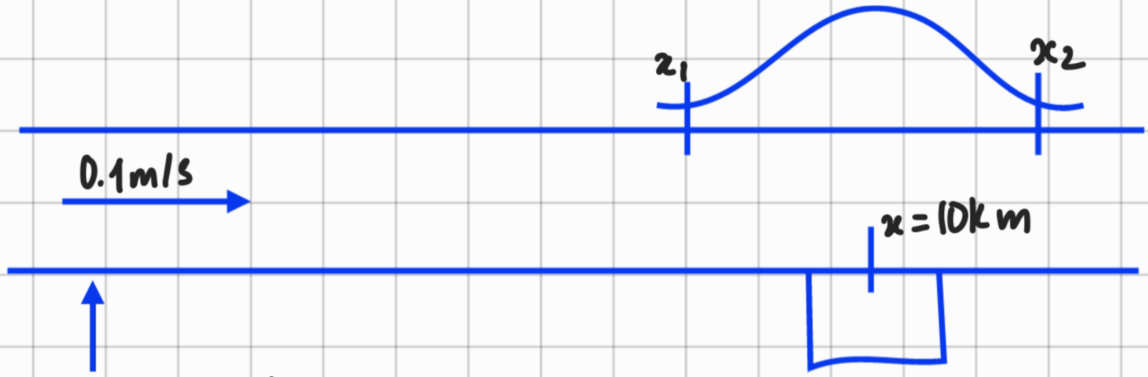
1.

Hoang Thu Minh



$x=0, t=0,$ 20kg

$D_m = 1.6 \times 10^{-9}\ m^2/s$

$B = 20m, \quad H = 5m$

(a) $B = 2\sigma = 2\sqrt{2D_m t}$

$\Rightarrow t = \left(\frac{B}{2\sqrt{2D_m}}\right)^2 = \left(\frac{20}{2\sqrt{2\times 1.6\times 10^{-9}}}\right)^2 = 3.125\times 10^{10}\ s$

(b) (i) $c_{max} = \frac{M}{\sqrt{4\pi D_m t}}$ when $\frac{-(x-x_0-ut)^2}{4D_{eff}t} = 0$

$\Rightarrow \frac{-(10\times 10^3 - 0 - 0.1t)^2}{4(0.05)t} = 0$

$\Rightarrow t = 100000\ s$

$M = \frac{20kg}{5m \times 20m} = 0.2\ kg/m^2$

$c_{max} = \frac{0.2}{\sqrt{4\pi \times 0.05 \times 100000}} = 7.98\times 10^{-4}\ kg/m^3$

$c_{max} = 798\ \mu g/L$

(ii) Advection flux: $J_{adv} = uc = (0.1\ m/s)(7.98\times 10^{-4}\ kg/m^3)$

$= 7.98\times 10^{-5}\ kg/m^2.s$

$J_m = (7.98\times 10^{-5}\ kg/m^2.s)(100m^2)$

$= 7.98\times 10^{-3}\ kg/s$

(iii) Size of cloud $= 4\sigma = 4\sqrt{2D_{eff}t}$

$$= 4\sqrt{2\times 0.05\times 10^5}$$

$$= 400\,m$$

Rate of increase $= \dfrac{d(4\sigma)}{dt} = \dfrac{d[4\sqrt{2D_{eff}t}]}{dt} = \dfrac{2\sqrt{2D_{eff}}}{\sqrt{t}}$

$$= \frac{2\sqrt{2\times 0.05}}{\sqrt{10^5}} = 2\times 10^{-3}\ m/s = 7.2\ m/h$$

(iv) $c(x, 10^5) = 200\ \mu g/L = 2\times 10^{-4}\ kg/m^3$

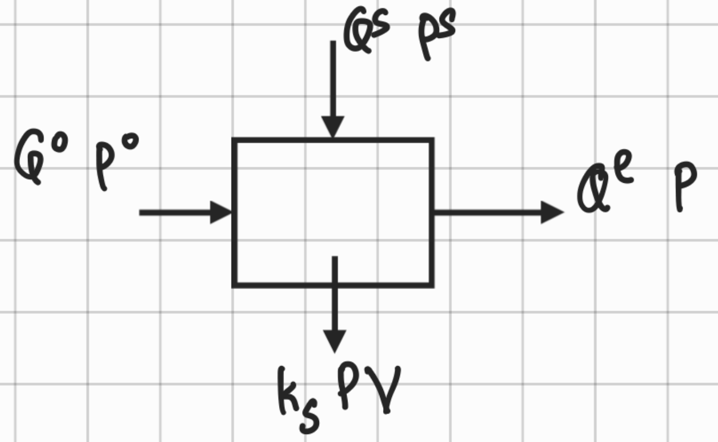
$$\frac{0.2}{\sqrt{4\pi\times 0.05\times 10^5}}\, e^{-\frac{(x-0-0.1\times 10^5)^2}{4\times 0.05\times 10^5}} = 2\times 10^{-4}$$

$x_2 = 10166.4\ m$
$x_1 = 9833.6\ m$

2.

(a) $A = 1.25\times 10^7\ m^2$
$H = 8\ m$
$Q^o = 2.5\ m^3/s$
$p^o = 0.01\ mg/L = 0.01\ g/m^3$

$Q^s = 1\ m^3/h = 0.000278\ m^3/s$
$Q^s p^s = 1.5\ g/s$



(i)

$$v_s = \frac{gd^2\left(\frac{\rho_s}{\rho_w} - 1\right)}{18\nu} = \frac{9.81 \times (8\times10^{-6})^2 \left(\frac{1.05}{1} - 1\right)}{18\,(0.89\times10^{-6})} = 1.958\times10^{-6}\ m/s$$

$$k_s = \alpha\frac{v_s}{H} = 0.65 \times \frac{1.958\times10^{-6}}{8} = 1.591\times10^{-7}\ s^{-1}$$

Mass balance: $\dfrac{d(PV)}{dt} = Q^o P^o + Q^s P^s - Q^e P - k_s PV$

$$V\frac{dP}{dt} + P\frac{dV}{dt} = Q^o P^o + Q^s P^s - Q^e P - k_s PV$$

At steady state: $\dfrac{dP}{dt} = 0$ and $\dfrac{dV}{dt} = 0$

$$\Rightarrow Q^o P^o + Q^s P^s - Q^e P - k_s PV = 0$$

$$\Rightarrow P = \frac{Q^o P^o + Q^s P^s}{Q^e + k_s V}$$

$V = 1.25\times10^7 \times 8 = 10^8\ m^3$

$Q^e \approx Q^o = 2.5\,m^3/s$ (since $Q^s << Q^o$)

$$\Rightarrow P = \frac{(2.5\,m^3/s)(0.01\,g/m^3) + 1.5\,g/s}{2.5\,m^3/s + (1.591\times10^{-7}\,s^{-1})(10^8\,m^3)}$$

$= 0.0828\ g/m^3$

$= 82.8\ \mu g/L$

(ii) $P = \dfrac{Q^o P^o + Q^s P^s}{Q^e + k_s V} \Rightarrow 0.02 = \dfrac{(2.5)(0.01) + Q^s P^s}{2.5 + (1.591\times10^{-7})(10^8)}$

$\Rightarrow Q^s P^s = 0.3432\ g/s$

Removal $= \dfrac{1.5 - 0.3432}{1.5} \times 100\% = 77\%$

(iii) In winter :

$$v_s = \frac{gd^2(\rho_s/\rho_w - 1)}{18\upsilon} = \frac{9.81 \times (8\times10^{-6})^2 (\frac{1.05}{1} - 1)}{18(1.31\times10^{-6})} = 1.33\times10^{-6} \text{ m/s}$$

$$k_s = \alpha \frac{v_s}{H} = 0.65 \times \frac{1.33\times10^{-6}}{8} = 1.08\times10^{-7} \ (s^{-1})$$

$$HRT \ \theta = \frac{V}{Q_0} = \frac{10^8 \, m^3}{2.5 \, m^3/s} = 4\times10^7 \, s$$

Fraction removed by the lake $= \dfrac{k_s\theta}{1+k_s\theta} = 0.812$

In summer

Fraction removed by the lake $= \dfrac{k_s\theta}{1+k_s\theta} = \dfrac{(1.591\times10^{-7})(4\times10^7)}{1+1.591\times10^{-7}\times4\times10^7}$

$= 0.864$

From summer to winter, the fraction of P removed by the lake will decrease.

(iv)

$$106\,CO_2 + 16\,NO_3^- + HPO_4^{2-} + 18H^+ + 122\,H_2O \underset{}{\overset{light}{\rightleftharpoons}} C_{106}H_{263}O_{110}N_{16}P + 138\,O_2$$

(b)

(i) $BOD = 70\% \times COD = 0.70 \times 500 \text{ mg/L} = 350 \text{ mg/L}$

$nbCOD = COD - BOD = 500 - 350 = 150 \text{ mg/L}$

(ii) Let L_t be the BOD at the end of t days

L_0 be the initial BOD $\Rightarrow L_0 = 350$ mg/L

$k = 0.2 \text{ day}^{-1} \Rightarrow \dfrac{dL_t}{dt} = -0.2L_t$

$\Rightarrow L_t = L_0 e^{-0.2t} = 350e^{-0.2t}$

At the end of 5 days: $L_5 = 350e^{-0.2\times 5} = 128.8 \text{ mg/L}$

Remaining total COD $= 500 - (350 - 128.8)$
$= 278.8 \text{ mg/L}$

(iii) At the end of Day 2: $L_2 = 350e^{-0.2\times 2} = 234.6 \text{ mg/L}$
From the beginning of Day 3:
$\dfrac{k'}{k} = \beta^{(T'-T)} \Rightarrow k' = 0.2 \times 1.065^{(20-25)} = 0.146 \text{ day}^{-1}$
$\Rightarrow L_t = L_2 e^{-0.146(t-2)}$
At the end of Day 5: $L_5 = 234.6 \times e^{-0.146(5-2)} = 151.4 \text{ mg/L}$

Remaining total COD $= 500 - (350 - 151.4)$
$= 301.4 \text{ mg/L}$

3.
(a)

C_{in}	C_{out}	$\dfrac{\Delta C}{\Delta t}$ (cu/min)	$\dfrac{\Delta C}{\Delta t} : C_{out}$ (min^{-1})
10	3.3	-0.67	-0.20
13	4.4	-0.86	-0.20
21	6.7	-1.43	-0.21

Since $\dfrac{\Delta C}{\Delta t} \approx -0.20\, C_{out}$, the rate expression $-kC$ is valid
$\therefore k = 0.20 \text{ min}^{-1}$

(b) Worst case scenario : $C_{in} = 21$ CU
$C_{out} = 3$ CU

Fraction removed $= \dfrac{k\bar{t}}{1+k\bar{t}}$

$\Rightarrow \dfrac{21-3}{21} = \dfrac{0.2\bar{t}}{1+0.2\bar{t}}$

$\Rightarrow \bar{t} = 30$ minutes

HRT of one tank is 10 min $\Rightarrow$ #tanks $= \dfrac{30}{10} = 3$

(c) - The system can still operate if one tank is out of order
- Avoid short circuiting

4.

(a) Total amount of X: $M_X = (500\ mg/L)(20\ m^3)(10^3 L/m^3)$
$= 10^7$ mg

(b) Consider the mass distribution between suspended solids and solution phases:

$$\frac{M_{SS}}{M_W} = K_d \cdot C_{TSS} = (0.01\ m^3/g)(10\ g/m^3) = \frac{1}{10}$$

$\Rightarrow M_{SS} = 0.1 M_W$

Consider the mass distribution between the solid and aqueous phases in the sludge:

$$\frac{M_S}{M_W} = K_d \cdot \rho_s \cdot \frac{V_s}{V_W} = (0.01\ m^3/g)(25\times10^3\ g/m^3)\left(\frac{0.2\ m^3}{20\ m^3}\right)$$

$= 2.5$

$\Rightarrow M_S = 2.5 M_W$

Consider the mass distribution between gaseous and aqueous phases:

$$\frac{M_G}{M_W} = \frac{V_G C_G}{V_W C_W} = \frac{V_G}{V_W} \times H_{cc} = \frac{2\,m^3}{20\,m^3} \times 5\times10^{-2} = 0.005$$

$$\Rightarrow M_G = 0.01\,M_W$$

$$M_W + M_S + M_{SS} + M_G = M_T = 10^7 \ (mg)$$

$$\Rightarrow M_W + 2.5M_W + 0.1M_W + 0.005M_W = 10^7$$

$$\Rightarrow M_W = 2.77\times10^6 \ mg$$

$$\Rightarrow M_S = 2.5M_W = 6.925\times10^6 \ mg$$

$$M_{SS} = 0.1\,M_W = 2.77\times10^5 \ mg$$

$$M_G = 0.005M_W = 13850 \ mg$$

$$F_W = \frac{M_W}{M_T} = 0.277$$

$$F_S = \frac{M_S}{M_T} = 0.693$$

$$F_{SS} = \frac{M_{SS}}{M_T} = 0.028$$

$$F_G = \frac{M_G}{M_T} = 0.001$$

(c) $$C_W = \frac{M_W}{V_W} = \frac{2.77\times10^6\,mg}{(20m^3)(10^3 L/m^3)} = 139\ mg/L$$

$$C_G = \frac{M_G}{V_G} = \frac{13850\,mg}{(2m^3)(10^3 L/m^3)} = 6.925\,mg/L$$

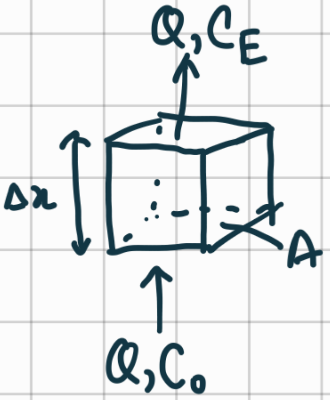
5.

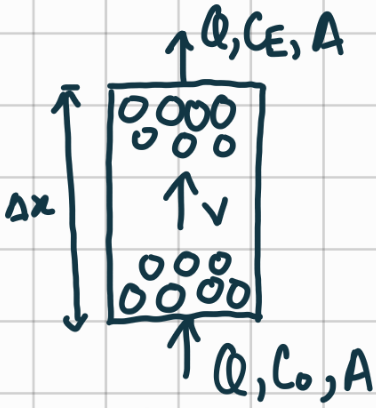
(a). Steady state – concentrations at any point in the system do not change with time.

- Equilibrium exist at the interface
- Sharp boundary – Interface between two phases is well-defined
- Laminar flow of fluid exists on the interface
- Only mass transfer processes occur at the interface; no other reactions

(b)

(i) Consider an infinitesimal control volume $(A.\Delta x)$ of the bed





Take the pore volume $(\varepsilon A \Delta x)$ and the cross-sectional pore area (ΔS) of the bed

Mass balance: $\varepsilon A \Delta x \dfrac{\Delta C}{\Delta t} = \varepsilon A v \left(C_x - C_{x+\Delta x}\right) - k \Delta S (C - C_s)$

$$\Rightarrow \quad \frac{dC}{dt} = -v\frac{dC}{dx} - \frac{k\Delta S}{\varepsilon A \Delta x}(C - C_s)$$

$$\Rightarrow \quad \frac{dC}{dt} = -v\frac{dC}{dx} - ka(C - C_s)$$

Steady state $\dfrac{dC}{dt} = 0 \Rightarrow v\dfrac{dC}{dx} = -ka(C - C_s)$

$$\int_{C_0}^{C_x} \frac{dC}{C - C_s} = \int_0^x \left(-\frac{ka}{v}\right) dx$$

$$\ln\left(\frac{C_s - C_x}{C_s - C_0}\right) = -\frac{kax}{v}$$

(ii) $Re = \frac{\varepsilon v \rho d}{\mu} = \frac{0.30\,(5.0\,cm/s)(1\,g/cm^3)(0.1\,cm)}{0.0131\,g/cm\text{-}s} = 11.45$

$Sc = \frac{\mu}{\rho D} = \frac{(0.0131\,g/cm\text{-}s)}{(1.2 \times 10^{-5}\,cm^2/s)(1\,g/cm^3)} = 1092$

$Sh = \frac{1.09}{\varepsilon} Re^{0.333}\, Sc^{0.333} = 84.3$

(iii) $C_E = 0.01$ mg/L
$C_0 = 10$ mg/L
Assume $C_s = 0$ mg/L (no flouride is attached on the surface of the beads)

$k = Sh \cdot \frac{D}{d} = 84.3 \times \frac{1.2 \times 10^{-5}\,cm^2/s}{0.1\,cm} = 0.0101$ cm/s

$a = \frac{6}{d}\frac{(1-\varepsilon)}{\varepsilon} = \frac{6}{0.1} \times \frac{(1-0.3)}{0.3} = 140\ cm^{-1}$

$$\ln\left(\frac{0 - 0.01}{0 - 10}\right) = -\frac{0.0101 \times 140 \times H}{5.0}$$

$\Rightarrow H = 24.4m$

(iv) The same exchange bed should not be used to treat it because the process will not give the same efficiency. Ion exchange process does not depend only on water quality parameters, but also the characteristics of the moleculars such as weight or size of the solute. Since borate anion is larger than flouride, the same ion exchange bed would likely yield a worse result.