

1. (a)

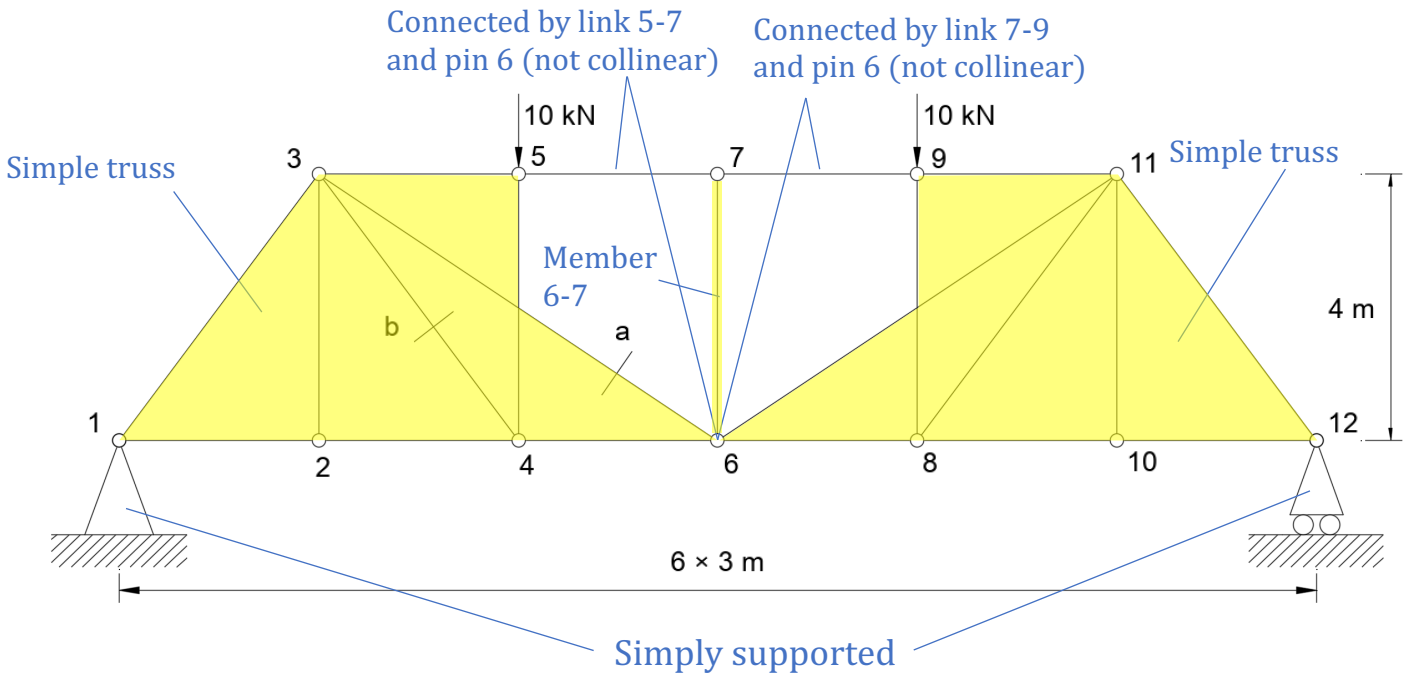
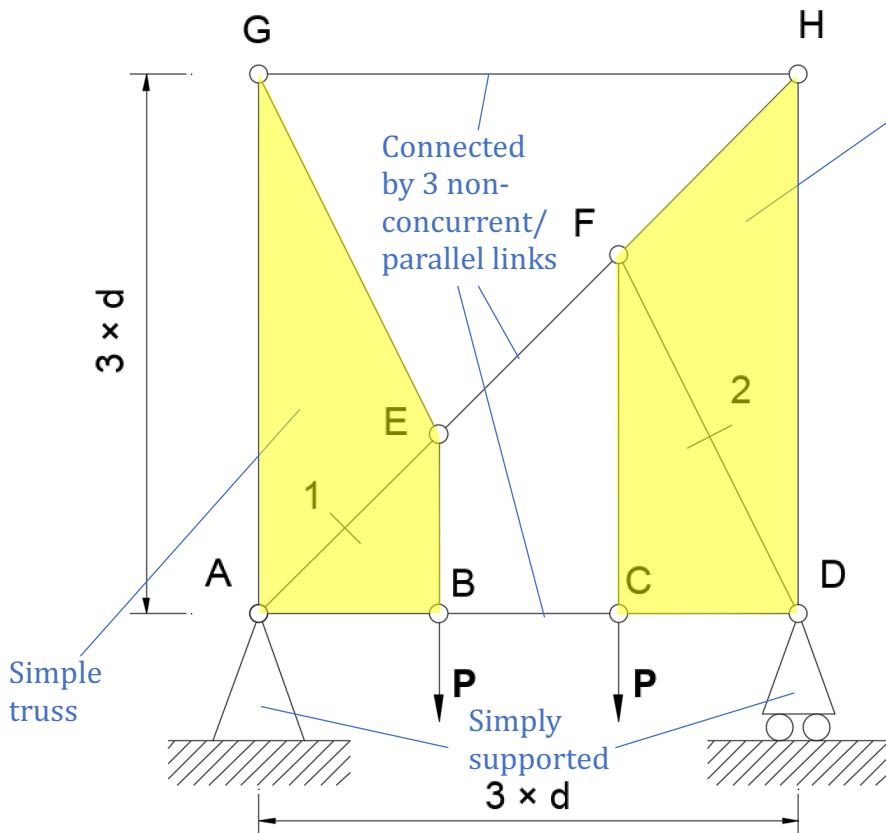


Figure Q1(a)

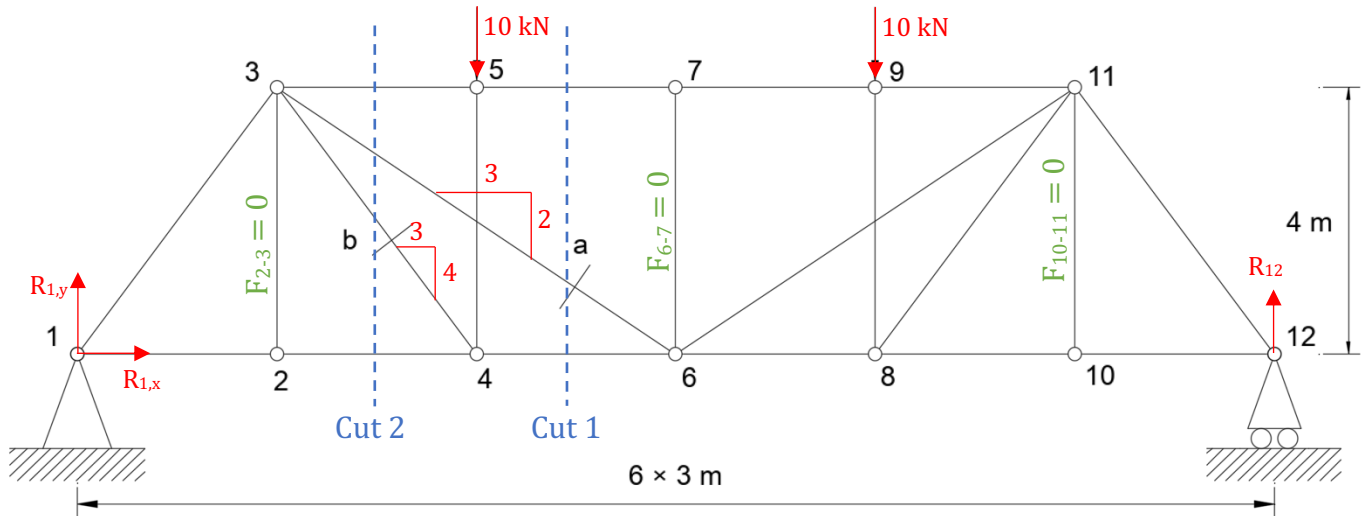
Joints: 12, Members: 21, Reactions: 3,  $2(12) = 21 + 3$ . No concurrent/parallel reactions, no collapse mechanism. Overall, statically determinate, stable. **Ans.**



Joints: 8, Members: 13, Reactions: 3,  $2(8) = 13 + 3$ . No concurrent/parallel reactions, no collapse mechanism. Overall, statically determinate, stable. **Ans.**

Figure Q1(b)

1. (b) Convert diagonal forces into x and y directions



Solving reactions:

$\rightarrow +, \Sigma F_x = 0:$

$R_{1,x} = 0 \text{ kN}$

By symmetry and  $\uparrow +, \Sigma F_y = 0:$

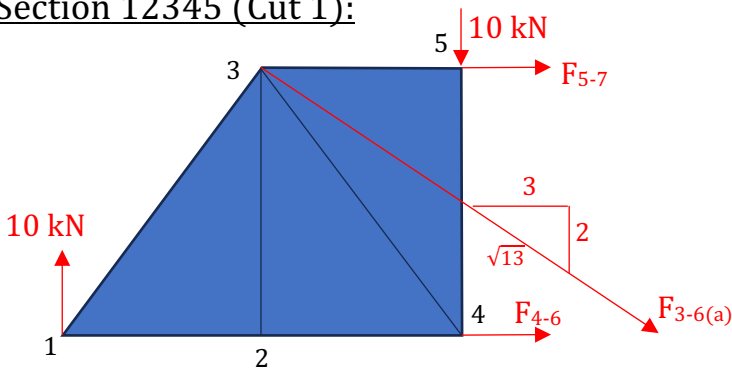
$R_{1,y} = R_{12}$

$R_{1,y} + R_{12} - 10 - 10 = 0$

$R_{1,y} = 10 \text{ kN (upwards)}$

$R_{12} = 10 \text{ kN (upwards)}$

Section 12345 (Cut 1):

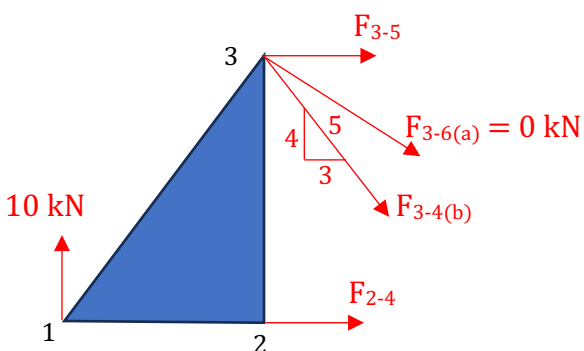


$\uparrow +, \Sigma F_y = 0:$

$10 - 10 - \frac{2}{\sqrt{13}} F_{3-6(a)} = 0$

$F_{3-6(a)} = 0 \text{ kN Ans.}$

Section 123 (Cut 2):

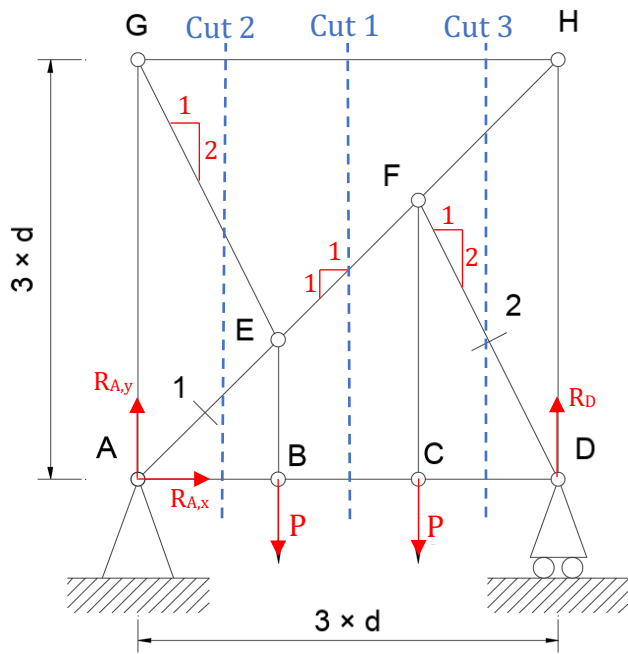


$\uparrow +, \Sigma F_y = 0:$

$10 - \frac{4}{5} F_{3-4(b)} = 0$

$F_{3-4(b)} = 12.5 \text{ kN (tension) Ans.}$

1. (c) Convert diagonal forces into x and y directions



Solving reactions:

$\rightarrow +, \Sigma F_x = 0:$

$R_{A,x} = 0 \text{ kN}$

By symmetry and  $\uparrow +, \Sigma F_y = 0:$

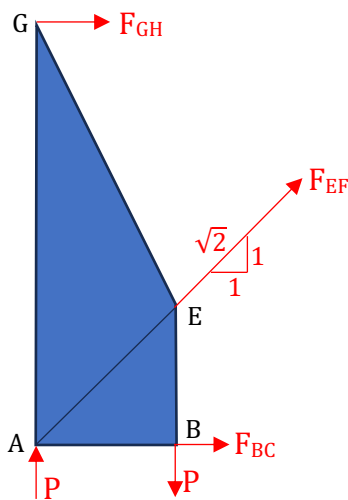
$R_{A,y} = R_D$

$R_{A,y} + R_D - P - P = 0$

$R_{A,y} = P \text{ (upwards)}$

$R_D = P \text{ (upwards)}$

Section ABEG (Cut 1):

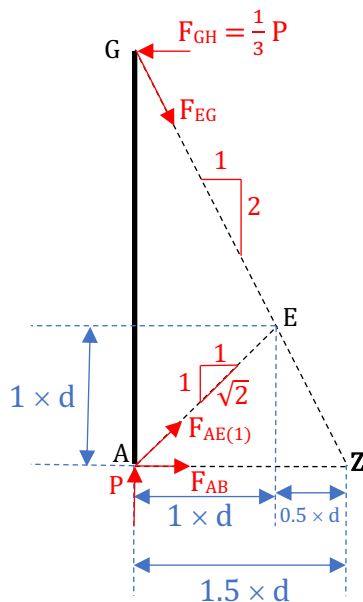


$\odot +, \Sigma M_A = 0:$

$- F_{GH} * (3d) - P * (d) = 0$

$F_{GH} = -\frac{1}{3} P \text{ (compression)}$

Section AG (Cut 2):



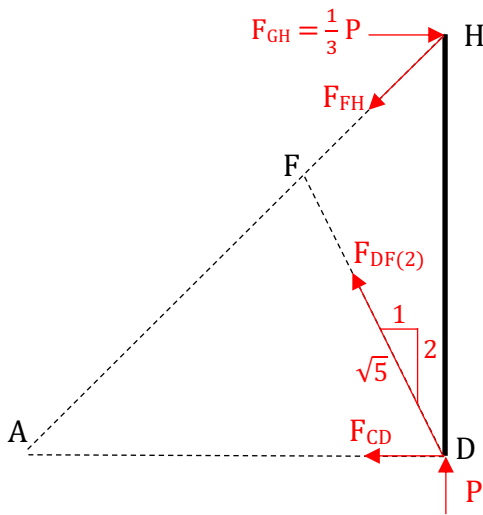
$\odot +, \Sigma M_z = 0:$

$\frac{1}{3} P * (3d) - P * (1.5d) - \frac{1}{\sqrt{2}} F_{AE} * (1.5d) = 0$

$F_{AE(1)} = -\frac{\sqrt{2}}{3} P \text{ (compression) **Ans.**}$

1. (c)

Section DH (Cut 3):

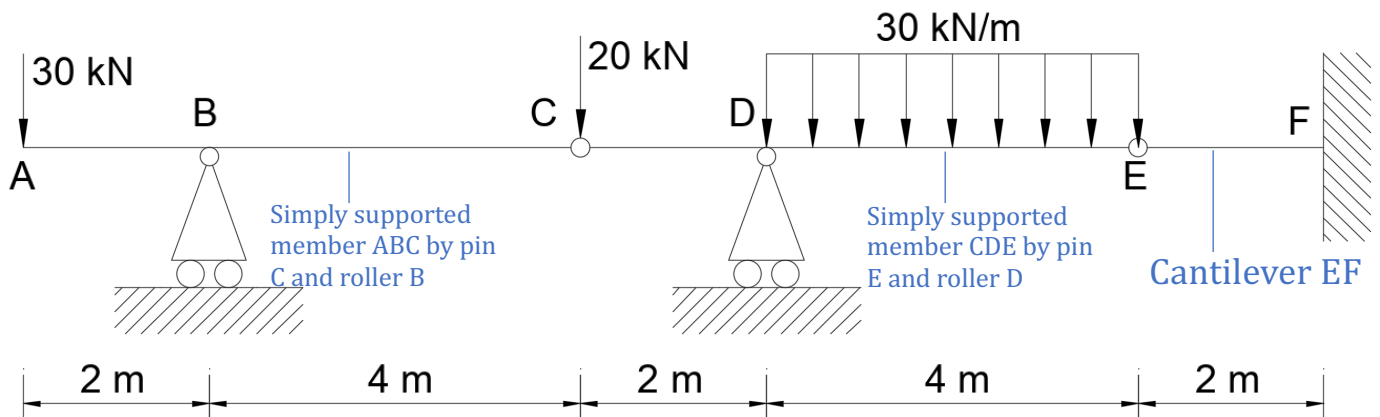


$$\odot+, \Sigma M_A = 0:$$

$$-\frac{1}{3}P(3d) + P(3d) + \frac{2}{\sqrt{5}}F_{DF}(2) = 0$$

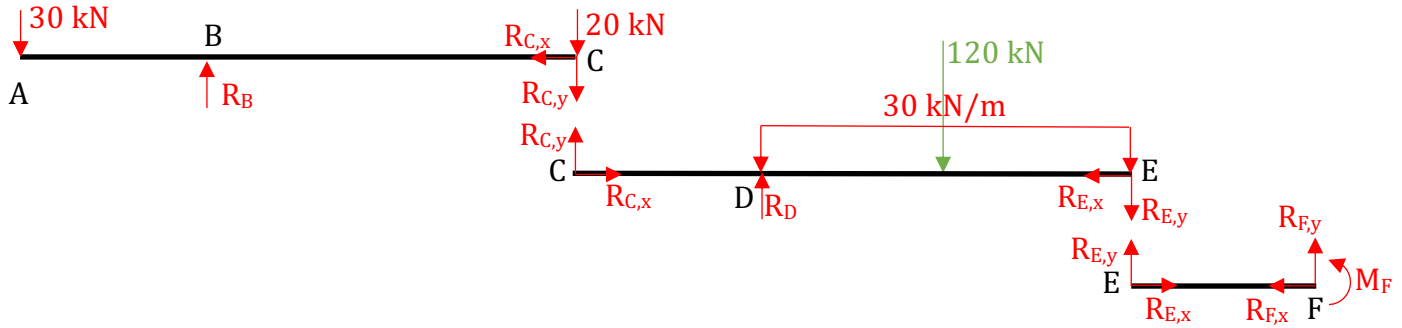
$$F_{DF(2)} = -\frac{\sqrt{5}}{3}P \text{ (compression) } \underline{\text{Ans.}}$$

2. (a)



Members: 3, Reactions:  $3(\text{fixed}) + 1 \cdot 2(2 \text{ rollers}) + 2 \cdot 2(2 \text{ hinge}) = 9$ ,  $3(3) = 9$ . No concurrent/parallel reactions, no collapse mechanism. Overall, statically determinate, stable. Ans.

2. (b)



Member ABC:

$$\odot+, \Sigma M_C = 0:$$

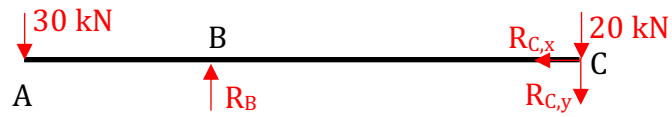
$$30 \cdot 6 - R_B \cdot 4 = 0$$

$$R_B = 45 \text{ kN (upwards) \underline{Ans.}}$$

$$\odot+, \Sigma M_B = 0:$$

$$30 \cdot 2 - R_{C,y} \cdot 4 - 20 \cdot 4 = 0$$

$$R_{C,y} = -5 \text{ kN (opposite direction)}$$



$$\rightarrow+, \Sigma F_x = 0:$$

$$R_{C,x} = 0 \text{ kN}$$

Member CDE:

$$\odot+, \Sigma M_E = 0:$$

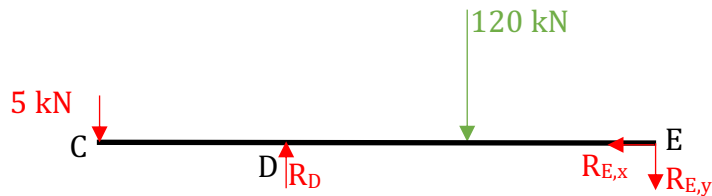
$$5 \cdot 6 + 120 \cdot 2 - R_D \cdot 4 = 0$$

$$R_D = 67.5 \text{ kN (upwards) \underline{Ans.}}$$

$$\odot+, \Sigma M_D = 0:$$

$$5 \cdot 2 - 120 \cdot 2 - R_{E,y} \cdot 4 = 0$$

$$R_{E,y} = -57.5 \text{ kN (opposite direction)}$$



$$\rightarrow+, \Sigma F_x = 0:$$

$$R_{E,x} = 0 \text{ kN}$$

Member EF:

$$\uparrow+, \Sigma F_y = 0:$$

$$-57.5 + R_{F,y} = 0$$

$$R_{F,y} = 57.5 \text{ kN (upwards) \underline{Ans.}}$$

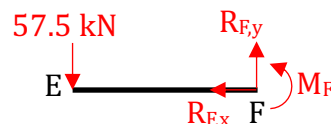
$$\rightarrow+, \Sigma F_x = 0:$$

$$R_{F,x} = 0 \text{ kN \underline{Ans.}}$$

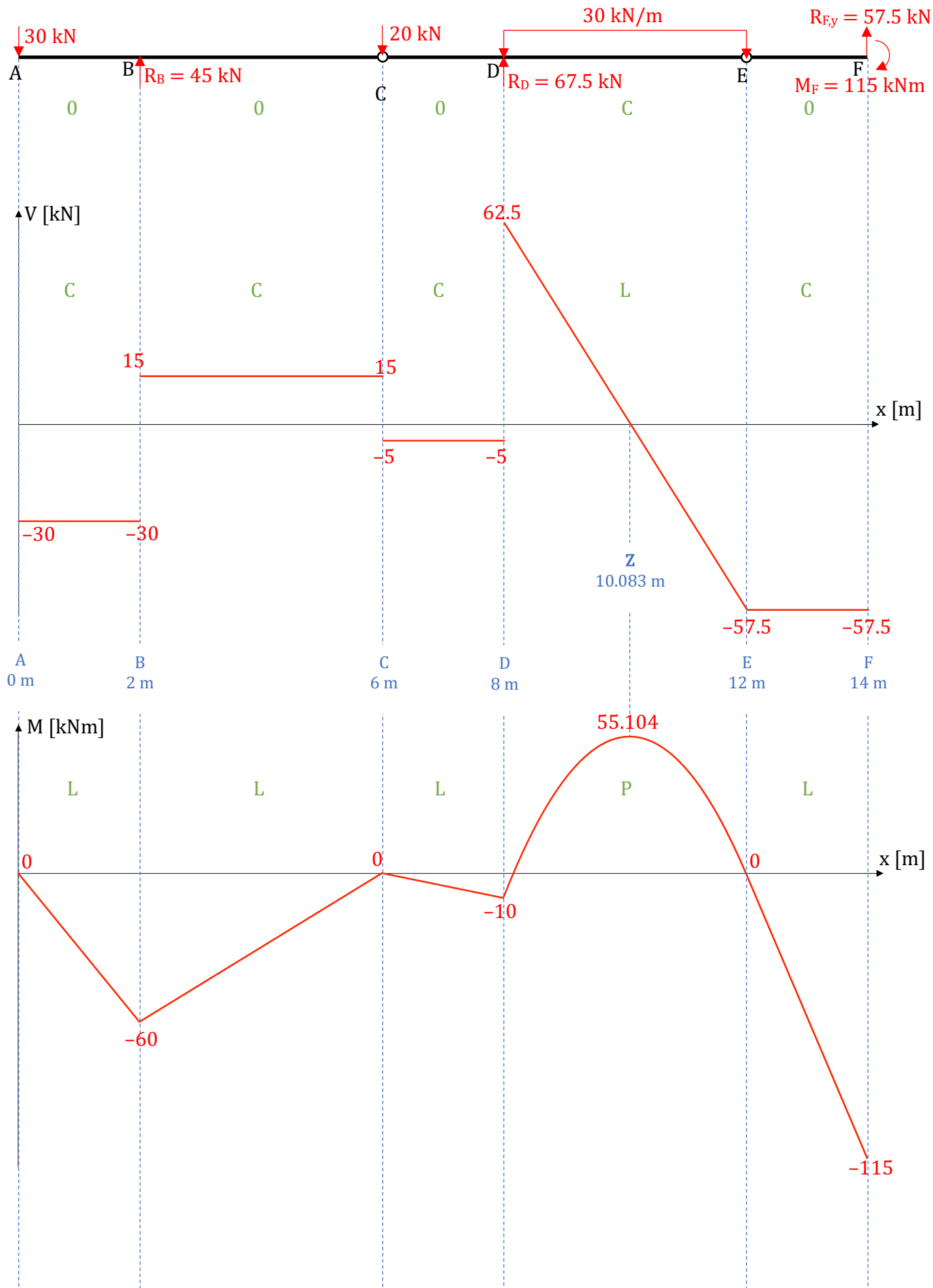
$$\odot+, \Sigma M_F = 0:$$

$$57.5 \cdot 2 + M_F = 0$$

$$M_F = -115 \text{ kNm (clockwise) \underline{Ans.}}$$



2. (c) **Ans.** 0 – zero, C – constant, L – linear(sloped), P – Parabola( $x^2$ ) 



## 2. (c) Calculations

### Shear force:

$$A: -30\text{kN}$$

$$B: -30\text{kN} + 45\text{kN} = 15\text{kN}$$

$$C: 15\text{kN} - 20\text{kN} = -5\text{kN}$$

$$D: -5\text{kN} + 67.5\text{kN} = 62.5\text{kN}$$

$$E: 62.5\text{kN} + \int_8^{12} w \, dx = 62.5\text{kN} + (-30\text{kN/m} * 4\text{m}) = -57.5\text{kN}$$

$$F: -57.5\text{kN} (R_{Fy})$$

### Bending moment:

$$A: 0\text{kNm}$$

$$B: 0\text{kNm} + \int_0^2 V \, dx = 0\text{kNm} + (-30\text{kN} * 2\text{m}) = -60\text{kNm}$$

$$C: -60\text{kNm} + \int_2^6 V \, dx = -60\text{kNm} + (15\text{kN} * 4\text{m}) = 0\text{kNm} \text{ (hinge)}$$

$$D: 0\text{kNm} + \int_6^8 V \, dx = 0\text{kNm} + (-5\text{kN} * 2\text{m}) = -10\text{kNm}$$

$$E: -10\text{kNm} + \int_8^{12} V \, dx = 0\text{kNm} \text{ (hinge)}$$

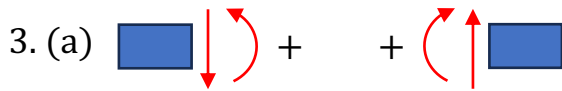
$$F: 0\text{kNm} + \int_{12}^{14} V \, dx = 0\text{kNm} + (-57.5\text{kN} * 2\text{m}) = -115\text{kNm} (M_F)$$

### Point Z:

$$\text{Distance: } 62.5\text{kN} - 30\text{kN/m} * d = 0\text{kN}$$

$$d = 2.083\text{m}, x = 8\text{m} + d = 10.083\text{m}$$

$$\begin{aligned} \text{Bending moment: } & -10\text{kNm} + \int_8^{10.083} V \, dx = -10\text{kNm} + (0.5 * 62.5\text{kN} * 2.083\text{m}) \\ & = 55.104\text{kNm} \end{aligned}$$



Solving reactions:

$$\uparrow+, \Sigma F_y = 0:$$

$$-15 + R_{A,y} = 0$$

$$R_{A,y} = 15 \text{ kN (upwards)}$$

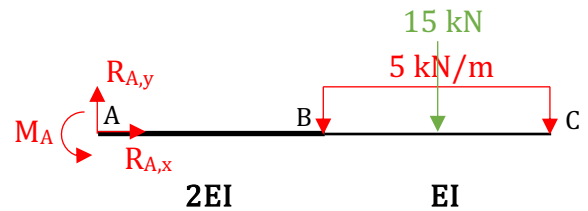
$$\rightarrow+, \Sigma F_x = 0:$$

$$R_{A,x} = 0 \text{ kN}$$

$$\odot+, \Sigma M_A = 0:$$

$$M_A - 15 \cdot 4.5 = 0$$

$$M_A = 67.5 \text{ kNm (anti-clockwise)}$$



Integration:

$$0 \leq x \leq 3:$$

$$w_1(x) = 0$$

$$V_1(x) = A_1$$

$$V_1(0) = A_1 = 15$$

$$A_1 = 15$$

$$V_1(x) = 15$$

$$M_1(x) = 15x + B_1$$

$$M_1(0) = 15(0) + B_1 = -67.5$$

$$B_1 = -67.5$$

$$M_1(x) = 15x - 67.5$$

$$2EI v_1'(x) = \frac{15}{2}x^2 - 67.5x + C_1$$

$$2EI v_1(x) = \frac{15}{6}x^3 - \frac{67.5}{2}x^2 + C_1x + D_1$$

$$3 \leq x \leq 6:$$

$$w_2(x) = -5$$

$$V_2(x) = -5x + A_2$$

$$V_2(6) = -5(6) + A_2 = 0$$

$$A_2 = 30$$

$$V_2(x) = -5x + 30$$

$$M_2(x) = -\frac{5}{2}x^2 + 30x + B_2$$

$$M_2(6) = -\frac{5}{2}(6)^2 + 30(6) + B_2 = 0$$

$$B_2 = -90$$

$$M_2(x) = -\frac{5}{2}x^2 + 30x - 90$$

$$EI v_2'(x) = -\frac{5}{6}x^3 + \frac{30}{2}x^2 - 90x + C_2$$

$$EI v_2(x) = -\frac{5}{24}x^4 + \frac{30}{6}x^3 - \frac{90}{2}x^2 + C_2x + D_2$$



3. (a)

Boundary conditions:

$$v_1'(0) = 0:$$

$$\frac{15}{2}(0)^2 - 67.5(0) + C_1 = 0$$

$$C_1 = 0$$

$$v_1(0) = 0:$$

$$\frac{15}{6}(0)^3 - \frac{67.5}{2}(0)^2 + 0(0) + D_1 = 0$$

$$D_1 = 0$$

Continuity conditions:

$$v_1'(3) = v_2'(3):$$

$$\frac{1}{2}\left(\frac{15}{2}(3)^2 - 67.5(3) + 0\right) = -\frac{5}{6}(3)^3 + \frac{30}{2}(3)^2 - 90(3) + C_2$$

$$C_2 = 90$$

$$v_1(3) = v_2(3):$$

$$\frac{1}{2}\left(\frac{15}{6}(3)^3 - \frac{67.5}{2}(3)^2 + 0(3) + 0\right) = -\frac{5}{24}(3)^4 + \frac{30}{6}(3)^3 - \frac{90}{2}(3)^2 + 90(3) + D_2$$

$$D_2 = -101.25$$

Slope equations [kN·m<sup>2</sup>/(EI)]:

$$v_1'(x) = \frac{1}{2EI}\left(\frac{15}{2}x^2 - 67.5x\right) \text{ for } 0 \leq x \leq 3$$

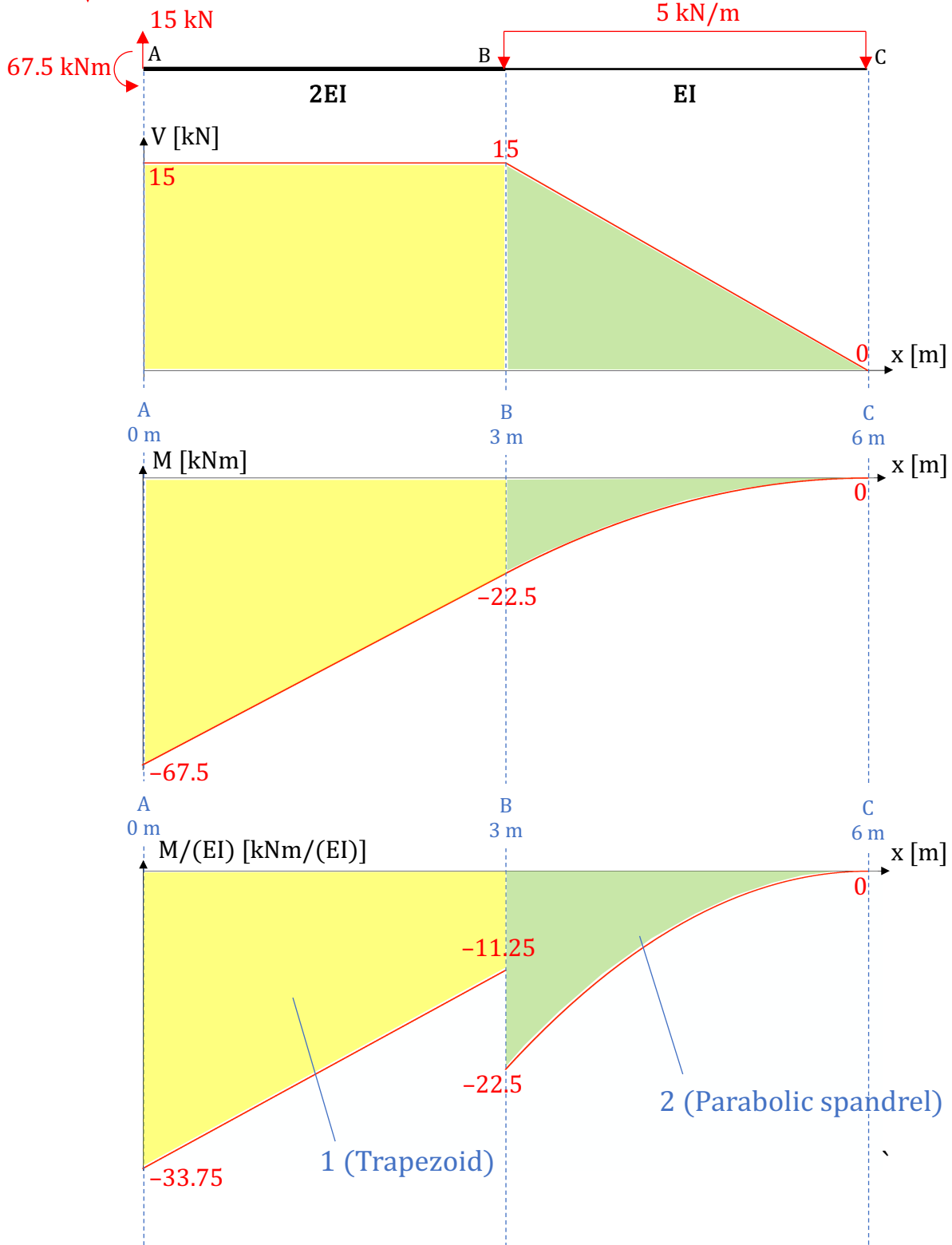
$$v_2'(x) = \frac{1}{EI}\left(-\frac{5}{6}x^3 + \frac{30}{2}x^2 - 90x + 90\right) \text{ for } 3 \leq x \leq 6$$

Deflection equations [kN·m<sup>3</sup>/(EI)]:

$$v_1(x) = \frac{1}{2EI}\left(\frac{15}{6}x^3 - \frac{67.5}{2}x^2\right) \text{ for } 0 \leq x \leq 3 \text{ **Ans.**}$$

$$v_2(x) = \frac{1}{EI}\left(-\frac{5}{24}x^4 + \frac{30}{6}x^3 - \frac{90}{2}x^2 + 90x - 101.25\right) \text{ for } 3 \leq x \leq 6 \text{ **Ans.**}$$

3. (b)  +



Area Index, $i$	Area, $A_i$ [kN·m <sup>2</sup> /(EI)]	Centroid rightwards from point A, $x_i$ [m]	Centroid leftwards from point C, $(x_c - x_i)$ [m]
1(A-B)	$-\frac{67.5}{EI}$	1.25	4.75
2(B-C)	$-\frac{22.5}{EI}$	3.75	2.25

3. (b)

Moment-Area Theorems:

$$\theta_A = 0,$$

$$\theta_{C/A} = \theta_C - \theta_A = \sum_A^C A_i = -\frac{90}{EI}$$

$$\theta_C = -\frac{90}{EI} + 0 = -\frac{90 \text{ kN}\cdot\text{m}^2}{EI} \text{ (clockwise) } \underline{\text{Ans.}}$$

$$v_A = 0, \theta_A = 0,$$

$$t_{C/A} = v_C - v_A - \theta_A(x_C - x_A) = \sum_A^C (x_C - x_i)A_i = -\frac{371.25}{EI}$$

$$v_C = -\frac{371.25}{EI} + 0 + 0 = -\frac{371.25 \text{ kN}\cdot\text{m}^3}{EI} \text{ (downwards) } \underline{\text{Ans.}}$$

3. (c)

Using equations:

$$v_2'(6) = \frac{1}{EI} \left( -\frac{5}{6}(6)^3 + \frac{30}{2}(6)^2 - 90(6) + 90 \right) = -\frac{90 \text{ kN}\cdot\text{m}^2}{EI} \text{ (clockwise) } \underline{\text{Ans.}}$$

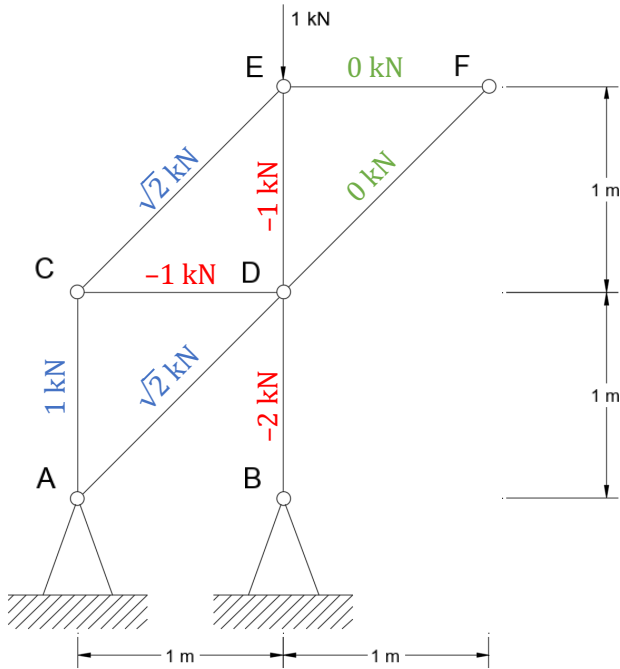
$$v_2(6) = \frac{1}{EI} \left( -\frac{5}{24}(6)^4 + \frac{30}{6}(6)^3 - \frac{90}{2}(6)^2 + 90(6) - 101.25 \right) = -\frac{371.25 \text{ kN}\cdot\text{m}^3}{EI} \text{ (downwards)}$$

Ans.

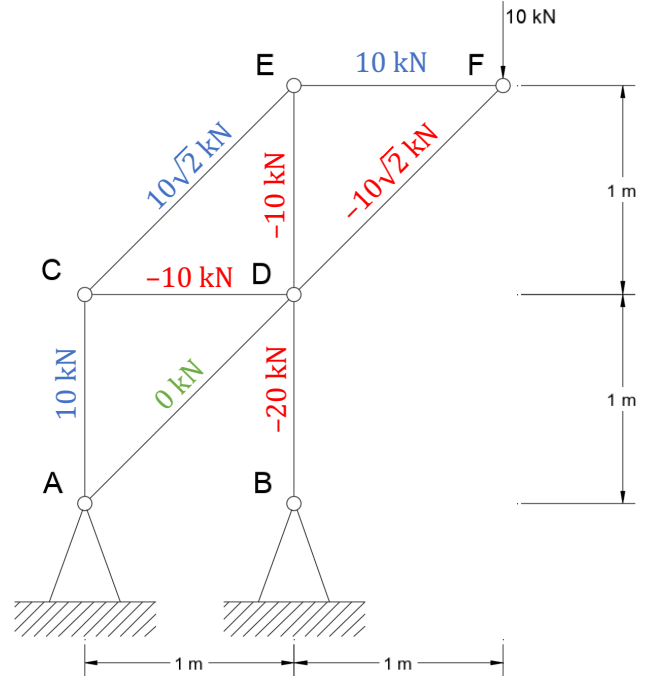
Both methods obtained the same results. Ans.

4. (a) Blue: Tension, Red: Compression, Green: Zero force member

Virtual forces n:



Real forces N:



Member	n [kN]	N [kN]	L [m]	nNL [(kN) <sup>2</sup> ·m]
AC	1	10	1	10
AD	$\sqrt{2}$	0	$\sqrt{2}$	0
BD	-2	-20	1	40
CD	-1	-10	1	10
CE	$\sqrt{2}$	$10\sqrt{2}$	$\sqrt{2}$	$20\sqrt{2}$
DE	-1	-10	1	10
DF	0	$-10\sqrt{2}$	$\sqrt{2}$	0
EF	0	10	1	0
				$\Sigma = 98.284$

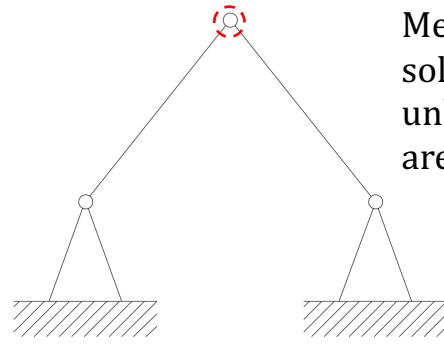
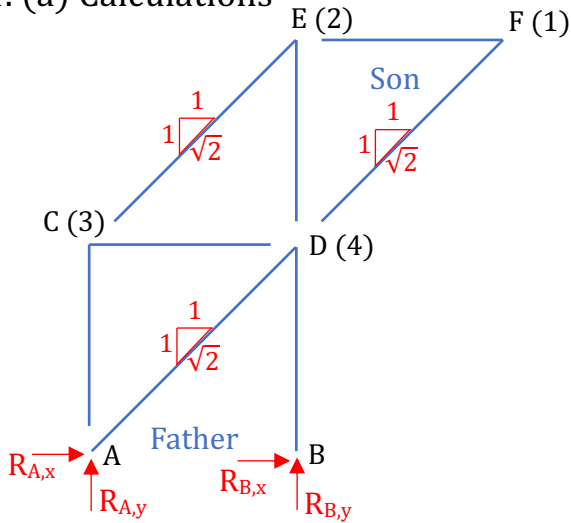
Virtual work equation:

$$EA = (200 \cdot 10^6 \text{ kN/m}^2)(300 (10^{-3} \text{ m})^2) = 6 \cdot 10^4 \text{ kN}$$

$$1 \text{ kN} \cdot \Delta_E = \frac{98.284 \text{ kN}^2 \cdot \text{m}}{6 \cdot 10^4 \text{ kN}} = 1.638 \cdot 10^{-3} \text{ kNm}$$

$$\Delta_E = 1.638 \cdot 10^{-3} \text{ m} = 1.638 \text{ mm (up) Ans.}$$

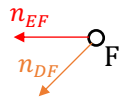
#### 4. (a) Calculations



Method of joints solves 2 unknowns that are not collinear.

#### Virtual forces n:

##### Joint F:



$$\uparrow +, \Sigma F_y = 0:$$

$$-\frac{1}{\sqrt{2}} n_{DF} = 0$$

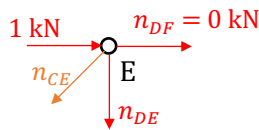
$$n_{DF} = 0 \text{ kN}$$

$$\rightarrow +, \Sigma F_x = 0:$$

$$-\frac{1}{\sqrt{2}} n_{DF} - n_{EF} = 0$$

$$n_{EF} = 0 \text{ kN}$$

##### Joint E:



$$\rightarrow +, \Sigma F_x = 0:$$

$$1 - \frac{1}{\sqrt{2}} n_{CE} = 0$$

$$n_{CE} = \sqrt{2} \text{ kN (tension)}$$

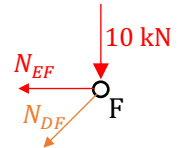
$$\uparrow +, \Sigma F_y = 0:$$

$$-\frac{1}{\sqrt{2}} n_{CE} - n_{DE} = 0$$

$$n_{DE} = -1 \text{ kN (compression)}$$

#### Real forces N:

##### Joint F:



$$\uparrow +, \Sigma F_y = 0:$$

$$-\frac{1}{\sqrt{2}} N_{DF} - 10 = 0$$

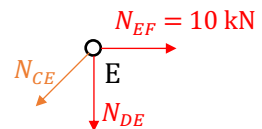
$$N_{DF} = -10\sqrt{2} \text{ kN (compression)}$$

$$\rightarrow +, \Sigma F_x = 0:$$

$$-\frac{1}{\sqrt{2}} N_{DF} - N_{EF} = 0$$

$$N_{EF} = 10 \text{ kN (tension)}$$

##### Joint E:



$$\rightarrow +, \Sigma F_x = 0:$$

$$N_{DF} - \frac{1}{\sqrt{2}} N_{CE} = 0$$

$$N_{CE} = 10\sqrt{2} \text{ kN (tension)}$$

$$\uparrow +, \Sigma F_y = 0:$$

$$-\frac{1}{\sqrt{2}} N_{CE} - N_{DE} = 0$$

$$N_{DE} = -10 \text{ kN (compression)}$$

#### 4. (a) Calculations

##### Virtual forces n:

###### Joint C:

$$\uparrow +, \Sigma F_y = 0:$$

$$\frac{1}{\sqrt{2}} n_{CE} - n_{AC} = 0$$

$$n_{AC} = 1 \text{ kN (tension)}$$

$$\rightarrow +, \Sigma F_x = 0:$$

$$\frac{1}{\sqrt{2}} n_{CE} + n_{CD} = 0$$

$$n_{CD} = -1 \text{ kN (compression)}$$

###### Joint D:

$$\rightarrow +, \Sigma F_x = 0:$$

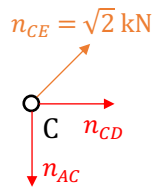
$$n_{CD} - \frac{1}{\sqrt{2}} n_{AD} = 0$$

$$n_{AD} = \sqrt{2} \text{ kN (tension)}$$

$$\uparrow +, \Sigma F_y = 0:$$

$$-n_{DE} - n_{BD} - \frac{1}{\sqrt{2}} n_{AD} = 0$$

$$n_{BD} = -2 \text{ kN (compression)}$$



##### Real forces N:

###### Joint C:

$$\uparrow +, \Sigma F_y = 0:$$

$$\frac{1}{\sqrt{2}} N_{CE} - N_{AC} = 0$$

$$N_{AC} = 10 \text{ kN (tension)}$$

$$\rightarrow +, \Sigma F_x = 0:$$

$$\frac{1}{\sqrt{2}} N_{CE} + N_{CD} = 0$$

$$N_{CD} = -10 \text{ kN (compression)}$$

###### Joint D:

$$\rightarrow +, \Sigma F_x = 0:$$

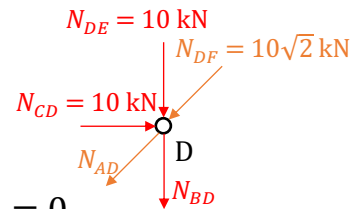
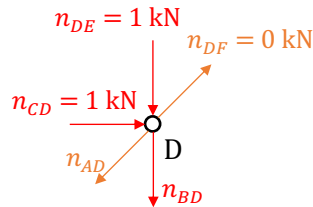
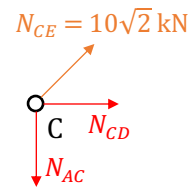
$$N_{CD} - \frac{1}{\sqrt{2}} N_{AD} - \frac{1}{\sqrt{2}} N_{DF} = 0$$

$$N_{AD} = 0 \text{ kN}$$

$$\uparrow +, \Sigma F_y = 0:$$

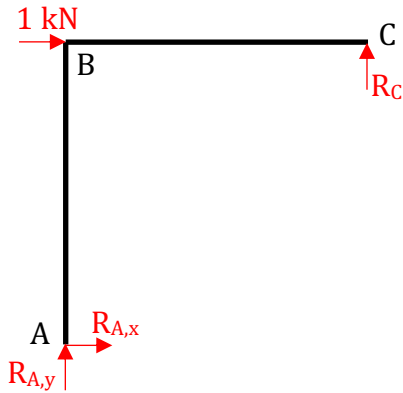
$$-N_{DE} - \frac{1}{\sqrt{2}} N_{DF} - \frac{1}{\sqrt{2}} N_{AD} - N_{BD} = 0$$

$$N_{BD} = -20 \text{ kN (compression)}$$



4. (b) (i)

Virtual horizontal force:



Solving reactions:

$$\circlearrowleft +, \Sigma M_A = 0:$$

$$R_C * 4 - 1 * 4 = 0$$

$$R_C = 1 \text{ kN (upwards)}$$

$$\uparrow +, \Sigma F_y = 0:$$

$$R_C + R_{A,y} = 0$$

$$R_{A,y} = -1 \text{ kN (downwards)}$$

$$\rightarrow +, \Sigma F_x = 0:$$

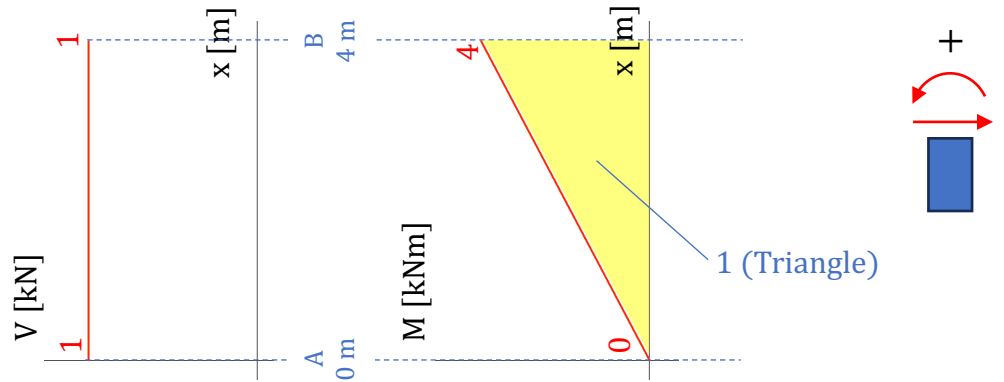
$$R_{A,x} + 1 = 0$$

$$R_{A,x} = -1 \text{ kN (leftwards)}$$

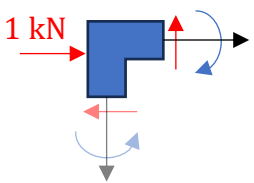
Section AB:

$$V_1(x) = 1 \text{ for } 0 \leq x \leq 4$$

$$M_1(x) = x \text{ for } 0 \leq x \leq 4$$



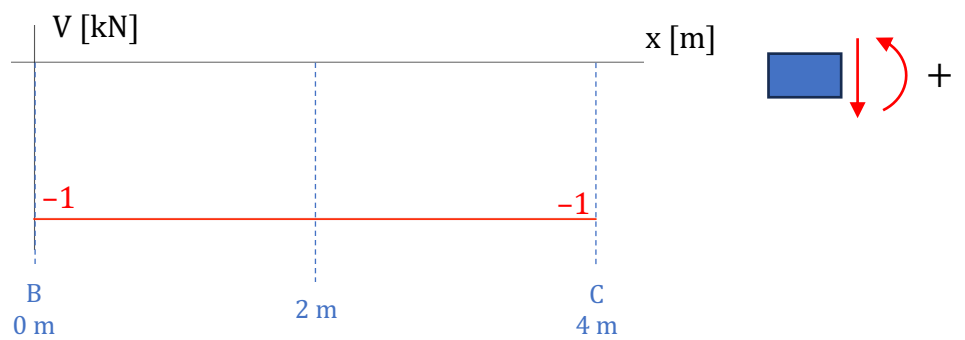
At B:



$$V = -1 \text{ kN}$$

$$M = 4 \text{ kNm}$$

$$N = 0 \text{ kN}$$



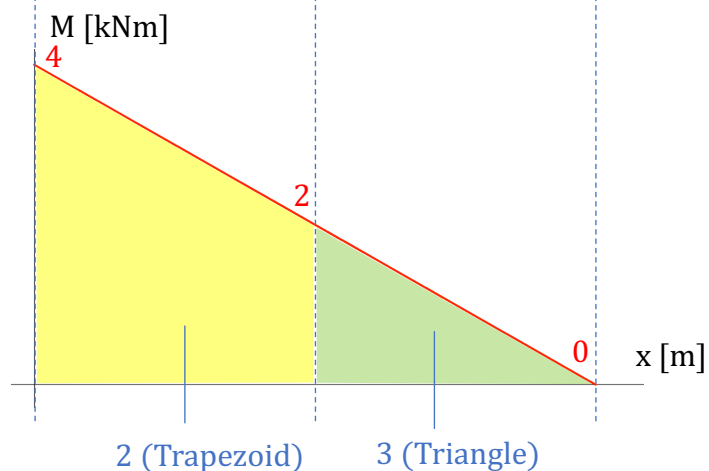
Section BC:

$$V_2(x) = -1 \text{ for } 0 \leq x \leq 2$$

$$M_2(x) = -x + 4 \text{ for } 0 \leq x \leq 2$$

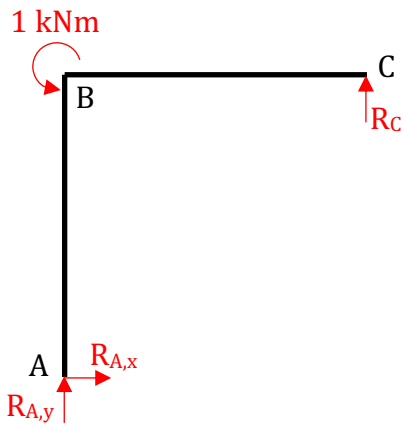
$$V_3(x) = -1 \text{ for } 2 \leq x \leq 4$$

$$M_3(x) = -x + 4 \text{ for } 2 \leq x \leq 4$$



4. (b) (i)

Virtual moment:



Solving reactions:

$$\circlearrowleft +, \Sigma M_A = 0:$$

$$R_C \cdot 4 + 1 = 0$$

$$R_C = -0.25 \text{ kN (downwards)}$$

$$\uparrow +, \Sigma F_y = 0:$$

$$R_C + R_{A,y} = 0$$

$$R_{A,y} = 0.25 \text{ kN (upwards)}$$

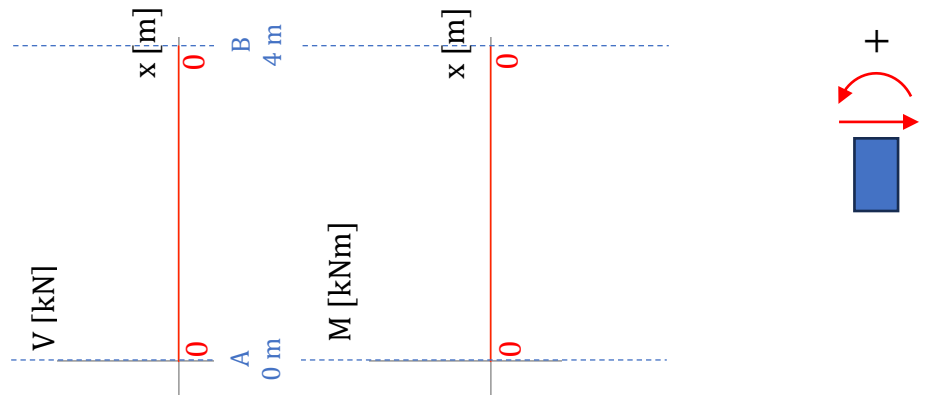
$$\rightarrow +, \Sigma F_x = 0:$$

$$R_{A,x} = 0 \text{ kN}$$

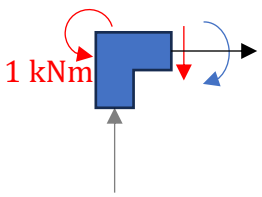
Section AB:

$$V_1(x) = 0 \text{ for } 0 \leq x \leq 4$$

$$M_1(x) = 0 \text{ for } 0 \leq x \leq 4$$



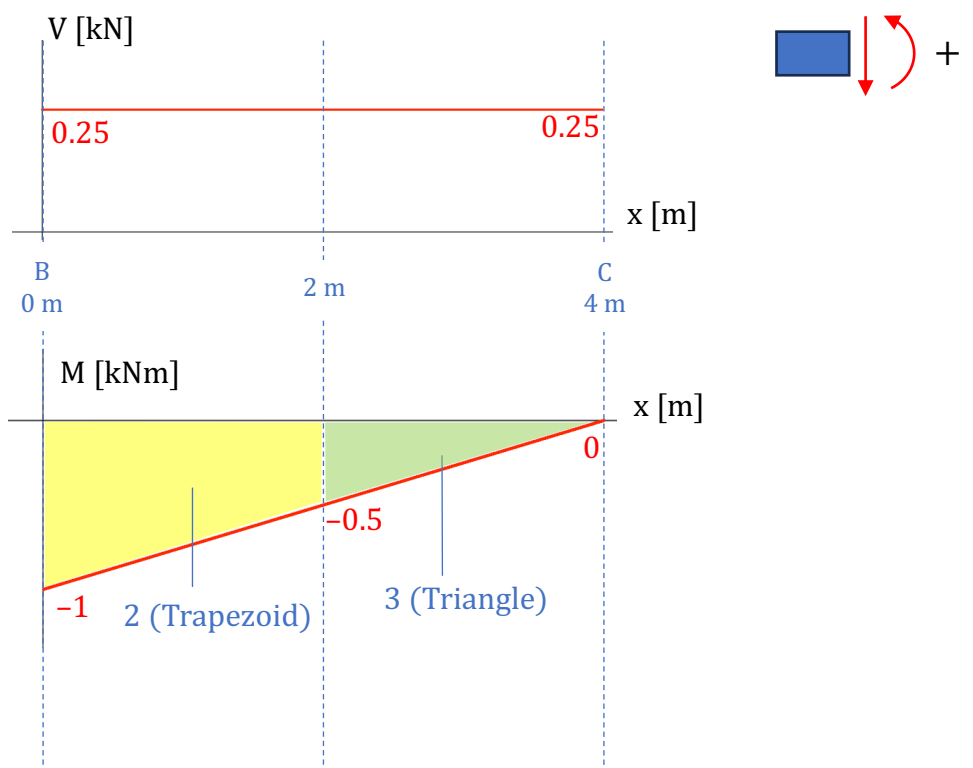
At B:



$$V = 0.25 \text{ kN}$$

$$M = -1 \text{ kNm}$$

$$N = 0 \text{ kN}$$



Section BC:

$$V_2(x) = 0.25 \text{ for } 0 \leq x \leq 2$$

$$M_2(x) = 0.25x - 1 \text{ for } 0 \leq x \leq 2$$

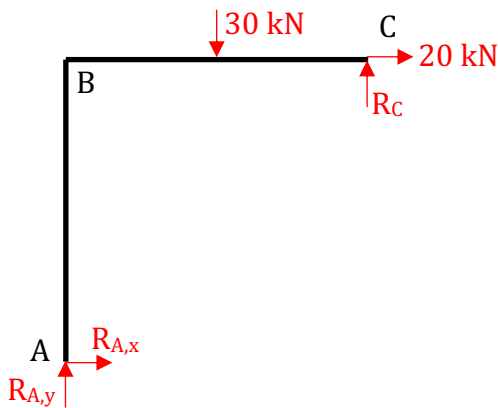
$$V_3(x) = 0.25 \text{ for } 2 \leq x \leq 4$$

$$M_3(x) = 0.25x - 1 \text{ for } 2 \leq x \leq 4$$



4. (b) (i)

Real forces:



Solving reactions:

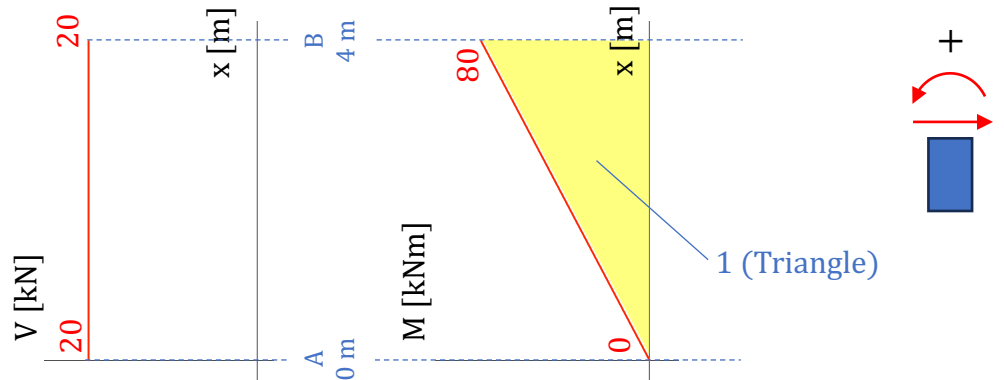
$$\begin{aligned} \circlearrowleft +, \Sigma M_A = 0: \\ R_C \cdot 4 - 30 \cdot 2 - 20 \cdot 4 = 0 \\ R_C = 35 \text{ kN (upwards)} \end{aligned}$$

$$\begin{aligned} \uparrow +, \Sigma F_y = 0: \\ R_C + R_{A,y} - 30 = 0 \\ R_{A,y} = -5 \text{ kN (downwards)} \end{aligned}$$

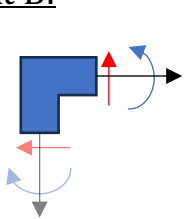
$$\begin{aligned} \rightarrow +, \Sigma F_x = 0: \\ R_{A,x} + 20 = 0 \\ R_{A,x} = -20 \text{ kN (leftwards)} \end{aligned}$$

Section AB:

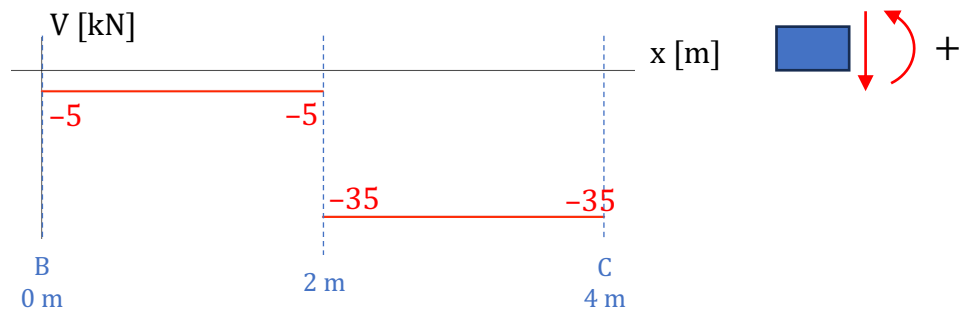
$$\begin{aligned} V_1(x) &= 20 \text{ for } 0 \leq x \leq 4 \\ M_1(x) &= 20x \text{ for } 0 \leq x \leq 4 \end{aligned}$$



At B:



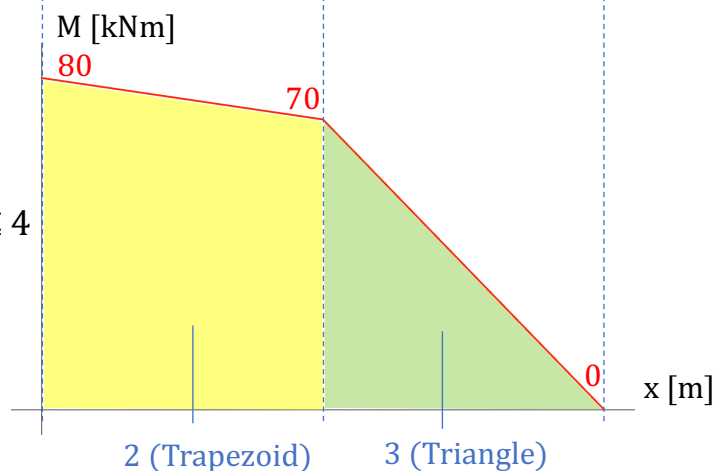
$$\begin{aligned} V &= -5 \text{ kN} \\ M &= 80 \text{ kNm} \\ N &= 20 \text{ kN} \end{aligned}$$



Section BC:

$$\begin{aligned} V_2(x) &= -5 \text{ for } 0 \leq x \leq 2 \\ M_2(x) &= -5x + 80 \text{ for } 0 \leq x \leq 2 \end{aligned}$$

$$\begin{aligned} V_3(x) &= -35 \text{ for } 2 \leq x \leq 4 \\ M_3(x) &= -35x + 140 \text{ for } 2 \leq x \leq 4 \end{aligned}$$



4. (b) (i)

$$EI = (200 \text{ GPa})(235 \cdot 10^6 \text{ (mm)}^4) = (200 \cdot 10^6 \text{ kN/m}^2)(235 \cdot 10^6 \text{ (10}^{-3} \text{ m)}^4) = 4.7 \cdot 10^4 \text{ kN} \cdot \text{m}^2$$

Virtual work equation:

Horizontal deflection:

$$\begin{aligned} 1 \text{ kN} \cdot \Delta_B &= \underbrace{\int_0^4 \frac{(x)(20x)}{EI} dx}_{426.667/(EI)} + \underbrace{\int_0^2 \frac{(-x+4)(-5x+80)}{EI} dx}_{453.333/(EI)} + \underbrace{\int_2^4 \frac{(-x+4)(-35x+140)}{EI} dx}_{93.333/(EI)} \\ &= 2.071 \cdot 10^{-2} \frac{(\text{kNm})^2}{\text{kN} \cdot \text{m}^2} \cdot \text{m} = 2.071 \cdot 10^{-2} \text{ kNm} \end{aligned}$$

$$\Delta_B = 2.071 \cdot 10^{-2} \text{ m} = 20.71 \text{ mm (rightwards) Ans.}$$

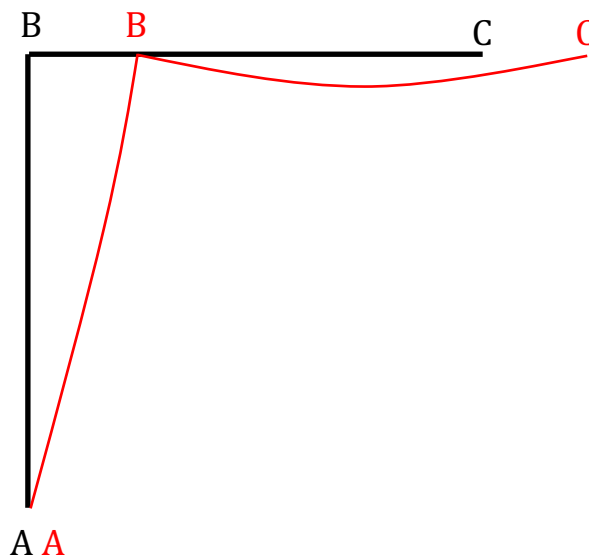
Rotation:

$$\begin{aligned} 1 \text{ kNm} \cdot \theta_B &= \underbrace{\int_0^4 \frac{(0)(20x)}{EI} dx}_{0/(EI)} + \underbrace{\int_0^2 \frac{(0.25x-1)(-5x+80)}{EI} dx}_{-113.333/(EI)} + \underbrace{\int_2^4 \frac{(0.25x-1)(-35x+140)}{EI} dx}_{-23.333/(EI)} \\ &= -2.908 \cdot 10^{-3} \frac{(\text{kNm})^2}{\text{kN} \cdot \text{m}^2} \cdot \text{m} = 2.908 \cdot 10^{-3} \text{ kNm} \end{aligned}$$

$$\theta_B = -2.908 \cdot 10^{-3} = -0.002908 \text{ rad (clockwise) Ans.}$$

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4. (b) (ii) Ans.



B moves right and rotates clockwise



**NANYANG TECHNOLOGICAL UNIVERSITY****SEMESTER 1 EXAMINATION 2023-2024****CV2011 – STRUCTURAL ANALYSIS I**

November / December 2023

Time Allowed: 2 hours

**INSTRUCTIONS**

1. This paper contains **FOUR (4)** questions and comprises **FIVE (5)** pages.
  2. Answer **ALL** questions.
  3. All questions carry equal marks.
  4. An Appendix of **TWO (2)** pages are attached to the Question Paper.
  5. This is a Closed-Book Examination.
  6. All answers must be written in the answer book provided. Answer each question beginning on the **FRESH** page of the answer book.
  7. Avoid illegible handwriting. Your writing must be **CLEAR** and **READABLE**.
- 

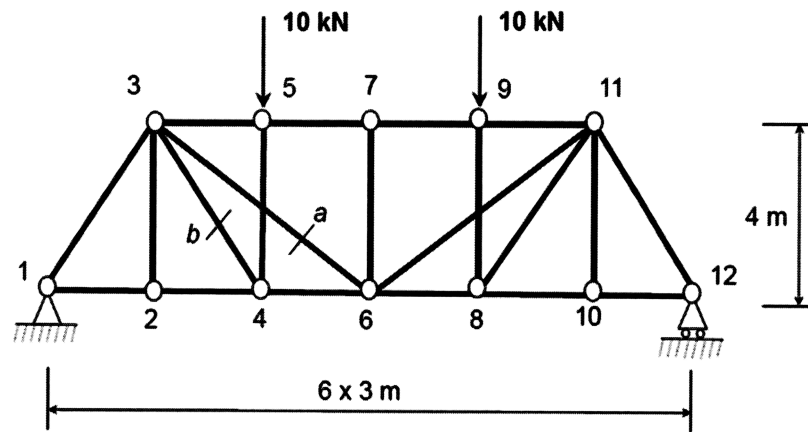
1. Two pin-jointed trusses are subjected to two points loads as shown in Figures Q1 (a) and (b).
  - (a) Prove that the two trusses are statically determinate.

(4 Marks)
  - (b) Calculate the internal forces of members 3-6(a) and 3-4(b) as shown in Figure Q1(a).

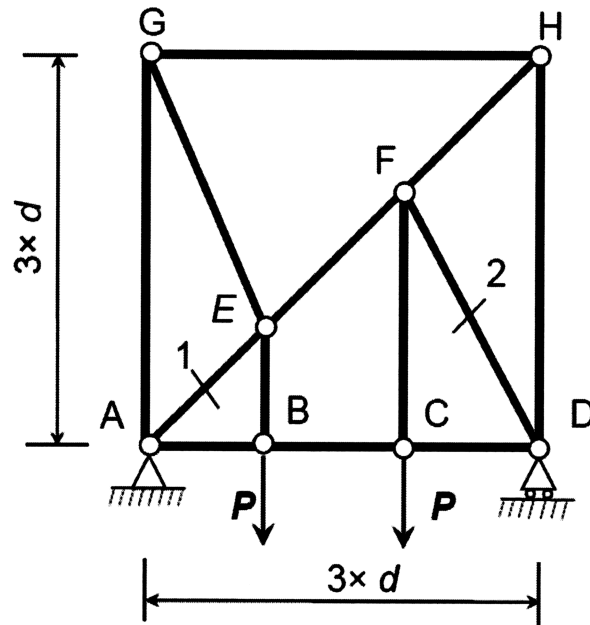
(6 Marks)
  - (c) Calculate the internal forces of members AE(1) and FD(2) as shown in Figure Q1(b).

(15 Marks)

**Note: Question No.1 continues on Page 2.**



**Figure Q1(a)**



**Figure Q1(b)**

2. A pin-jointed beam as shown in Figure Q2 is fix-supported at F, and supported by rollers at B and D. The beam is subjected to a set of uniformly distributed loading of 30 kN/m at section DE. Two point loads of 30 kN and 20 kN are applied at A and C, respectively, as shown in Figure Q2.

(a) Verify that the structure is stable and determinate.

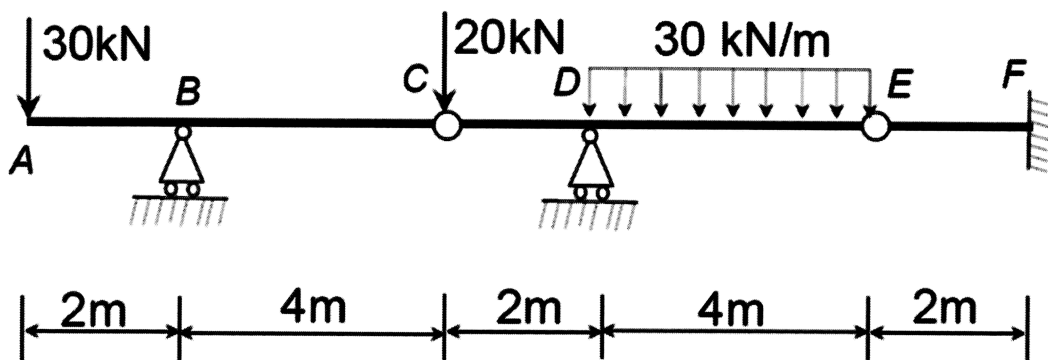
(3 Marks)

(b) Calculate the reactions at the supports B, D and F.

(6 Marks)

(c) Draw the bending moment and shear force diagrams of the beam. Indicate the values of the bending moments and shear force at points A, B, C, D, E and F.

(16 Marks)



**Figure Q2**

3. The cantilever beam ABC is loaded as shown in Figure Q3. Support A is a fixed end. A uniform distributed load of 5 kN/m is applied at the segment of BC. The beam has a flexural rigidity of  $2EI$  in the segment of AB and  $EI$  in the segment of BC.

- (a) Determine the deflection curve of the cantilever in terms of  $EI$  using the **direct integration method**.

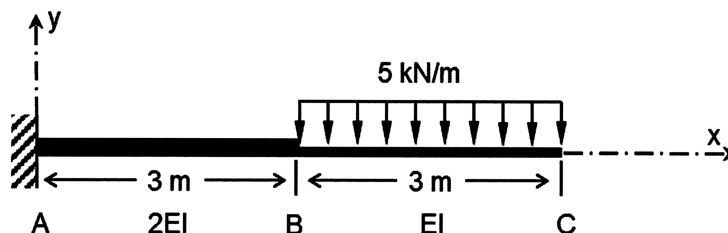
(11 Marks)

- (b) Determine the vertical deflection and rotation at C using **moment area methods**.

(11 Marks)

- (c) Obtain the vertical deflection and rotation at C based on the results in (a) and compare them with the results in (b).

(3 Marks)

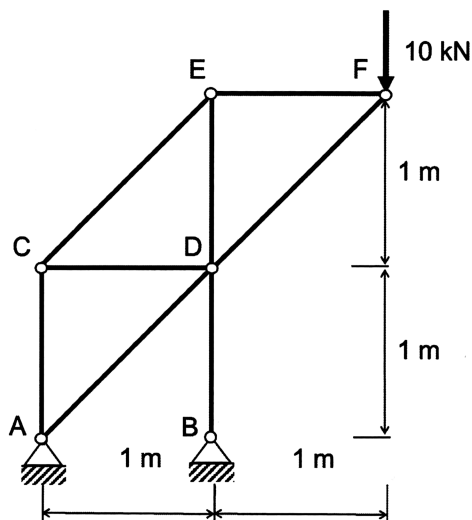


**Figure Q3**

4. (a) As shown in Figure Q4(a), a truss ABCDEF is supported by two pinned ends at A and B. One point load of 10 kN is applied at Joint F vertically. Each truss member has the same Young's modulus of  $E = 200$  GPa and the same cross-sectional area of  $A = 300$  mm<sup>2</sup>. By using **principle of virtual work**, determine the horizontal deflection at Joint E.

(12 Marks)

**Note: Question No. 4 continues on Page 5.**



**Figure Q4(a)**

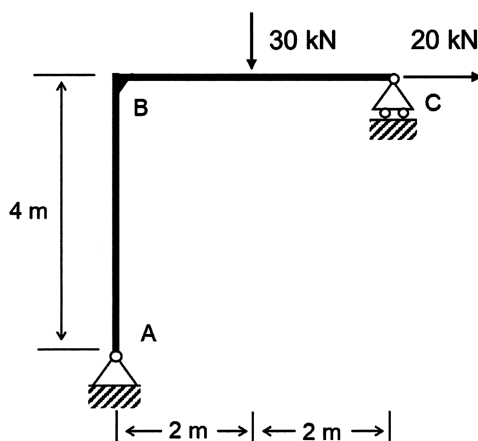
- (b) The frame ABC is loaded as shown in Figure Q4(b), with a pin support at A and a roller support at C. Each member has the same Young's modulus of  $E = 200 \text{ GPa}$  and the same moment of inertia of  $I = 235 \times 10^6 \text{ mm}^4$ .

- (i) By using **principle of virtual work**, determine the horizontal deflection and rotation at Joint B.

(11 Marks)

- (ii) Based on the results in (i), draw the deformed shape of the frame in a qualitative manner.

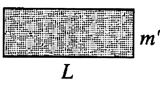


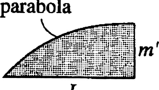

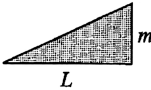
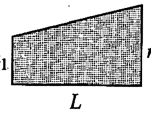
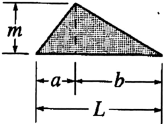
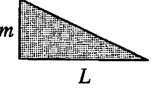
(2 Marks)



**Figure Q4(b)**

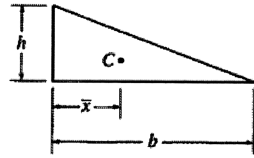
**END OF PAPER**

Appendix: Values of Product Integrals  $\int_0^L mm' dx$

$\int_0^L m m' dx$				
	$mm'L$	$\frac{1}{2}mm'L$	$\frac{1}{2}m(m_1 + m_2)L$	$\frac{2}{3}mm'L$
	$\frac{1}{2}mm'L$	$\frac{1}{3}mm'L$	$\frac{1}{6}m(m_1 + 2m_2)L$	$\frac{5}{12}mm'L$
	$\frac{1}{2}m'(m_1 + m_2)L$	$\frac{1}{6}m'(m_1 + 2m_2)L$	$\frac{1}{6}[m_1'(2m_1 + m_2) + m_2'(m_1 + 2m_2)]L$	$\frac{1}{12}[m_1'(3m_1 + 5m_2)]L$
	$\frac{1}{2}mm'L$	$\frac{1}{6}mm'(L + a)$	$\frac{1}{6}m_1[m_1'(L + b) + m_2(L + a)]$	$\frac{1}{12}mm'\left(3 + \frac{3a}{L} - \frac{a^2}{L^2}\right)L$
	$\frac{1}{2}mm'L$	$\frac{1}{6}mm'L$	$\frac{1}{6}m(2m_1 + m_2)L$	$\frac{1}{4}mm'L$



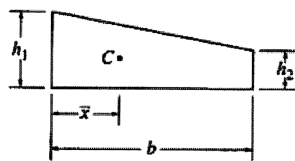
Appendix: Geometric Properties of Areas



$$A = \frac{1}{2}bh$$

$$\bar{x} = \frac{1}{3}b$$

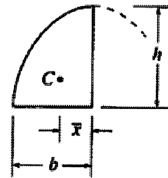
Triangle



$$A = \frac{1}{2}b(h_1 + h_2)$$

$$\bar{x} = \frac{b(2h_2 + h_1)}{3(h_1 + h_2)}$$

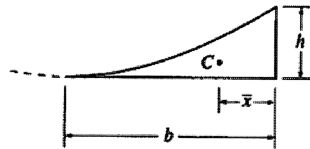
Trapezoid



$$A = \frac{2}{3}bh$$

$$\bar{x} = \frac{3}{8}b$$

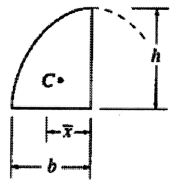
Semi Parabola



$$A = \frac{1}{3}bh$$

$$\bar{x} = \frac{1}{4}b$$

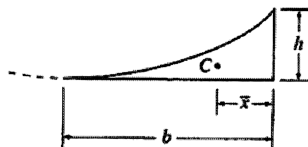
Parabolic spandrel



$$A = bh \left( \frac{n}{n+1} \right)$$

$$\bar{x} = \frac{b(n+1)}{2(n+2)}$$

Semi-segment of  $n$ th degree curve



$$A = bh \left( \frac{1}{n+1} \right)$$

$$\bar{x} = \frac{b}{(n+2)}$$

Spandrel of  $n$ th degree curve

# CV2011 STRUCTURAL ANALYSIS I

Please read the following instructions carefully:

- 1. Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**
2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.
3. Please write your Matriculation Number on the front of the answer book.
4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.