AY 18/19 - S1 - MH2814

NO Hiew Wng Yian

1a) If $p(A \cap B) = 0$

- A and B are mutually exclusive
- $p(A) = 0$ or $p(B) = 0$
- $A \cap B$ may not be an impossible event if the random variable is continuous

b) $p(C \mid A \cap B) = 1$

$$\frac{p(C \cap A \cap B)}{p(A \cap B)} = 1$$

$$p(C \cap A \cap B) = p(A \cap B)$$

probability ≤ 1

$\therefore p(A \cup B) \leq 1$

$p(A \cap B) = p(A) + p(B) - p(A \cup B)$

$\therefore p(A \cap B) \geq p(A) + p(B) - 1$

$p(C) \geq p(A) + p(B) - 1$

c) Given: ~~For~~ x, y, $E(XY) = E(X)E(Y)$

$\sigma_{xy} = \cancel{E(X)E} \; E(XY) - E(X)E(Y) = 0$

$\sigma_{x+y}^2 = \sigma_x^2 + \sigma_y^2 + 2\sigma_{xy}$

$\therefore var(X+Y) = var(X) + var(Y)$

d) $p(X<a) = \int_0^a 2x\,dx = [x^2]_0^a = a^2$

Given: $p(X>a) = p(X<a)$

$p(X>a) + p(X<a) = 1$

$2a^2 = 1$

$a^2 = \frac{1}{2}$

$a = \frac{\sqrt{2}}{2} \; (a>0)$

e) $\sigma_x^2 = 4$

$\sigma_y^2 = 9$

$\rho_{xy} = 0.6$

$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$

$0.6 = \frac{\sigma_{xy}}{\sqrt{(4)(9)}}$

$\sigma_{xy} = \cancel{3.6} \; 3.6$

$\sigma_{3x-2y}^2 = 3^2\sigma_x^2 + (-2)^2\sigma_y^2 + 2(3)(-2)\sigma_{xy}$

$= 28.8$

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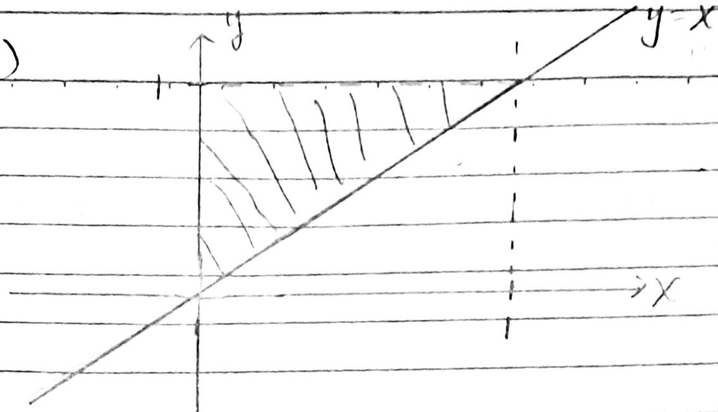
2) a)



b) $g(x)=\int_x^1 \frac{1}{y}\,dy$

$= [\ln y]_x^1$

$= -\ln x$

$g(x)=0$ (elsewhere)

$h(y)=\int_0^y \frac{1}{y}\,dx$

$= \left[\frac{x}{y}\right]_0^y$

$= 1$

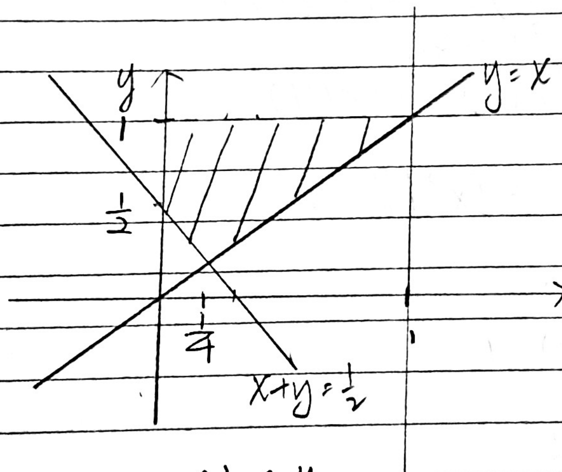
$h(y)=0$ (elsewhere)

c) $f(x,y)=\frac{1}{y}$

$g(x)h(y)=-\ln x$ $\because f(x,y)\neq g(x)h(y)$

$\therefore$ They are not independent

d)



$$P(X+Y>\tfrac{1}{2})=\int_{\frac{1}{4}}^{\frac{1}{2}}\int_{1/2-y}^{y}\frac{1}{y}\,dx\,dy+\int_{1/2}^{1}\int_{0}^{y}\frac{1}{y}\,dx\,dy$$

$$=\int_{1/4}^{1/2}\int_{1/2-y}^{y}\frac{1}{y}\,dx\,dy+\int_{1/2}^{1}\frac{1}{y}(y)\,dy$$

$$=\int_{1/4}^{1/2} 2-\frac{1}{2y}\,dy+\int_{1/2}^{1}1\,dy$$

$$=\left[2y-\tfrac{1}{2}\ln y\right]_{1/4}^{1/2}+[y]_{1/2}^{1}$$

$$=1-\tfrac{1}{2}\ln 2$$

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3a (i) $E(X^2) = 0 + 1^2(0.08 + 0.32 + 2) = 0.6$

$E(Y^2) = (-1)^2(0.07 + 0.08) + 0 + (1)^2(0.15 + \cancel{z}\, 0.2) = 0.5$

$E(X^2Y^2) = (1)^2(-1)^2(0.08) + (1)^2(1)^2(0.2) + 0$

$= 0.28$

(ii) $\sigma_{x^2y^2} = E(X^2Y^2) - E(X^2)E(Y^2)$

$= 0.28 - (0.5)(0.6)$

$= -0.02$

(iii) $E(X) = 0.08 + 0.32 + 0.2$

$= 0.6$

$E(Y) = (-1)(0.07 + 0.08) +$

$1(0.15 + 0.2)$

$= 0.2$

$E(XY) = (1)(-1)(0.08) + (1)(1)(0.2) + 0$

$= 0.12$

$\sigma_{xy} = E(XY) - E(X)E(Y)$

$= 0.12 - (0.6)(0.2)$

$= 0$

$\rho_{xy} = 0$ (They are independent)

b) (i) $p(X=3) = {}^{10}C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^7 = \frac{15}{128}$

$p(X=4) = {}^{10}C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^6 = \frac{105}{512}$

$p(X=5) = {}^{10}C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^5 = \frac{63}{256}$

$p(X=6) = {}^{10}C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^4 = \frac{105}{512}$

$p(3 \le X \le 6) = \frac{99}{128}$

(ii) ~~Normal approximation~~

$\mu = np$

$= (10)(0.5)$

$= 5$

$\sigma = \sqrt{npq}$

$= 1.58$

Apply continuity correction:

$p(3 \le X \le 6) = p(2.5 < X < 6.5)$

$= p\left(\frac{2.5-5}{1.58} < Z < \frac{6.5-5}{1.58}\right)$

$= p(-1.58 < Z < 0.95)$

$= 0.7718$

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4) (i) $\bar{X}_A \sim N(\mu, \frac{100}{9})$

(ii) $\bar{X}_B \sim N(\mu, \frac{100}{16})$

(iii) $\bar{X}_A - \bar{X}_B \sim N(0, 100[\frac{25}{144}]))$

(iv) $p(\bar{X}_A - \bar{X}_B > 2) = p(Z > \frac{2-0}{\sqrt{\frac{100}{9}+\frac{100}{16}}})$

$= 0.3156$

b) $n=50$
$\bar{x}=82$
$z=1.96$

$82-1.96(\frac{10}{\sqrt{50}}) < \mu_C < 82+1.96(\frac{10}{\sqrt{50}})$

$79.23 < \mu_C < 84.77$

c) $|\bar{x}-\mu| < z(\frac{\sigma}{\sqrt{n}})$

$n > (\frac{z_{\alpha/2} \cdot \sigma}{\varepsilon})^2$

$n > 42.7$

$n = 43$

(roundoff to the larger integer)

5) a) $H_0: \mu=70$

b) $H_1: \mu \neq 70$

c) $\alpha = 0.05$

d) $z = \frac{67-70}{12/\sqrt{50}} = -1.77$

p-value $= p(|z| > 1.77)$
$= 0.0384 \times 2$
$= 0.0768$ (>0.05)

e) $\because$ p-value $> \alpha$

$\therefore$ There is Not enough evidence to reject the claim

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f) $H_0: \mu \geq 70$

$H_1: \mu < 70$

To find p-value

$S = 12$ $n = 50$

$\bar{x} = 67$

$z = \frac{67-70}{12/\sqrt{50}} = -1.77$

$p(|z| > 1.77) = 0.0384$

$\because$ p-value $< \alpha$

$\therefore$ There is enough evidence to reject $H_0: \mu \geq 70$

~~$\therefore$ The claim is~~

6) a) $S_{xx} = \sum x^2 - \left(\frac{\bar{x}}{n}\right)^2 n$

$= 46000 - \left(\frac{1200}{40}\right)^2 40$

$= 10000$

$S_{xy} = \sum xy - \frac{\sum x \sum y}{n}$

$= 180000 - \frac{(1200)(5000)}{40}$

$= 30000$

(b) $\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}}$

$= 3$

$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$

$= \frac{5000}{40} - 3 \times \frac{1200}{40}$

$= 35$

$E(y) = 35 + 3x$

c) When $x = 35$, $E(y) = 35 + 3(35)$

$= 140$

$S_{yy} = \sum y^2 - \frac{1}{n}\left[\sum y\right]^2$

$= 725000 - \frac{5000^2}{40}$

$= 100000$

$S^2 = \frac{S_{yy} - \hat{\beta}_1 \cdot S_{xy}}{n-2}$

$= \frac{100000 - 3 \times 30000}{38}$

$= 263.16$

$S = 16.22$

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$$E(y_0) \pm Z_{\frac{\alpha}{2}} \cdot s \cdot \sqrt{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}}$$

$$= 140 \pm 1.96 \times 16.22 \times \sqrt{\frac{1}{40} + \frac{(3530)^2}{10000}}$$

$$= (134.7, 145.3)$$

Wng Yian