AY 19/20 _ S1 - MH2814

Hiew Wing Yian

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1) $p(X=0) = \frac{^3C_3}{^7C_3} = \frac{1}{35}$

$p(X=1) = \frac{^3C_2 \times {^4C_1}}{^7C_3} = \frac{12}{35}$

$p(X=2) = \frac{^3C_1 \times {^4C_2}}{^7C_3} = \frac{18}{35}$

$p(X=3) = \frac{^3C_0 \times {^4C_3}}{^7C_3} = \frac{4}{35}$

2a) $p(X=2) = {^{10}C_2} (0.1)^2 (0.9)^8$

$= 0.1937$

$p = 0.1$

$q = 0.9$

b) $x = 2$

$\lambda t = np$

$= 10(0.1)$

$= 1$

$p = \frac{e^{-1}(1)^2}{2!}$

$= 0.1839$

3a) $g(x) = \int_0^2 \frac{1}{8}(x+y)\,dy$

$= \left[\frac{xy}{8} + \frac{y^2}{16}\right]_0^2$

$= \frac{x}{4} + \frac{1}{4}$

$= \frac{(x+1)}{4}$ for $0 \le x \le 2$

$g(x) = 0$, elsewhere

$h(y) = \int_0^2 \frac{1}{8}(x+y)\,d\cancel{y}x$

$= \left[\frac{xy}{8} + \frac{x^2}{16}\right]_0^2$

$= \frac{y}{4} + \frac{1}{4}$

$= \frac{y+1}{4}$ for $0 \le y \le 2$

$h(y) = 0$, elsewhere

b) $f(x,y) = \frac{1}{8}(x+y)$

$g(x)h(y) = \frac{(x+1)(y+1)}{16}$

$f(x,y) \neq g(x) \cdot h(y)$

$\therefore$ x and y are independent

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c) $E(X) = \int_0^2 x g(x)\,dx$

$= \int_0^2 \frac{x(x+1)}{4}\,dx$

$= \left[\frac{x^3}{12} + \frac{x^2}{8}\right]_0^2$

$= \frac{7}{6}$

$E(X^2) = \int_0^2 x^2 g(x)\,dx$

$= \left[\frac{x^4}{16} + \frac{x^3}{12}\right]_0^2$

$= \frac{5}{3}$

$var(X) = E(X^2) - [E(X)]^2$

$= \frac{11}{36}$

$E(y) = \int_0^2 y\,h(y)\,dy$

$= \int_0^2 \frac{y(y+1)}{4}\,dy$

$= 7/6$

$E(y^2) = \int_0^2 y^2 h(y)\,dy$

$= \int_0^2 \frac{y^2}{4}(y+1)\,dy$

$= \frac{5}{3}$

$var(Y) = E(Y^2) - [E(Y)]^2 = \frac{11}{36}$

d) $E(xy) = \int_0^2\int_0^2 xy\,f(x,y)\,dx\,dy$

$= \int_0^2\int_0^2 \frac{xy(x+y)}{8}\,dx\,dy$

$= \int_0^2\int_0^2 \frac{x^2y}{8} + \frac{xy^2}{8}\,dx\,dy$

$= \int_0^2 \left[\frac{x^3y}{24} + \frac{x^2y^2}{16}\right]_0^2 dy$

$= \int_0^2 \frac{y}{3} + \frac{y^2}{4}\,dy$

$= \left[\frac{y^2}{6} + \frac{y^3}{12}\right]_0^2 = \frac{4}{3}$

$\sigma_{xy} = E(XY) - E(X)E(Y)$

$= \frac{4}{3} - \frac{7}{6}\times\frac{7}{6}$

$= \frac{-1}{36}$

$\rho_{xy} = \frac{\frac{-1}{36}}{\frac{11}{36}} = \frac{-1}{11}$

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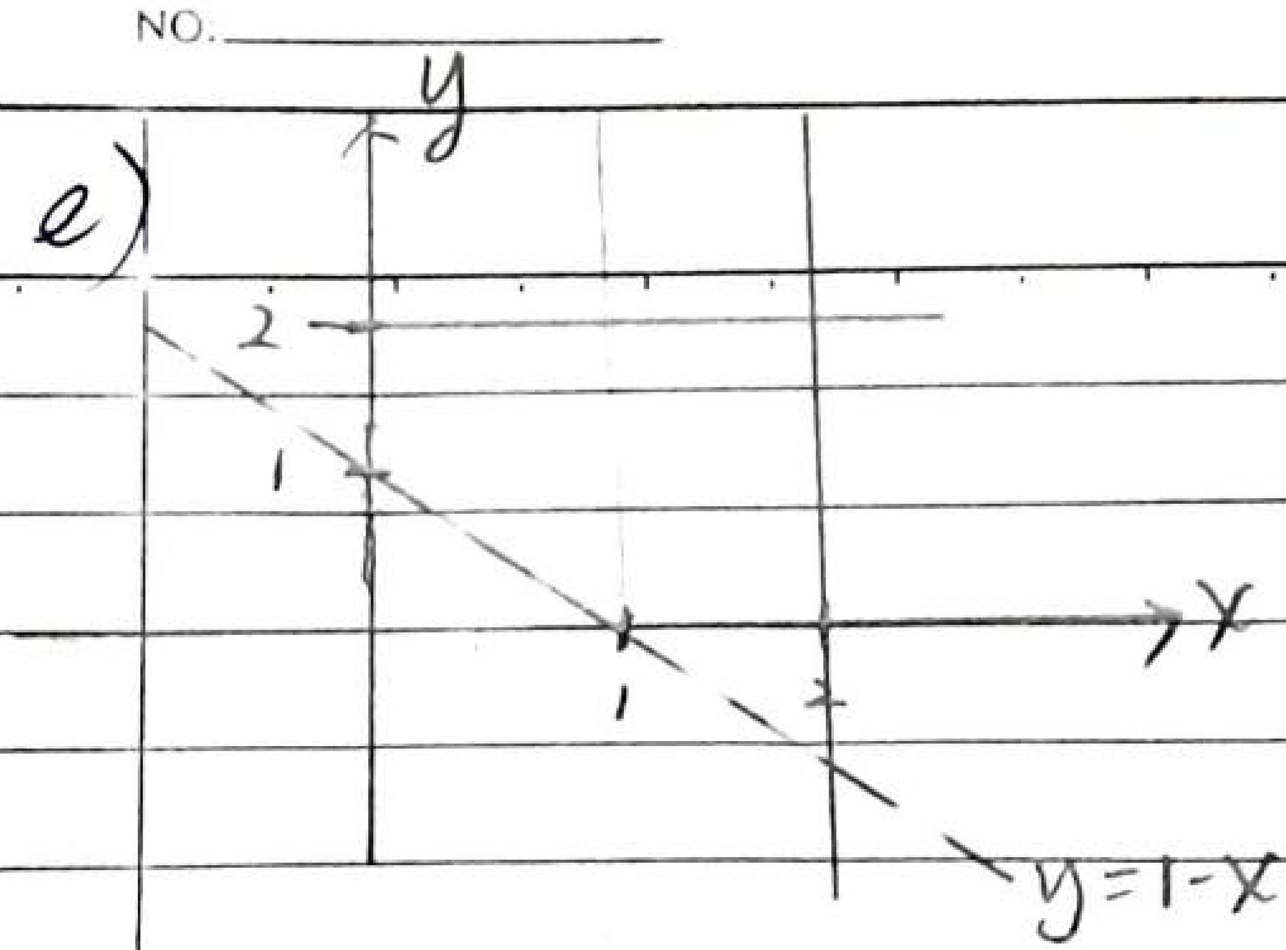
e)



$x+y>1$

$y>1-x$

$p(x+y>1) = 1-p(x+y\le 1) = 1-\frac{1}{24} = \frac{23}{24}$

$p(x+y\le 1) = \int_0^1\int_0^{1-y} \frac{1}{8}(x+y)\,dx\,dy$

$= \frac{1}{8}\int_0^1 \left[\frac{x^2}{2}+xy\right]_{x=0}^{x=1-y} dy$

$= \frac{1}{8}\int_0^1 \left[\frac{(1-y)^2}{2}+y(1-y)\right]dy$

$= \frac{1}{16}\int_0^1 [1-y^2]\,dy = \frac{1}{16}\left[y-\frac{y^3}{3}\right]_0^1$

$= \frac{1}{24}$

4a) $\bar{X}_A \sim N\left(\mu_A, \frac{5000^2}{16}\right)$

$\bar{X}_B \sim N\left(\mu_B, \frac{5000^2}{20}\right)$

b) $\bar{X}_A - \bar{X}_B \sim N\left(\mu_A-\mu_B, 5000^2\left(\frac{9}{80}\right)\right)$

c) $p(\bar{X}_A-\bar{X}_B > 2500) = p\left(\frac{\bar{X}_A-\bar{X}_B-2000}{5000\times\sqrt{9/80}} > \frac{500}{5000\times\sqrt{9/80}}\right)$

$= p(z>0.298)$

$= 0.383$

d) $\bar{X}_A - \bar{X}_B = 2500$

$2500-1.96\times 5000\times\sqrt{\frac{9}{80}} < \mu_A-\mu_B < 2500+1.96\times 5000\sqrt{\frac{9}{80}}$

$-787 < \mu_A-\mu_B < 5787$

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5) a) $H_0: \mu \geq 22$

b) $H_1: \mu < 22$

c) $\alpha = 0.05$

d) $\bar{x} = 21.7$
$S = 1.2$
$n = 100$

$z = \frac{21.7 - 22}{1.2/\sqrt{100}}$
≈ -2.5

using normal approximation
$p(Z < -2.5) = p(Z > 2.5)$
$= 0.00621$
p-value: 0.00621

e) $\because$ p-value $< \alpha$
~~$\therefore$ we reject the~~
$\therefore$ There is enough evidence to reject the claim

6a) $\beta_1 = \frac{S_{xy}}{S_{xx}}$
$= \frac{4500}{37500}$
$= 0.12$

$\beta_0 = \bar{y} - \beta_1 \bar{x}$
$= 87 - 0.12(225)$
$= 60$

$E(Y) = 60 + 0.12X$

b) $H_0: \beta_1 = 0.1$
$H_1: \beta_1 \neq 0.1$

$t = \frac{0.12 - 0.10}{1.414/\sqrt{37500}}$
$= 2.739$

$S^2 = \frac{560 - 0.12 \times 4500}{10}$
$= 2$
$S = 1.414$

$\nu = n - 2$
$= 10$

p-value $= 2p(T > 2.739)$
$= 0.02 < 0.05$

~~$\therefore$ we reject the claim that~~
$\therefore$ There is enough evidence to reject the claim

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c) when $x = 300$

$E(y_0) = 96$

~~$E(Y_0) \pm t_{\frac{\alpha}{2}} \cdot S$~~

$$E(Y_0) \pm t_{\frac{\alpha}{2}} \cdot S \cdot \sqrt{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}}$$

$$= 96 \pm (2.228)(1.414)\left(\sqrt{\frac{1}{12} + \frac{(300-225)^2}{37500}}\right.$$

$$= (94.48, 97.52)$$

Ming Yian