Name: Owen

Paper: CV1012 – Fluid Mechanics 2018–2019 Sem 2

1. (a)

Volume at 10°C $= \frac{20}{999.7}$

$= \underline{\underline{0.020006002\ m^3}}$

Volume at 50°C $= \frac{20}{988.0}$

$= \underline{\underline{0.020242915\ m^3}}$

$\Delta P = K\left(\frac{\rho_1 - \rho_0}{\rho_0}\right)$

$= 2150*10^6\left(\frac{999.7-988.0}{988.0}\right)$

$= \underline{\underline{25460526\ Pa}}$

1. (b)

Acceleration at 0m/s $= 9.81\sin 30°$

$= \underline{\underline{4.905\ m/s^2}}$

At terminal velocity V_T, $\sum F = 0$.

$mg\sin 30° = \tau\,(0.3)^2$

$\tau = \mu\left(\frac{V_T}{y}\right)$

$V_T = \frac{(100)(9.81)(\sin 30°)(0.005*10^{-3})}{(0.3)^2(0.4)}$

$= \underline{\underline{0.068125\ m/s}}$

1. (c)

$(P_1 - P_2) = (1300)(9.81)(1) - (1000)(9.81)(1)$

$= \underline{\underline{2943\ N/m^2}}$

$A_1 = (\pi/4)(0.4)^2$

$A_2 = (\pi/4)(0.2)^2$

$A_1V_1 = A_2V_2$

$V_2 = 4V_1$

1. (c)

Assuming $z_1 = z_2$, no head loss,
Bernoulli's equation 1 to 2:

$$\frac{P_1}{\rho g} + \frac{{V_1}^2}{2g} = \frac{P_2}{\rho g} + \frac{(4V_1)^2}{2g}$$

$$(P_1 - P_2) = 7.5\rho(V_1)^2$$

$$V_1 = \sqrt{\frac{2943}{7.5(1000)}}$$

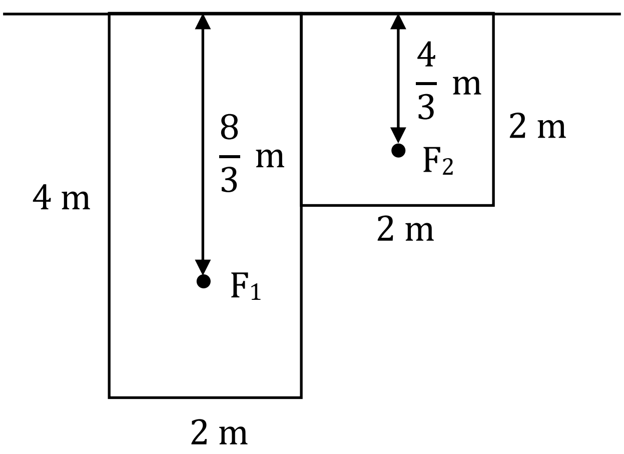
$$= 0.62642 \text{ m/s}$$

$$Q = V_1 A_1$$

$$= (0.62642)(\pi/4)(0.4)^2$$

$$= \underline{\underline{0.787 \text{ m}^3\text{/s}}}$$

1. (d)



$$F_1 = \rho g A h_c$$

$$= (940)(9.81)(4)(2)(2)$$

$$= 147542.4 \text{ N} \left(\frac{8}{3} \text{ m below surface}\right)$$

$$F_2 = (940)(9.81)(2)(2)(1)$$

$$= 36885.6 \text{ N} \left(\frac{4}{3} \text{ m below surface}\right)$$

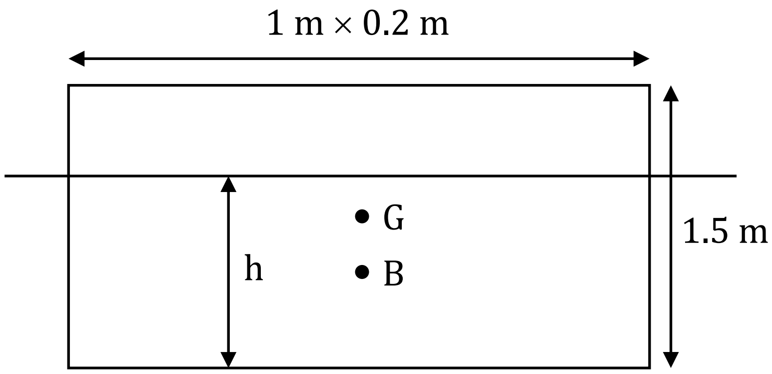
$$F_R = 147542.4 + 36885.6$$

$$= \underline{\underline{184428 \text{ N}}}$$

$$\text{Distance of } F_R \text{ below surface} = \frac{\left(\frac{8}{3}\right)(147542.4) + \left(\frac{4}{3}\right)(36885.6)}{184428}$$

$$= \underline{\underline{2.4 \text{ m}}}$$

1. (e)



Upthrust = Weight

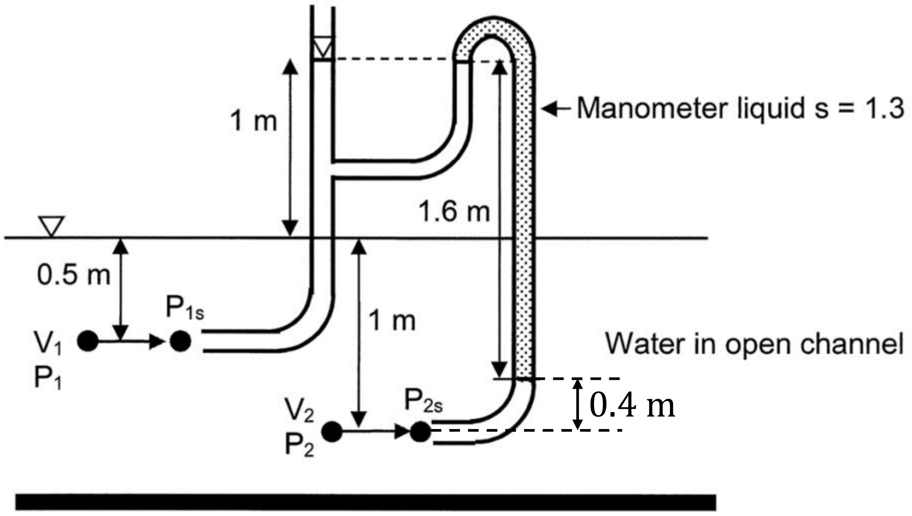
$\rho g V_i = 2*10^3$
$V_i = 0.19890 \text{ m}^3$

$$h = \frac{0.19890}{(1)(0.2)}$$
$$= 0.99451 \text{ m}$$

$$BG = (0.5)(1.5) - (0.5)(0.99451)$$
$$= 0.25275 \text{ m}$$

$$BM = \frac{I_{yy}}{V_i}$$
$$= \frac{\frac{(1)(1.5)^3}{12}}{0.19890}$$
$= 1.4140 \text{ m} > BG$, <u>therefore block is in stable equilibrium.</u>

2. (a)



$P_{1s} = (1000)(9.81)(1.5)$
$= \underline{\underline{14715 \text{ Pa}}}$

$P_{2s} = (1300)(9.81)(1.6)+(1000)(9.81)(0.4)$
$= \underline{\underline{24328.8 \text{ Pa}}}$

$P_1 = (1000)(9.81)(0.5)$
$= 4905 \text{ Pa}$

$P_2 = (1000)(9.81)(1)$
$= 9810 \text{ Pa}$

Bernoulli's equation outside tube to inside tube:

$$\frac{P}{\rho g} + \frac{V^2}{2g} = \frac{P_s}{\rho g}$$

$$V = \sqrt{\frac{2(P_s - P)}{\rho}}$$

$$V_1 = \sqrt{\frac{2(14715-4905)}{1000}}$$

$= \underline{\underline{4.43 \text{ m/s}}}$

$$V_2 = \sqrt{\frac{2(24328.8-9810)}{1000}}$$

$= \underline{\underline{5.39 \text{ m/s}}}$

2. (b)

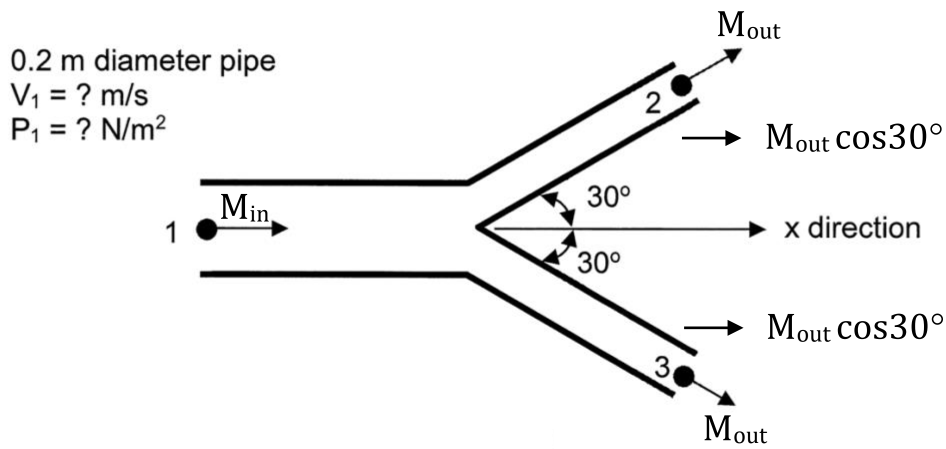
$Q_1 = Q_2 + Q_3$
$= (5)(\pi/4)(0.1)^2 + (5)(\pi/4)(0.05)^2$
$= 0.049087\ m^3/s$

$V_1 = \frac{Q}{A}$
$= \frac{0.049087}{(\pi/4)(0.2)^2}$
$= 1.5625\ m/s$

Assuming $P_{atm} = 0$, $z_1 = z_2$,
Bernoulli's equation 1 to 2:
$\frac{P_1}{\rho g} + \frac{{V_1}^2}{2g} = \frac{{V_2}^2}{2g}$
$P_1 = (1000)(0.5)(5^2 - 1.5625^2)$
$= 11279\ Pa$

Momentum equation in x-direction (right as positive)
$F_{net} = M_{out} - M_{in}$
$= [\rho(\pi/4)(0.1)^2(5)^2 + \rho(\pi/4)(0.05)^2(5)^2](\cos 30°) - \rho(\pi/4)(0.2)^2(1.5625)^2$
$= 135.86\ N$

$F_{net} = P_1A_1 + F_x$
$135.86 = (11279)(\pi/4)(0.2)^2 + F_x$
$F_x = 135.86 - (11279)(\pi/4)(0.2)^2$
$= -218.5\ N$ (on water)
F_x on nozzle = $\underline{\underline{218.5\ N}}$



3. (a)

$f = \text{function}(\rho, \mu, V, D, \varepsilon)$

Their dimesions are:

$f = [M^0L^0T^0]$

$\rho = [ML^{-3}]$

$\mu = [ML^{-1}T^{-1}]$

$V = [LT^{-1}]$

$D = [L]$

$\varepsilon = [L]$

Using μ, V, D as repeating variables, 3 ∏ terms:

$\prod_1 = \underline{\underline{f}}$

$\prod_2 = \rho\mu^aV^bD^c$

$\prod_3 = \underline{\underline{\frac{\varepsilon}{D}}}$

Using MLT system:

$ML^{-3}M^aL^{-a}T^{-a}L^bT^{-b}L^c = M^0L^0T^0$

$1 + a = 0$

$-3 - a + b + c = 0$

$- a - b = 0$

Solving: $a = -1$, $b = 1$, $c = 1$

$\prod_2 = \frac{\rho VD}{\mu}$

$= \underline{\underline{Re}}$

3. (a) (ii)

$f = \text{function}(Re, \frac{\varepsilon}{D})$

1. $Re < 2100$, in laminar flow regime, f is a function of Re only & is independent of ε/D, the equation is $f = 64/Re$.
2. $2100 < Re < 4000$, in transition range, f is uncertain as flow may be laminar or turbulent.
3. $Re > 4000$ but not in wholly turbulent flow regime, f depends on both Re and ε/D.
4. Re very large and in the wholly turbulent flow regime, f depends on ε/D only, independent of Re.

3. (b) (i)

From Bernoulli's equation:

$$H = \frac{8fLQ^2}{g\pi^2 D^5}$$

$$Q = \sqrt{\frac{D^5}{f}}\sqrt{\frac{g\pi^2 H}{8L}}$$

Q in pipe 1 = Q in pipe 2

$$\sqrt{\frac{(0.2)^5}{(0.04)}}\sqrt{\frac{g\pi^2 H}{8L}} = \sqrt{\frac{{D_2}^5}{(0.01)}}\sqrt{\frac{g\pi^2 H}{8L}}$$

$$D_2 = \sqrt[5]{\frac{(0.01)(0.2)^5}{(0.04)}}$$

$$= \underline{\underline{0.152 \text{ m}}}$$

3. (b) (ii)

2Q in old pipe = Q in new pipe

$$2\sqrt{\frac{(0.2)^5}{0.04}}\sqrt{\frac{g\pi^2 H}{8L}} = \sqrt{\frac{{D_{new}}^5}{0.01}}\sqrt{\frac{g\pi^2 H}{8L}}$$

$$D_{new} = \sqrt[5]{\frac{(0.2)^5(2)^2(0.01)}{(0.04)}}$$

$$= \underline{\underline{0.2 \text{ m}}}$$

4. (a)

$\mathrm{Fr} = \frac{V}{\sqrt{gL}}$

For Fr similarity,

$\frac{V_p}{\sqrt{L_p}} = \frac{V_m}{\sqrt{L_m}}$, if $g_m = g_p$

$\frac{V_p}{V_m} = \sqrt{\frac{L_p}{L_m}}$

$\lambda_v = (\lambda_L)^{0.5}$ (shown)

4. (b) (i)

Spillway flow is governed by similarity of Froude number.

For Froude number similarity:

$\frac{Q_p}{Q_m} = \frac{V_p L_p{}^2}{V_m L_m{}^2} = \left(\frac{L_p}{L_m}\right)^{\frac{5}{2}}$

$L_p = L_m\left(\frac{Q_p}{Q_m}\right)^{\frac{2}{5}}$

$= 0.6\left(\frac{50}{0.35}\right)^{\frac{2}{5}}$

$= \underline{\underline{4.37 \text{ m}}}$

4. (b) (ii)

For similarity of Drag coefficient:

$\frac{F_p}{0.5\rho_p V_p{}^2 L_p{}^2} = \frac{F_m}{0.5\rho_m V_m{}^2 L_m{}^2}$

$F_p = \left(\frac{V_p{}^2 L_p{}^2}{V_m{}^2 L_m{}^2}\right) F_m$

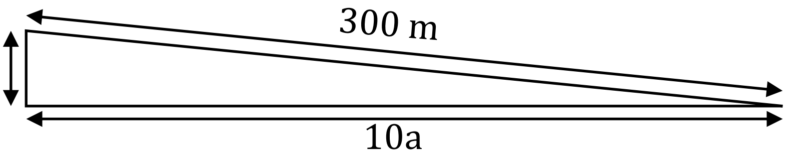
$= \left(\frac{L_p}{L_m}\right)^3 F_m$

$= \left(\frac{4.3663}{0.6}\right)^3 (1.5)$

$= \underline{\underline{578 \text{ N}}}$

4. (c) (i)

$300^2 = (a)^2 + (10a)^2$



$a = \sqrt{\frac{300^2}{101}}$

$= 29.851\text{m}$

$V = \frac{Q}{A}$

$= \frac{0.025}{(\pi/4)(0.2)^2}$

$= 0.79577 \text{ m/s}$

$\frac{V^2}{2g} = 0.032276 \text{ m}$

$\text{Re} = \frac{VD}{\upsilon}$

$= \frac{(0.79577)(0.2)}{1.19*10^{-4}}$

$= 1337.4$ (laminar flow regime)

Bernoulli's equation upper to lower:

$\frac{P_{upper}}{\rho g} + \frac{V^2}{2g} + a = \frac{P_{lower}}{\rho g} + \frac{V^2}{2g} + h_f$

$h_f = \frac{128\upsilon LQ}{g\pi D^4}$

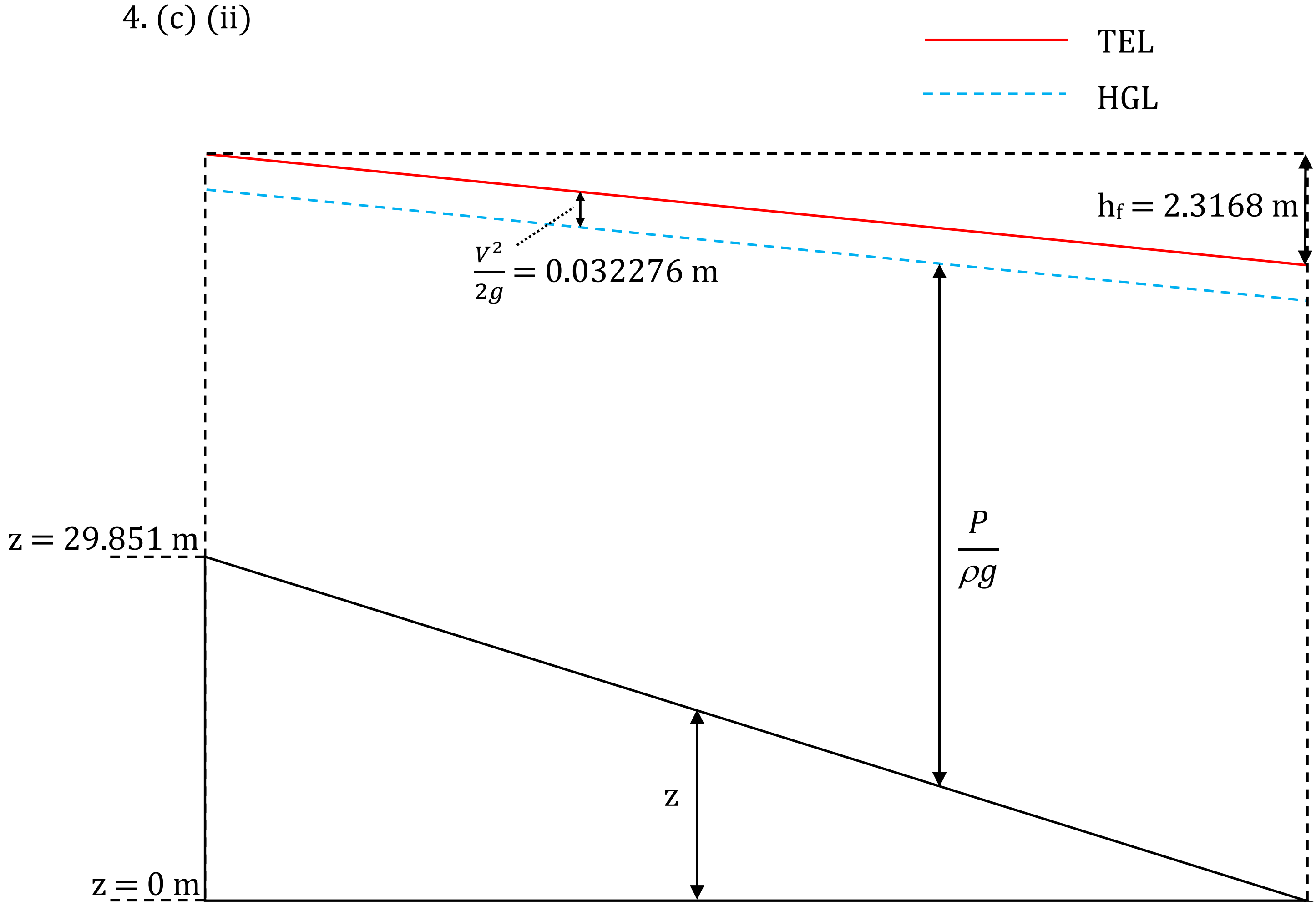
$= \frac{128(1.19*10^{-4})(300)(0.025)}{(9.81)\pi(0.2)^4}$

$= 2.3168 \text{ m}$

$P_{lower} - P_{upper} = (850)(9.81)(29.851 - 2.3168)$

$= \underline{\underline{229.6 \text{ kPa}}}$

4. (c) (ii)


TEL
HGL
$h_f = 2.3168$ m
$\frac{V^2}{2g} = 0.032276$ m
$\frac{P}{\rho g}$
z = 29.851 m
z
z = 0 m

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2018-2019

CV1012 - FLUID MECHANICS

April / May 2019 **Time Allowed: 2½ hours**

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **EIGHT (8)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. An **Appendix** of **ONE (1)** page is attached to the paper.
5. All answers must be written in the answer book provided. Answer each question beginning on a **FRESH** page of the answer book.
6. This is a Closed-Book Examination.

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1. (a) A rigid cylinder contains 20 kg of water as shown in Figure Q1(a). What is the volume of the water at 10° C when its density is 999.7 kg/m^3? What is its volume as its temperature is raised to 50^0 C, when its density drops to 988.0 kg/m^3? What pressure should be applied to the water to maintain it at the same volume as its temperature is raised from 10° C to 50° C? Given the bulk modulus of elasticity of water = 2,150 MPa.

(5 Marks)

Useful formula:

$$\Delta P = -K\frac{\Delta V}{V}$$

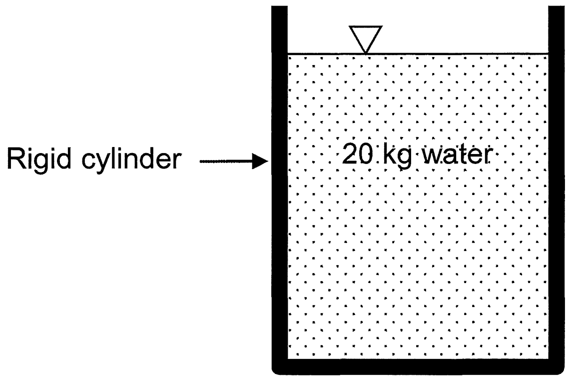


Figure Q1(a)

Note: Question No. 1 continues on page 2.

(b) A 0.3 m cube with a mass of 100 kg slides down a plane inclined at 30° to the horizontal, as shown in Figure Q1(b). The block slides on top of a 0.005 mm oil film. What is the initial acceleration of the block dV/dt when its velocity V = 0? What is its terminal velocity V_T? Given the oil has a dynamic viscosity μ = 0.4 N.s/m^2.

(5 Marks)

Useful formulas:

$$\tau = \mu \frac{dV}{dy}$$

$$F = m.a = m\frac{dV}{dt}$$

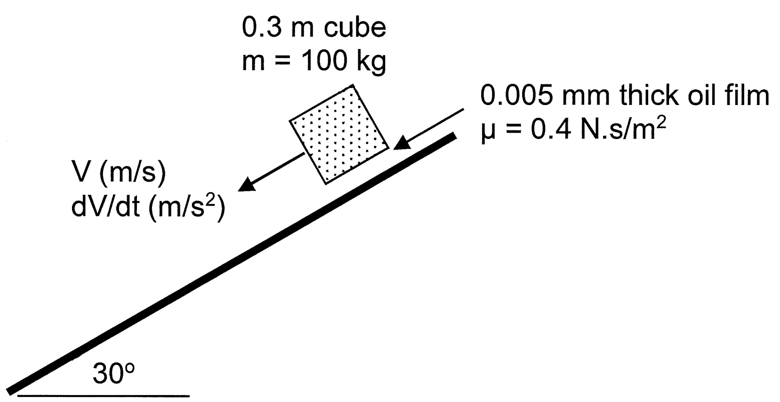


Figure Q1(b)

Note: Question No. 1 continues on page 3.

(c) Water flows from a bigger 0.4 m diameter pipe into a smaller 0.2 m diameter pipe, as shown in Figure Q1(c). For the manometer reading as shown in the figure, what is $P_1 - P_2$ in N/m^2 and the flow rate Q in m^3/s? The pipe axis lies in a horizontal plane. Ignore energy losses in flow.

(5 Marks)

Useful Formula:

$$\Delta P = \rho g \Delta h$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

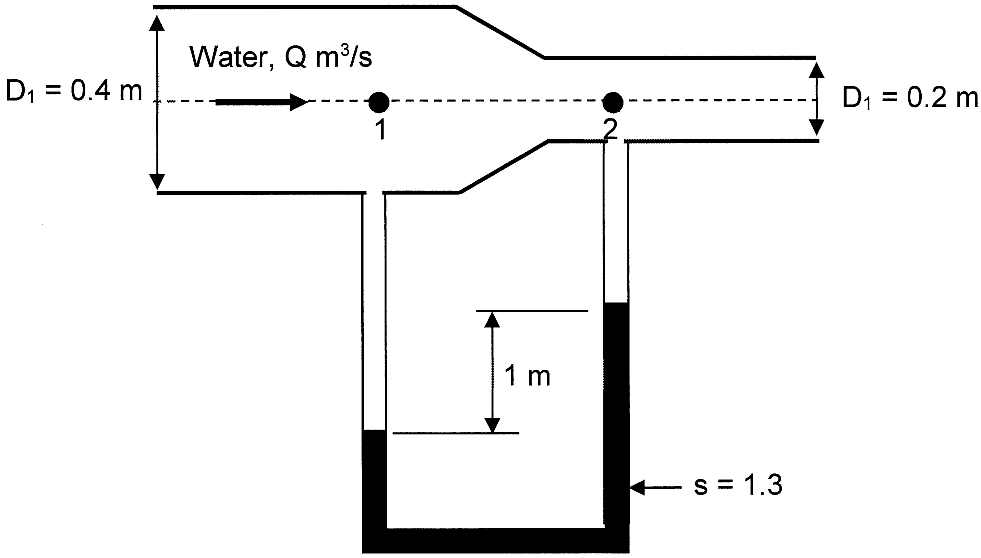


Figure Q1(c)

Note: Question No. 1 continues on page 4.

(d) The vertical Utah shaped plate in Figure Q1(d) is submerged in oil with its top at the free oil surface. What is the resulting hydrostatic thrust acting on one side of the plate, and distance of the resultant thrust below the free surface? (Hint: Subdivide the plate into two rectangles)

(5 Marks)

Useful Formulas:

$F = \rho\, g\, A\, h_c$
$\Delta y = I_c / (A\, y_c)$
$I_c = (b\, h^3)/12$ for a rectangle

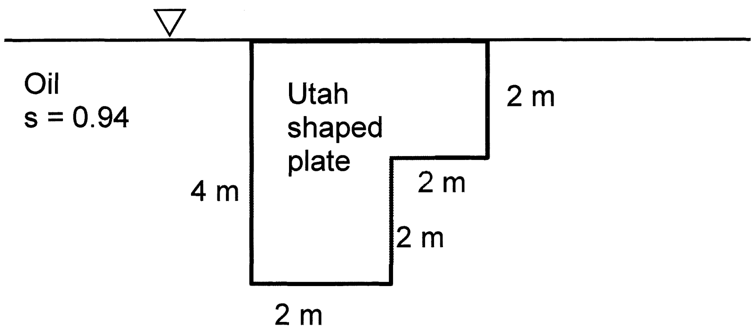


Figure Q1(d)

(e) A rectangular block of uniform material, 1 m x 0.2 m x 1.5 m height, has a weight of 2 kN and floats in sea water as shown in Figure Q1(e). Determine if the block will float in stable equilibrium.

(5 Marks)

Useful Formulas:

$F_b = \rho\, g\, V_i$
$BM = I_{yy}/V_i$
$I_{yy} = (b\, h^3/12)$ for a rectangle

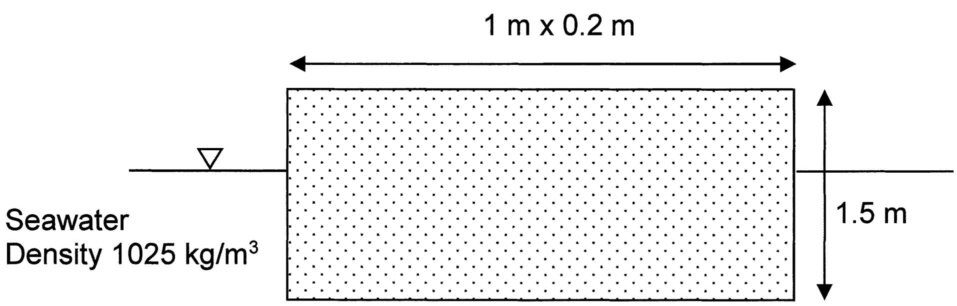


Figure Q1(e)

2. (a) Flow velocities at two points in an open channel are measured using pitot static tubes and a manometer as shown in Figure Q2(a). Compute the stagnation pressures P_{1s} and P_{2s}, and the flow velocities V_1 and V_2. You can assume hydrostatic pressure distribution in the open channel.

(12 Marks)

Useful Formula:

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2$$

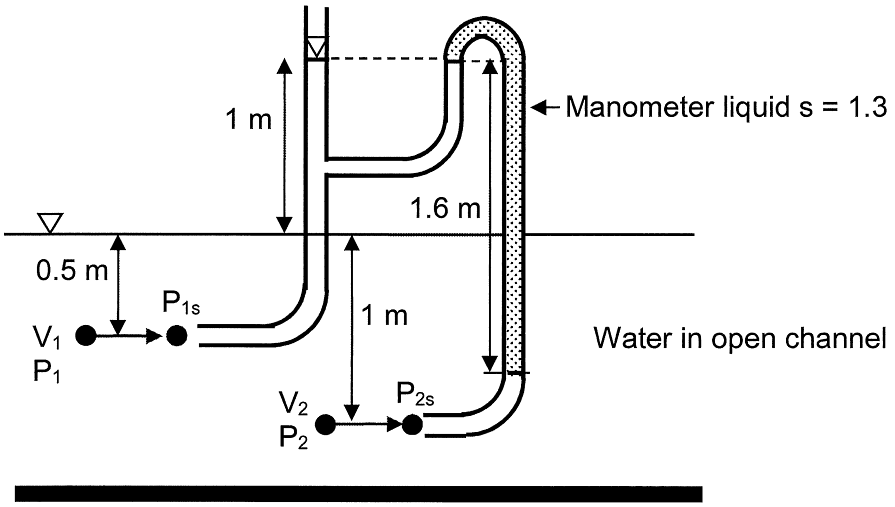


Figure Q2(a)

Note: Question No. 2 continues on page 6.

(b) Water flows out of a pipe through a double nozzle as shown in Figure Q2(b). Determine the resultant hydrodynamic force acting on the nozzle in the x-direction. The velocity of both nozzle jets is 5 m/s. The axes of the pipe and both nozzles lie in a horizontal plane. Ignore energy losses in flow.

(13 Marks)

Useful Formulas:

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2$$

$$F_{net}=\dot{M}_{out}-\dot{M}_{in}$$

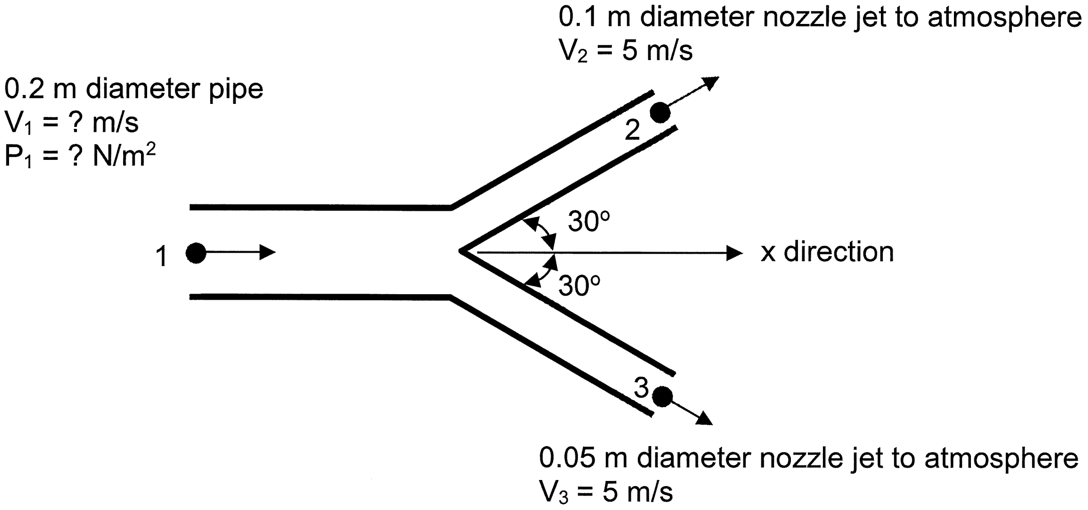


Figure Q2(b)

3. (a) The Darcy-Weisbach friction factor, f for pipe flow depends on the following variables:

$$f = \text{function}\ (\rho, \mu, V, D, \varepsilon)$$

where ρ = fluid density, μ = dynamic viscosity of fluid, V = pipe flow velocity, D = pipe diameter, and ε = pipe roughness.

(i) Using dimensional analysis, derive the appropriate dimensionless Π groups.

(ii) Explain briefly how f varies with the functional relationship you have derived.

(10 Marks)

(b) Figure Q3 shows two reservoirs are connected by an existing old pipeline with a diameter of 0.2 m with a rather rough friction factor, f = 0.04. Due to increased demand for water supply it is decided to increase the flow rate to twice the existing value. The kinematic viscosity of water, $\nu = 1 \times 10^{-6}$ m^2/s.

(i) To increase the flow rate, one option is to construct an additional pipeline parallel to the existing one. If the second pipeline has the same length with a friction factor of 0.01, calculate the diameter of the new pipeline.

(ii) The second option is to abandon the existing pipeline and to install a completely new pipeline with the same length and a friction factor of 0.01. Calculate the new single pipeline diameter.

[Hint: Ignore all minor losses in your computation, and you may assume the head difference, H between the two reservoirs remains unchanged.]

(15 Marks)

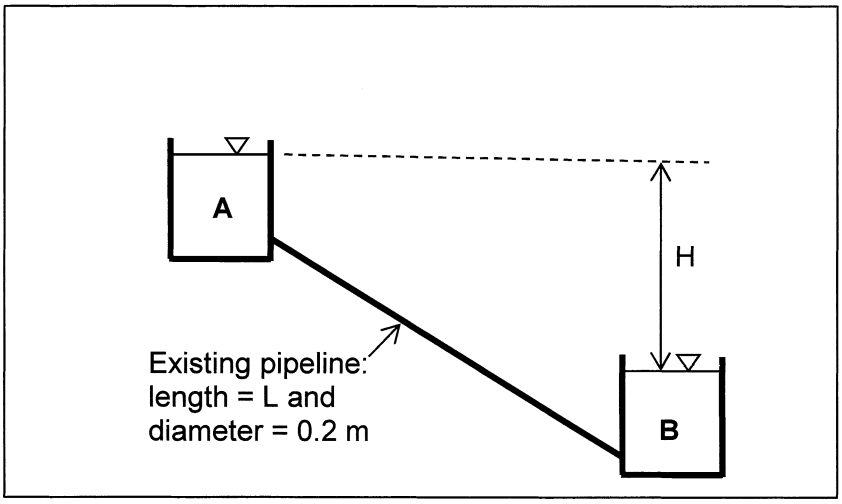


Figure Q3

Useful Formulas:
Moody's diagram is given in the Appendix.

$$h_f = \frac{f\ L\ V^2}{2\ g\ D} = \frac{8\, f\, L\, Q^2}{g\, \pi^2\, D^5}$$

4. (a) For dynamic similarity between model and prototype, show that for flow governed by gravity and inertia force, the velocity scale λ_v (= V_m/V_p) is equal to the length scale, $(\lambda_L)^{0.5}$.

(4 Marks)

(b) Water (density ρ = 1000 kg/m^3, kinematic viscosity, ν = 1 x 10^{-6} m^2/s) flows over a model spillway that is 0.6 m high. The flow rate in the prototype spillway is Q_p = 50 m^3/s.

(i) What is the height of the prototype spillway if the flow in the model is tested at a rate of 0.35 m^3/s?

(ii) On a part of the model, a force of 1.5 N was measured. What is the corresponding force on the prototype spillway if viscosity and surface tension effects are neglected?

(8 Marks)

(c) Oil with a density, ρ_{oil} = 850 kg/m^3 and a kinematic viscosity ν_{oil} = 1.19 x 10^{-4} m^2/s flows downhill at a rate of 0.025 m^3/s through a 0.2 m diameter pipe (pipe roughness, ε = 0.8 mm). A stretch of the pipeline with a length of 300 m is lying on a 1:10 slope.

(i) What is the difference in pressure between the upper and lower ends of the 300 m long pipeline?

(ii) Sketch the Total Energy Line (TEL) and the Hydraulic Grade Line (HGL) for the pipeline. Include in your sketch all the relevant values of the head loss, pressure heads, velocity heads and potential heads.

(13 Marks)

Useful Formulas:

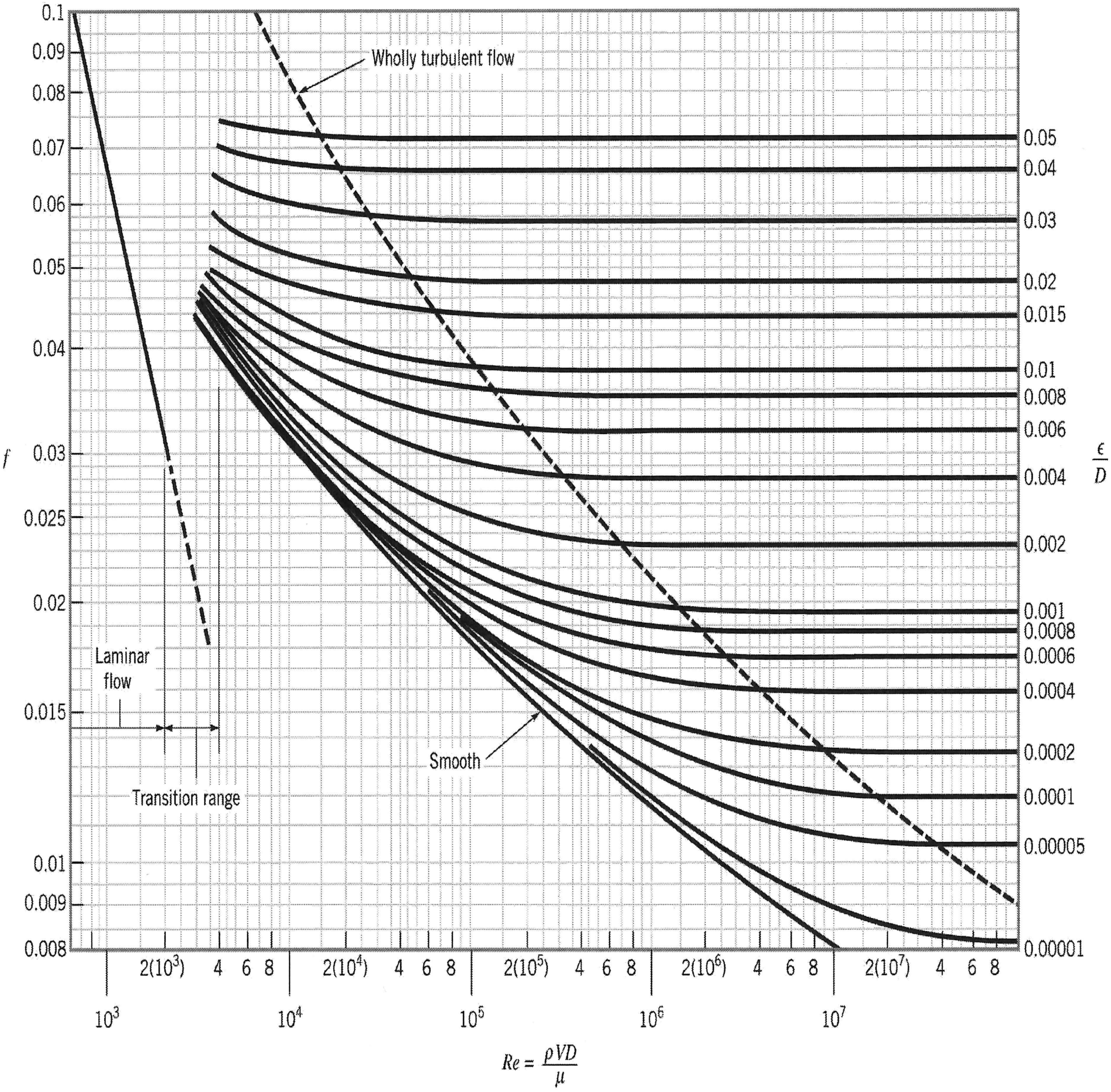
Darcy-Weisbach Eq: $h_f = \frac{f\ L\ V^2}{2\ g\ D} = \frac{8\, f\, L\, Q^2}{g\, \pi^2\, D^5}$

Poiseuille's Eq: $h_f = \frac{32\, \mu\, L\, V}{\rho\, g\, D^2} = \frac{128\, \mu\, L\, Q}{\rho\, g\, \pi\, D^4}$

$f = \frac{64}{Re}$

END OF PAPER

Appendix: Moody Diagram



Moody Diagram

CV1012 FLUID MECHANICS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.