CV1012 Semester 2 2017/2018 Examination

1. (a) A prism is submerged in water, which comprises a horizontal face (ABFE), three vertical faces (ABCD, BCF, ADE) and an inclined face (CDEF). The magnitudes of the average pressures acting on the different faces have the following relation:

(A) $p_{ABFE} > p_{BCF} = p_{ABCD} = p_{CDEF}$.
(B) $p_{ABFE} < p_{BCF} < p_{ABCD} < p_{CDEF}$.
(C) $p_{ABFE} < p_{BCF} = p_{ABCD} = p_{CDEF}$.
(D) $p_{ABFE} < p_{BCF} < p_{ABCD} = p_{CDEF}$.

(5 Marks)

In this case, p depends only on h_c. Therefore, comparing h_c of each surface, we get (D) as the answer.

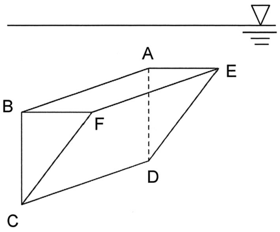


Figure Q1(a)

(b) A large plate is moving between two large fixed plates, which contain a Newtonian fluid. The velocity distributions above and below the moving plate are all linear. The gap b_1 is greater than the gap b_2. The relationship among the magnitudes of the shear stresses exerting on the plate surface at the four locations can be expressed as

(A) $\tau_1 > \tau_2 > \tau_3 > \tau_4$.
(B) $\tau_1 = \tau_2 > \tau_3 = \tau_4$.
(C) $\tau_1 < \tau_2 < \tau_3 < \tau_4$.
(D) $\tau_1 = \tau_2 < \tau_3 = \tau_4$.

(5 Marks)

$\tau = \mu \frac{V}{b}$, so as y increases, τ decreases

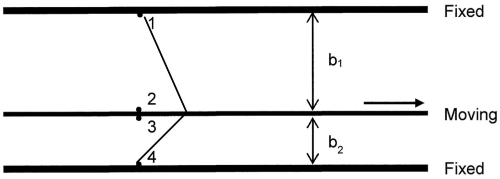


Figure Q1(b)

(c) Three half-ball cups are positioned at the water surface in a water tank, with the opening upwards. Two of them are filled with water and oil, respectively, and the other is empty. All free surfaces of the fluids are of the same level. The wall thickness of each cup is very small. The oil density is less than the water density. The magnitudes of the fluid-induced pressure forces exerting on the inner and external wall are denoted by F_{in} and F_{out}, respectively. Which of the following statements is correct?

(A) $F_{out,1} = F_{out,2} = F_{out,3} = F_{in,1} < F_{in,2} < F_{in,3}$.
(B) $F_{out,1} = F_{out,2} = F_{out,3} = F_{in,1} > F_{in,2} > F_{in,3}$.
(C) $F_{out,1} > F_{out,2} > F_{out,3}$ and $F_{in,1} > F_{in,2} > F_{in,3}$.
(D) $F_{out,1} = F_{out,2} = F_{out,3}$ and $F_{in,1} < F_{in,2} < F_{in,3}$.

(5 Marks)

The pressure on the bottom of each cup is constant (same depth below the surface). Since pressure is equal, F is also equal.

F exerted by equal volumes of liquid depends on its density. Since $\rho_{water} > \rho_{oil} > \rho_{air}$, $F_{water} > F_{oil} > F_{air}$

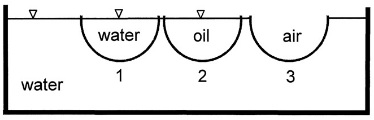


Figure Q1(c)

(d) A manometer is used to measure the pressure difference between point A and point B in a closed pipe. Three elevation differences, a, b and c, have the following possible relation:

(A) a > b > c.
(B) c > a > b.
(C) a = c > b.
(D) a + b = c.

(5 Marks)

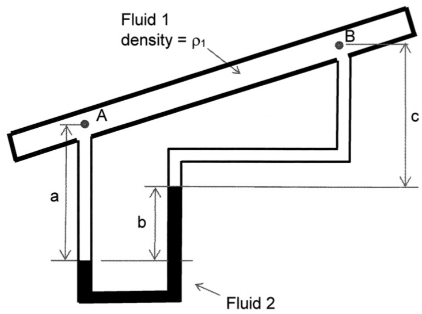


Figure Q1(d)

$P_B + \rho_1 g(c+b-a) = P_A$

$P_A + \rho_1 g a - \rho_2 g b - \rho_1 g c = P_B$

$\cancel{\rho_1}\cancel{g}(a-c) - \frac{\rho_2}{\rho_1}\cancel{g} b = -\cancel{\rho_1}\cancel{g}(c+b-a)$

$-(a-c) + \frac{\rho_2}{\rho_1} b = c+b-a$

$\rho_2 b = \rho_1 b$

Since $\rho_2 \neq \rho_1$, thus $b=0$

$P_B + \rho_1 g(c-a) = P_A$

Since $P_A > P_B$

$P_A - P_B > 0$

$c - a > 0$

$c > a$

(e) A Venturi Meter is useful for measuring the flow rate in a flow system. The flow rate can be calculated based on

(A) the continuity equation only.
(B) the energy equation only.
(C) the continuity equation and energy equation.
(D) the continuity equation, energy equation and momentum equation.

(5 Marks)

② (a) Bernoulli Eq. between (1) and (4)

$$0 + \frac{V_4^2}{2g} + 0 = 0 + 0 + 2h \quad \text{where} \quad V_3 = V_4 \text{ (Continuity eq.)}$$

$$2h = \frac{V_3^2}{2g}$$

Bernoulli's eq. between (1) and (3)

$$\frac{P_3}{\gamma} + \frac{V_3^2}{2g} + h = 0$$

$$\frac{P_3}{\gamma} = -2h - h = -3h$$

Bernoulli's eq. between (1) and (2)

$$0 + 0 + h = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + 0 \quad \text{where} \quad \frac{P_2}{\gamma} = \frac{P_3}{\gamma}$$

$$h = -3h + \frac{V_2^2}{2g}$$

$$4h = \frac{1}{2g}\left(\frac{D}{d}\right)^4 V_3^2$$

$$4\cancel{h} = 2\cancel{h}\left(\frac{D}{d}\right)^4$$

$$\left(\frac{D}{d}\right)^4 = 2$$

$$\frac{D}{d} = 1.189 \;//$$

Continuity eq. between (2) and (3)

$$\cancel{\frac{\pi}{4}} d^2 V_2 = \cancel{\frac{\pi}{4}} D^2 V_3$$

$$V_2 = \frac{D^2}{d^2} V_3$$

② (b) $0 + 0 + h = 0 + \frac{V_1^2}{2g} + 0$

$$\frac{V_1^2}{2g} = h$$

$$V_1^2 = 2gh$$

$$0 + 0 + 2h = 0 + \frac{V_2^2}{2g} + 0$$

$$\frac{V^2}{2g} = 2h$$

$$V^2 = 4gh$$

For tank to remain stationary, $\Sigma F = 0$

$$\Sigma F = [\rho Q V_2 - \rho Q V_1] - 0 = 0$$

$$\cancel{\rho} Q_2 V_2 = \cancel{\rho} Q_1 V_1$$

$$\cancel{\frac{\pi}{4}} d_2^2 V_2^2 = \cancel{\frac{\pi}{4}} d_1^2 V_1^2$$

$$d_2^2 (4g\cancel{h}) = d_1^2 (2g\cancel{h})$$

$$\left(\frac{d_1}{d_2}\right)^2 = 2$$

$$\frac{d_1}{d_2} = 1.414 \;//$$

③ (a) (i) $\nu_m = \nu_p$ (same fluid)

$$\frac{V_m L_m}{\cancel{\nu_m}} = \frac{V_p L_p}{\cancel{\nu_p}}$$

$$\frac{V_m}{V_p} = \frac{L_p}{L_m} \;// \quad \text{(PROVEN)}$$

(ii) $$\frac{V_m}{\sqrt{g L_m}} = \frac{V_p}{\sqrt{g L_p}}$$

$$\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}} \;// \quad \text{(PROVEN)}$$

(iii) For the model to satisfy both the Reynold number and Freude number's similarity, we use the viscosity scale obtained from Froude's number in the fluid kinematic viscosity scale from Reynold number's similarity

$$\frac{\nu_m}{\nu_p} = \frac{V_m L_m}{V_p L_p}$$

$$\frac{\nu_m}{\nu_p} = \left(\frac{L_m}{L_p}\right)^{3/2} \;//$$

③ (b) $n=6 \quad m=3$

$n-m=3$

$\pi_1 = \tau_w \rho^a V^b D^c$

$M^0L^0T^0 = [ML^{-1}T^{-2}][ML^{-3}]^a[LT^{-1}]^b[L]^c$

$0=1+a \quad 0=-2-b \quad 0=-1-3a+b+c$

$a=-1 \quad b=-2 \quad c=0$

$\therefore \pi_1 = \dfrac{\tau_w}{\rho V^2}$

$\therefore \pi_2 = \dfrac{\rho V D}{\mu}$

$\therefore \pi_3 = \dfrac{\varepsilon}{D}$

$\therefore \dfrac{\tau_w}{\rho V^2} = f_1\left(\dfrac{\rho V d}{\mu}, \dfrac{\varepsilon}{D}\right)$ // (SHOWN)

$\pi = \dfrac{\tau_w}{\rho V^2}$

$\pi = \sqrt{\dfrac{\tau_w}{\rho}} \cdot \dfrac{1}{V}$

$\pi = \dfrac{V}{u_*}$ => V and u_* has the same dimensions

Therefore, $\dfrac{\rho V d}{\mu}$ can be written down as $\dfrac{\rho u_* D}{\mu}$

$\therefore \dfrac{V}{u_*} = f_2\left(\dfrac{\rho u_* D}{\mu}, \dfrac{\varepsilon}{D}\right)$ // (SHOWN)

③ (c) (i) $\dfrac{\varepsilon}{D} = \dfrac{8\times10^{-5}}{0.1} = 0.0008$

$Re = \dfrac{2.5\times0.1}{10^{-6}} = 250000$

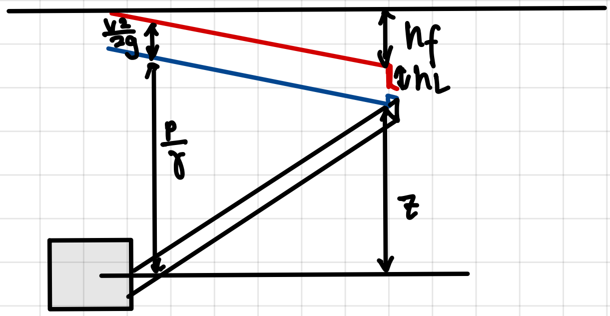
From M.D. $f=0.02$

$\dfrac{P_h}{\gamma} = \dfrac{V^2}{2g} + 50 + h_f + h_L$

$\dfrac{P_h}{\gamma} = \dfrac{40^2}{2\times9.81} + 50 + \dfrac{(0.02)(60)(2.5)^2}{2\times9.81\times0.1} + 0.04\left(\dfrac{40^2}{2\times9.81}\right)$

$P_h = 1.36$ MPa //

(ii)



④ (a) 1. $Re < 2100$, in laminar flow regime, f depends on Re only

2. $2100 < Re < 4000$, in transition range, f is uncertain

3. $Re > 4000$, but not in wholly turbulent flow regime, f depends on Re and $\frac{\varepsilon}{D}$

4. Re very large, in wholly turbulent flow regime, f depends on $\frac{\varepsilon}{D}$ only

④ (b) $Q = \frac{\pi}{4} d^2 V$

$V = \frac{4Q}{\pi d^2}$

$Re = \frac{Vd}{\nu} = \frac{4Q}{\pi d^2} \cdot \frac{d}{\nu} = \frac{4Q}{\pi d \nu}$

$13.22 = \frac{f(1000)(8 \times 0.5^2)}{\pi^2 g d^5}$

$13.22 = 20.657 \frac{f}{d^5}$

Assume $f_1 = 0.02$

$d^5 = \frac{20.657 \times 0.02}{13.22}$

$d = 0.5$ m

$\frac{\varepsilon}{D} = 0.0011$ m

$Re = \frac{4 \times 0.5}{\pi \times 0.5 \times 10^{-6}} = 1.27 \times 10^6$

From M.D. $f_2 = 0.0195$

$d^5 = \frac{20.657 \times 0.0195}{13.22}$

$d = 0.497$

$\frac{\varepsilon}{D} = 0.0011$

$Re = 1.28 \times 10^{-6}$

From MD. $f_3 = 0.0195 = f_2$

$\therefore d = 0.497$ m //

④ (c) (i) $Q_2 + Q_3 + Q_4 = 0.8$ m^3/s

$h_{f_2} = h_{f_3} = h_{f_4}$

$K_2 Q_2^2 = k_3 Q_3^2 = K_4 Q_4^2$

$300 Q_2^2 = 600 Q_3^2 = 150 Q_4^2$

$Q_3^2 = \frac{1}{2} Q_2^2$ $\quad$ $Q_4^2 = 2 Q_2^2$

$Q_3 = \frac{1}{\sqrt{2}} Q_2$ $\quad$ $Q_4 = \sqrt{2} Q_2$

$\rightarrow Q_2 + \frac{1}{\sqrt{2}} Q_2 + \sqrt{2} Q_2 = 0.8$

$Q_2 = 0.256$ m^3/s //

$Q_3 = 0.181$ m^3/s //

$Q_4 = 0.363$ m^3/s //

(ii) $Q_{new} = 0.8$ m^3/s $\quad$ $Q = \frac{\pi}{4} d^2 V$

$h_{f_2} = \frac{f L 8 Q^2}{g \pi^2 d^5}$

$300(0.256)^2 = (0.02) \frac{(800)(8 Q_n^2)}{g d^5 \pi^2}$

$d^5 = 0.0428$

$d = 0.532$ m //