.81 \times 6^2} = 0.014$, $\theta_0 = 45°$ $\Rightarrow K_R K_s = 0.84$

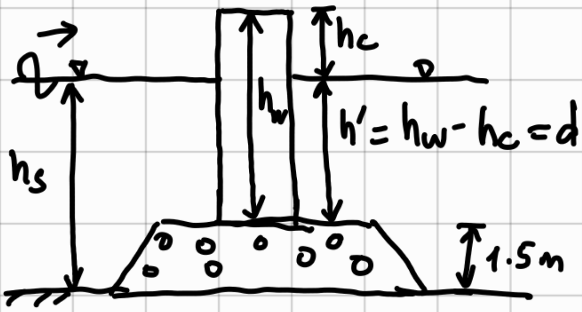
$\Rightarrow H_s = K_R K_s H_{so} = 0.84 \times 0.8 = 0.672 \text{ m}$

$L_0 = \frac{gT^2}{2\pi} = \frac{9.81 \times 6^2}{2\pi} = 56.2 m$

From CEM Fig II-1-7 : $\frac{d}{gT^2} = 0.014 \Rightarrow L/L_0 = 0.68$

$\Rightarrow L = 0.68 \times 56.2 = 38.2 m$

(b) Goda's method



$H_{des} = 1.8 H_s = 1.21 m$

$\beta = 0°$

$h_s = 5m$

$d = h' = 5 - 1.5 = 3.5m$

no overtopping $\Rightarrow \eta^* < h_c$

$h_b = h_s = 5m$

$\lambda_1 = \lambda_2 = \lambda_3 = 1$

$\eta^* = 0.75 (1 + \cos\beta) H_{design} = 0.75 \times 2 \times 1.21 = 1.815 m$

$\alpha_1 = 0.6 + 0.5 \left[\frac{4\pi h_s / L}{\sinh(4\pi h_s / L)}\right]^2 = 0.818$

$\alpha_2 = \min\left\{ \frac{h_b - d}{3h_b} \left(\frac{H_{design}}{d}\right)^2, \frac{2d}{H_{design}} \right\} = \min\{0.0120, 5.785\}$

$= 0.0120$

$\alpha_* = \alpha_2 = 0.0120$

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$$\alpha_3 = 1 - \frac{h_w - h_c}{h_s}\left[1 - \frac{1}{\cosh(2\pi h_s/L)}\right] = 0.816$$

$$p_1 = 0.5(1+\cos\beta)(\lambda_1 \alpha_1 + \lambda_2 \alpha_* \cos^2\beta)\rho_w g H_{design}$$
$$= 0.5 \times 2 \ (0.818 + 0.012) \times 10^4 \times 1.21$$
$$= 10\,043 \ N/m^2$$

$p_2 = 0 \quad (\eta^* < h_c)$

$p_3 = \alpha_3 p_1 = 0.816 \times 10043 = 8195 \ N/m^2$

(c) Linear wave theory: $P_{dynamic} = \rho g \eta \dfrac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}$

At $z = -3.5m$ and $d = 5m$:

$$P_{dynamic} = 10^4 \times \frac{1.21}{2} \times \frac{\cosh[2\pi(1.5)/38.2]}{\cosh(2\pi \times 5/38.2)} = 4592 \ N/m^2$$

$\Rightarrow$ From linear wave theory: $p_u = 4592 \ N/m^2$

Reduction factor $= \dfrac{p_u}{p_1} = \dfrac{4592}{10043} = 0.457$

Moment from uplift force adds up to the moment by horizontal force, hence reducing the stability of the structure