CV2012 - STRUCTURAL ANALYSIS II

AY18/19 S2

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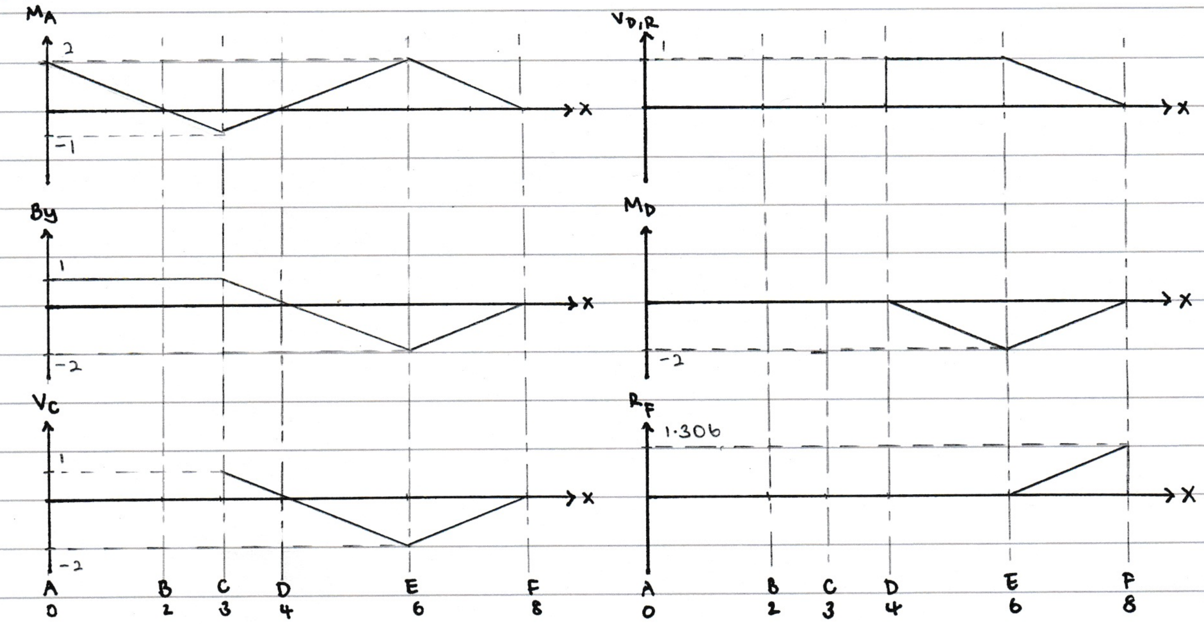
Q1. (a)



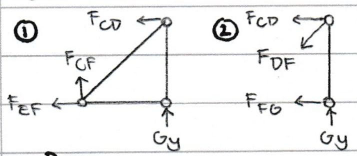
$$M_{A,max\,(+ve)} = \frac{1}{2}(2)(2)(12) - \frac{1}{2}(2)(1)(12) + \frac{1}{2}(4)(2)(12) + \frac{1}{2}(2)(2)(20) + \frac{1}{2}(4)(2)(20) + 2(30) = 240\text{ kNm}$$

(b)(i) To ease calculation, make table for different reaction force for support G due to unit load application.

X	0	2	4	6
Gy(↑)	-0.5	0	0.5	1.0

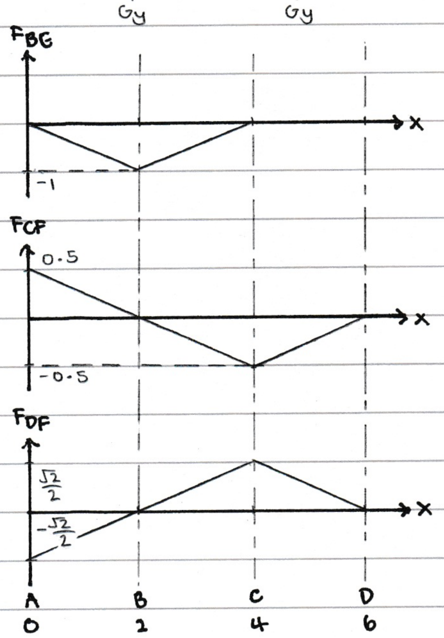
** - Take note of ZFMS when applying unit loads. For example, when unit load applied at x = 6(D), all members are ZFM except for DG, as such, FBD ① and ② should not be used.

FBDs used:

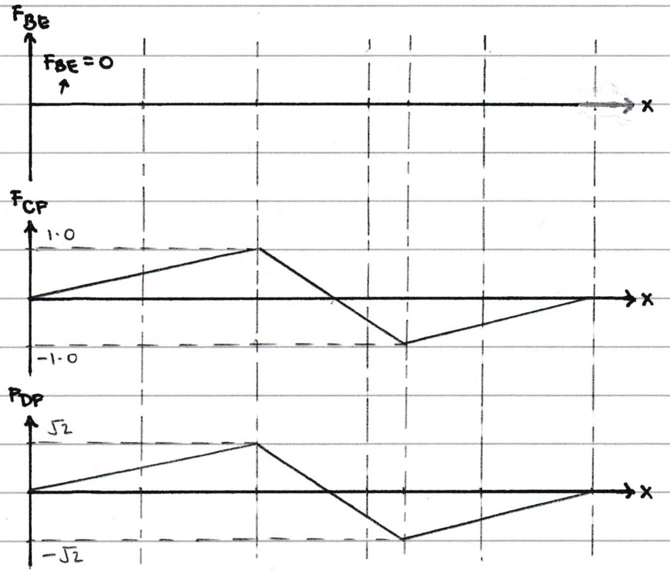


For ①: $\Sigma F_y = 0$, $F_{CF} = -G_y$

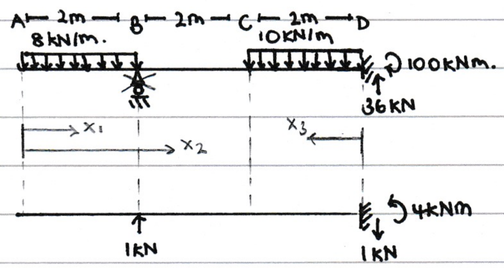
For ②: $\Sigma F_y = 0$, $\frac{1}{\sqrt{2}} F_{DF} = G_y \rightarrow F_{DF} = \sqrt{2}\,G_y$.



(b)(ii)



Q2. (a)(i) Take B_y as redundant and assume acts upward.



$M(x_1) = -(x)(8)(x/2) = -4x^2$, $m(x_1) = 0$

$M(x_2) = -16(x-2+1) = (-16x+16)$, $m(x_2) = (x-2)$

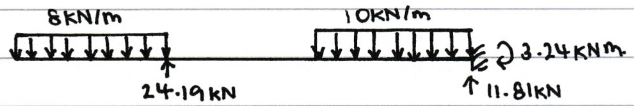
$M(x_3) = -10x(x/2) - 100 + 36x = (-5x^2 - 100 + 36x)$, $m(x_3) = (4-x)$.

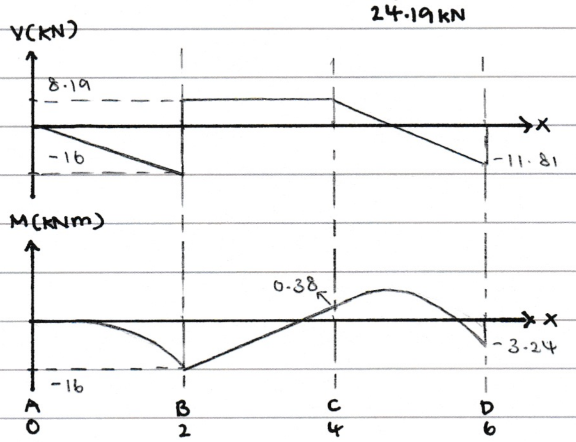
$-|\Delta_B| + B_y f_{BB} = 0$

$\Delta_B = \sum \int \frac{mM}{EI} dx = -\frac{516}{EI}$, $f_{BB} = \sum \int \frac{m^2}{EI} dx = \frac{64}{3EI}$

$-\frac{516}{EI} + B_y\left(\frac{64}{3EI}\right) = 0 \Rightarrow B_y = 24.1875\text{ kN} \Rightarrow B_y = 24.19\text{ kN} (\uparrow)$

(a)(ii) new FBD:





(b) Using results from part (a),

$$-|\Delta_{B}| + f_{BB} B_y = -\frac{B_y}{K} - 0.04$$

$$-\frac{516}{2000} + \frac{64}{3 \times 2000} B_y = -\frac{B_y}{600} - 0.04$$

Solving for B_y gives, $B_y = 17.676\,kN = 17.7\,kN$.

Q3 (a) $(FEM)_{AB} = -\frac{80}{3}\,kNm$, $(FEM)_{BA} = \frac{80}{3}\,kNm$, $(FEM)_{BC} = (FEM)_{CB} = 0\,kNm$, $(FEM)_{CD} = -75\,kNm$.

$$M_{AB} = 2(3E)(\tfrac{I}{4})[2(0) + \theta_B - 3(0)] - \frac{80}{3} = \frac{3EI}{2}\theta_B - \frac{80}{3}$$

$$M_{BA} = 2(3E)(\tfrac{I}{4})[2\theta_B + 0 - 3(0)] + \frac{80}{3} = 3EI\theta_B + \frac{80}{3}$$

$$M_{BC} = 2(2E)(\tfrac{I}{4})[2\theta_B + \theta_C - 3(0)] = EI(2\theta_B + \theta_C)$$

$$M_{CB} = 2(2E)(\tfrac{I}{4})[2\theta_C + \theta_B - 3(0)] = EI(2\theta_C + \theta_B)$$

$$M_{CD} = 3E(\tfrac{I}{4})[\theta_C - 0] - 75 = \frac{3EI}{4}\theta_C - 75$$

$M_{BA} + M_{BC} = 0$ | $M_{CB} + M_{CD} = 0$

$3EI\theta_B + \frac{80}{3} + EI(2\theta_B + \theta_C) = 0$ | $EI(2\theta_C + \theta_B) + \frac{3EI}{4}\theta_C - 75 = 0$

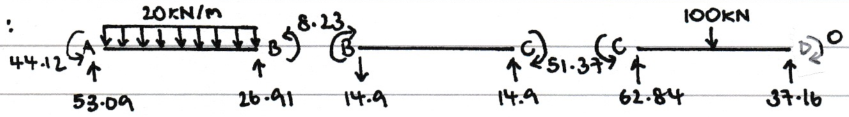
$5EI\theta_B + EI\theta_C + \frac{80}{3} = 0$ | $2.75EI\theta_C + EI\theta_B - 75 = 0$

$EI\theta_C = \frac{300}{11} - \frac{4EI}{11}\theta_B$

$$5EI\theta_B + \frac{300}{11} - \frac{4EI}{11}\theta_B + \frac{80}{3} = 0 \rightarrow \theta_B = -\frac{1780}{153EI}, \quad \theta_C = \frac{4820}{153EI}$$

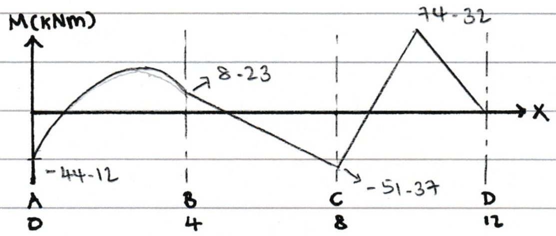
$M_{AB} = -44.12\,kNm$, $M_{BA} = -8.23\,kNm$, $M_{BC} = 8.23\,kNm$, $M_{CB} = 51.37\,kNm$, $M_{CD} = -51.37\,kNm$.

FBD:



$R_C = 14.9 + 62.84 = 77.74\,kN\,(\uparrow)$

$R_D = 37.16\,kN\,(\uparrow)$.



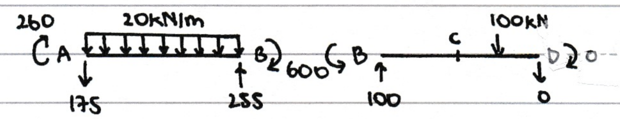
(b) $(FEM)_{AB} = -\frac{80}{3}\,kNm$, $(FEM)_{BA} = \frac{80}{3}\,kNm$, $(FEM)_{BD} = -100 \times 6 = -600\,kNm$.

$M_{AB} = \frac{3EI}{2}\theta_B - \frac{80}{3}$, $M_{BA} = 3EI\theta_B + \frac{80}{3}$, $M_{BD} = -600$

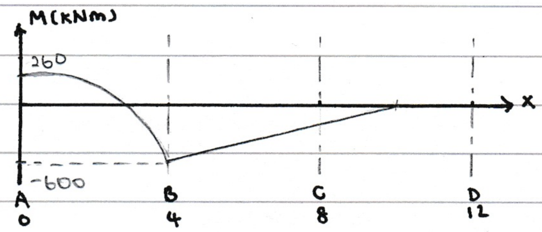
$M_{BA} + M_{BD} = 0$: $3EI\theta_B + \frac{80}{3} - 600 = 0 \rightarrow \theta_B = \frac{1720}{9EI}$

$M_{AB} = 260\,kNm$, $M_{BA} = 600\,kNm$, $M_{BD} = -600\,kNm$.

FBD:



(b) [cont.].



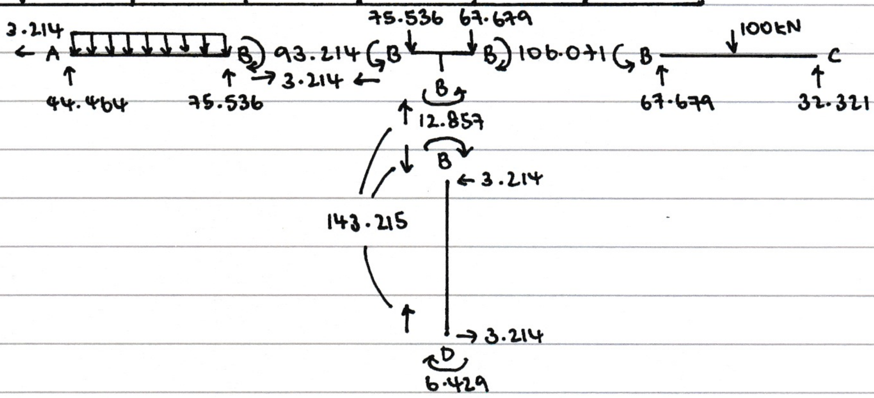
By inspection / geometry : $M_{CB} = 200\,kNm$, $M_{CD} = -200\,kNm$.

Q4. (a) $K_{BA} = \frac{3EI}{6} = \frac{EI}{2}$, $K_{BC} = \frac{3(2EI)}{6} = EI$, $K_{BD} = \frac{4(3EI)}{6} = 2EI \rightarrow \Sigma_i = \frac{7}{2}EI$

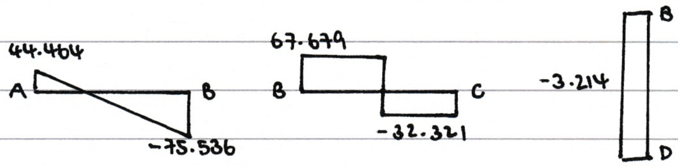
$DF_{BA} = \frac{1}{7}$, $DF_{BC} = \frac{2}{7}$, $DF_{BD} = \frac{4}{7}$. ; $(FEM)_{BA} = 90\,kNm$, $(FEM)_{BC} = -112.5\,kNm$.

Joint	A	B			C	D
Member	AB	BA	BC	BD	CB	DB
DF	1	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{4}{7}$	1	0
FEM		90	-112.5	0		
Dist.		3.214	6.429	12.857		
CO.						6.429
Sum.	0	93.214	-106.071	12.857	0	6.429

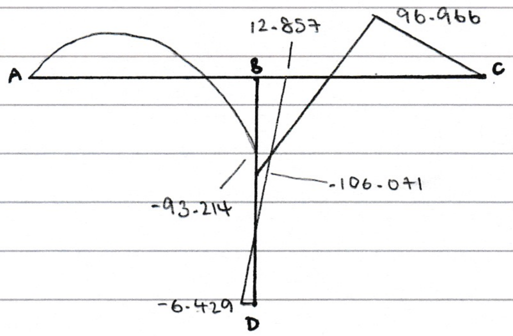
FBD:



SFD:



BMD:



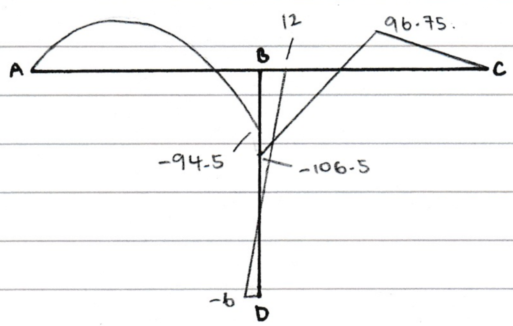
(b) Reuse intermediate working from (a),

$DF_{BA} = \frac{1}{7}$, $DF_{BC} = \frac{2}{7}$, $DF_{BD} = \frac{4}{7}$

new $(FEM)_{BA} = 90 + \frac{3EI}{6^2}\left(\frac{18}{EI}\right) = 91.5\,kNm$, $(FEM)_{BC} = -112.5\,kNm$.

Joint	A	B			C	D
Member	AB	BA	BC	BD	CB	DB
DF	1	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{4}{7}$	1	0
FEM	0	91.5	-112.5	0	0	0
Dist.		3	6	12		
CO						6
Sum.		94.5	-106.5	12		6

BMD:



(c) Given that structure is statically determinate,

$\Sigma M_A = 0,\ 20(6)(6/2) + 100(9) = C_y(12).$

$C_y = 105\text{kN} (\uparrow).$

$\Sigma F_y = 0,\ A_y + C_y = 20(6) + 100$

$A_y = 115\text{kN} (\uparrow)$

$\Sigma M_B = 0,\ M_{BA} + 20(6)(3) = 115(6)$

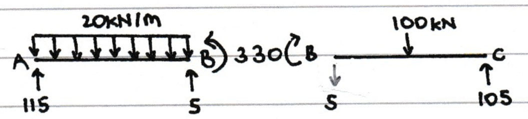
$M_{BA} = 330 \text{ kNm (ACW)}.$

$M_{BC} + 100(3) = 105(6)$

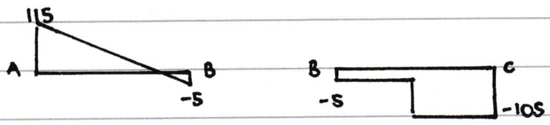
$M_{BC} = 330 \text{ kNm (CW)}.$

$M_{AB} = M_{CB} = 0 \text{ kN}.$

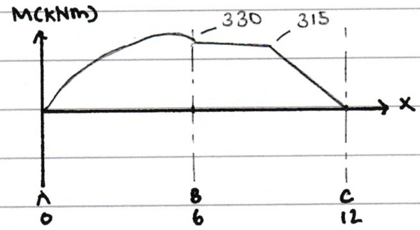
FBD:



SFD:



BMD:



(NOTE: solution of 300 kNm seems to be wrong, my solution checked with VisualFEA and should be correct).

provided by school