

1. (a) For a dilute NaCl solution at 15 °C, the molecular diffusivity of NaCl is $1.24 \times 10^{-9} \text{ m}^2/\text{s}$. If the NaCl solution is heated from 15 °C to 40 °C, what is the molecular diffusivity of NaCl in the solution at 40 °C? Assume that the viscosities of the dilute NaCl solution at 15 °C and 40 °C are $1.1375 \times 10^{-3} \text{ Ns/m}^2$ and $0.6527 \times 10^{-3} \text{ Ns/m}^2$, respectively.

15°C → 40°C (4 Marks)

$$D_m = 1.24 \times 10^{-9} \text{ m}^2/\text{s}$$

$$\mu = 1.1375 \times 10^{-3} \text{ Ns/m}^2$$

$$D_m = \frac{kT}{3\pi\mu d_p}$$

$$\frac{k}{3\pi d_p} = \frac{D_m \mu}{T}$$

$$\left(\frac{k}{3\pi d_p}\right)_{20^\circ\text{C}} = \left(\frac{k}{3\pi d_p}\right)_{40^\circ\text{C}}$$

$$\left(\frac{D_m \mu}{T}\right)_{20^\circ\text{C}} = \left(\frac{D_m \mu}{T}\right)_{40^\circ\text{C}}$$

$$\frac{1.24 \times 10^{-9} (1.1375 \times 10^{-3})}{15 + 273} = \frac{D (0.6527 \times 10^{-3})}{40 + 273}$$

$$D_m = 2.349 \times 10^{-9} \text{ m}^2/\text{s} \#$$

- (b) A warm water with a temperature of 40 °C flows in a tube. The tube has a diameter of 20 mm and the water flow velocity in the tube is 0.1 mm/s along the positive x direction. If a very small amount of NaCl is injected to the tube, what is the effective longitudinal dispersion coefficient (in the unit of m^2/s) in the tube? Assume after the injection of NaCl in the water flow in the tube, the molecular diffusivity of NaCl equals to that determined in part (a).

$$r = \frac{20}{2} = 10 \text{ mm} = 10 \times 10^{-3} \text{ m}$$

$$u = 0.1 \text{ mm/s} = 0.1 \times 10^{-3} \text{ m/s}$$

(4 Marks)

$d = 20 \text{ mm}$
 $u = 0.1 \text{ mm/s}$
 $D = ?$
 $D_m = 2.349 \times 10^{-9} \text{ m}^2/\text{s}$
 $\approx 2.35 \times 10^{-9} \text{ m}^2/\text{s}$

$$D_{\text{eff}} = \frac{r^2 u^2}{48 D_m} = \frac{(10 \times 10^{-3} \text{ m})^2 (0.1 \times 10^{-3} \text{ m/s})^2}{48 (2.35 \times 10^{-9} \text{ m}^2/\text{s})} = 8.865 \times 10^{-6} \text{ m}^2/\text{s} \#$$

- (c) For the case in part (b) and in the first scenario, there is only one injection of NaCl (salt) of $1 \times 10^{-5} \text{ g}$, which is made at the location $x = 0$ and time $t = 0$. At time $t = 10$ hours, $m = 1 \times 10^{-5} \text{ g}$ $u = 0.1 \times 10^{-3} \text{ m/s}$

- (i) What is the location of the maximum salt concentration? What is the maximum concentration?

$$u = \frac{x}{t}, x = ut = (0.1 \times 10^{-3} \text{ m/s})(10 \text{ h} \times 60^2) = 3.6 \text{ m} \#$$

$$M = \frac{m}{\text{Area}} \leftarrow C_{\text{max}} = \frac{m}{\sqrt{4\pi D_{\text{eff}} t}} = \frac{1 \times 10^{-5}}{\sqrt{4\pi (8.865 \times 10^{-6})(10 \times 60^2)}} = 0.0159 \text{ g/m}^3 = 1.59 \times 10^{-5} \text{ kg/m}^3 \#$$

- (ii) What is the length of salt cloud?

$$\text{Length} = 4\sigma = 4\sqrt{2 D_{\text{eff}} t} = 4\sqrt{2 (8.865 \times 10^{-6})(10 \times 60^2)} = 3.196 \text{ m} \#$$

- (iii) What is the salt concentration at the location $x = 3.5 \text{ m}$?

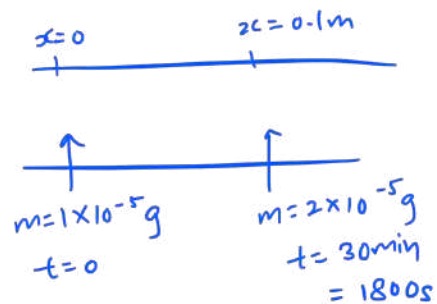
$$C = \frac{m}{\sqrt{4\pi D_{\text{eff}} t}} e^{-\frac{(x-x_0-ut)^2}{4 D_{\text{eff}} t}} = 1.59 \times 10^{-5} e^{-\frac{[3.5 - 0 - (0.1 \times 10^{-3})(10 \times 60^2)]^2}{4 (8.865 \times 10^{-6})(10 \times 60^2)}}$$

$$= 1.578 \times 10^{-5} \text{ kg/m}^3 \#$$

C_{max} from C(i)

(d) For the case in part (b) and in the second scenario, there are two injections of NaCl (salt) into the tube. The first injection of NaCl (salt) of 1×10^{-5} g is made at the location $x = 0$ and the time $t = 0$. The second injection of NaCl (salt) of 2×10^{-5} g is made at the location $x = 0.1$ m and the time $t = 30$ minutes. At time $t = 10$ hours and location $x = 3.5$ m,

(i) What will be the salt concentration?



1st injection $C_1 = 1.578 \times 10^{-5} \text{ kg/m}^3$ [from c (ii)]

2nd injection $t_2 = 10\text{h} - 30\text{min} = 9.5\text{h} = 34200\text{s}$

$$C_2 = \frac{2 \times 10^{-5}}{\pi (10 \times 10^{-3})^2} e^{-\frac{[3.5 - 0 - (0.1 \times 10^{-3})(34200)]^2}{4(8.865 \times 10^{-6})(34200)}}$$

$$= 0.0324 \text{ g/m}^3$$

$$= 3.24 \times 10^{-5} \text{ kg/m}^3$$

$$C_{\text{Total}} = C_1 + C_2$$

$$= (1.578 \times 10^{-5}) + (3.24 \times 10^{-5})$$

$$= 4.822 \times 10^{-5} \text{ kg/m}^3 \#$$

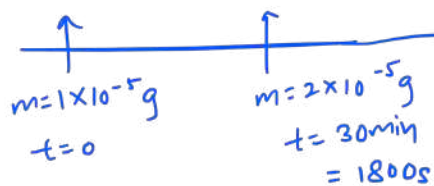
(ii) What will be the advection flux in kg/s?

$$\text{Advection flux} = uCA$$

$$= (0.1 \times 10^{-3} \text{ m/s}) (4.822 \times 10^{-5} \text{ kg/m}^3) [\pi (10 \times 10^{-3})^2 \text{ m}^2]$$

$$= 1.515 \times 10^{-12} \text{ kg/s}$$

(iii) Describe how you can derive the diffusive flux due to the two injections at this location. Would the diffusive flux due to the two injections be in the same or opposite direction? Explain the reasons.



The diffusive flux due to 2 injections are in the same direction (from left to right) as NaCl diffuse from high concentration to low concentration.

2. (a) Under optimum condition, an algae strain can grow at $\mu_{max} = 1.3/d$. This strain is the dominant species in a pond where the total nitrogen and phosphorus concentrations are $100 \mu\text{g N/L}$ and $10 \mu\text{g P/L}$. Given the K_s for N and P are $25 \mu\text{g/L}$ and $5 \mu\text{g/L}$ respectively. If both N and P are considered limiting, what will be the estimated growth rate?

$\mu_{max} = 1.3 \text{ d}^{-1}$
 Total N = $100 \mu\text{g/L}$
 Total P = $10 \mu\text{g/L}$
 $K_N = 25 \mu\text{g/L}$
 $K_P = 5 \mu\text{g/L}$

$$\mu = \mu_{max} \left(\frac{N}{K_N + N} \right) \left(\frac{P}{K_P + P} \right) = 1.3 \left(\frac{100}{25 + 100} \right) \left(\frac{10}{5 + 10} \right) = 0.693/d \quad (4 \text{ Marks})$$

- (b) A river has a width of 10 m and a depth of 4 m. The flow velocity of the river is 0.1 m/s. Two factories at different locations discharge wastewater streams that flow into the river (refer to Figure Q2). The first factory discharges a wastewater stream at $x = 0$, while the second factory discharges a wastewater stream at 20 km downstream of the first factory discharging point (i.e., at $x = 20 \text{ km}$). The characteristics of the two wastewater streams are listed below.

$H = \text{depth} = 4 \text{ m}$ $u = 0.1 \text{ m/s}$

- Upstream of river before the discharge of wastewater streams:
 $Q_r = 0.5 \text{ m}^3/\text{s}$, $BOD_r = 20 \text{ mg/L}$, $DO_r = 6.0 \text{ mg/L}$, $T_r = 25^\circ\text{C}$
- Wastewater stream discharged from the 1st factory:
 $Q_{d,1} = 0.05 \text{ m}^3/\text{s}$, $BOD_{d,1} = 30 \text{ mg/L}$, $DO_{d,1} = 2.0 \text{ mg/L}$, $T_{d,1} = 25^\circ\text{C}$
- Wastewater stream discharged from the 2nd factory:
 $Q_{d,2} = 0.05 \text{ m}^3/\text{s}$, $BOD_{d,2} = 50 \text{ mg/L}$, $DO_{d,2} = 1.0 \text{ mg/L}$, $T_{d,2} = 25^\circ\text{C}$

Assume the two wastewater streams flowing into the river achieve complete mixing with the river water immediately at the discharge locations. Given that $k_1 = 0.15/d$ and $DO_{sat} = 8.4 \text{ mg/L}$.

- (i) Determine the BOD and DO of the mixed water at the 1st discharging location.

Figure Q2

$DO_0 = \frac{Q_{d1} DO_{d1} + Q_r DO_r}{Q_{d1} + Q_r}$

$$= \frac{(0.05 \text{ m}^3/\text{s} \times 2) + (0.5 \times 1000 \times 6)}{(0.05 \times 1000) + (0.5 \times 1000)}$$

$DO_0 = 5.64 \text{ mg/L} \#$

$BOD = \frac{(0.05 \text{ m}^3/\text{s} \times 1000 \times 30) + (0.5 \times 1000 \times 20)}{(0.05 \times 1000) + (0.5 \times 1000)}$

$BOD = 20.91 \text{ mg/L} \#$

similar formula

- (ii) Determine the BOD and DO of the mixed water at the 2nd discharging location.

Total $Q_r = 0.05 + 0.5 = 0.55 \text{ m}^3/\text{s} = 550 \text{ L/s}$

Injection $Q = 0.05 \text{ m}^3/\text{s} = 50 \text{ L/s}$

$t = \frac{x}{u} = \frac{20000}{0.1} = 2 \times 10^5 \text{ s} = 2.31 \text{ day}$

$BOD = BOD_0 e^{-k_1 t} = 20.91 e^{-0.15(2.31)} = 14.79 \text{ mg/L}$

$BOD = \frac{(550 \times 14.79) + (50 \times 50)}{550 + 50} = 17.72 \text{ mg/L} \#$

$$k_{2,20C} = \frac{3.9u^{0.5}}{H^{1.5}}, \quad K_{2,T} = k_{2,20C}\theta^{(T-20)} \quad (25-20)$$

$$= \frac{3.9(0.1)^{0.5}}{4^{1.5}} \quad k_{2,25} = 0.154(1.024)^5$$

$$= 0.154 \quad = 0.173$$

$u = 0.1 \text{ m/s} = 8640 \text{ m/day}$ ← k unit: day^{-1} , u unit must be m/day

$$PO = DO_{\text{sat}} - (DO_{\text{sat}} - DO_0) e^{-\frac{k_2}{u}x} - \frac{k_1 L_0}{k_2 - k_1} \left(e^{-\frac{k_1}{u}x} - e^{-\frac{k_2}{u}x} \right)$$

$$= 8.4 - (8.4 - 5.64) e^{-\frac{0.173}{8640}(20000)} - \frac{0.15(20.91)}{0.173 - 0.15} \left(e^{-\frac{0.15(20000)}{8640}} - e^{-\frac{0.173}{8640}(20000)} \right)$$

$$= 6.5508 - 4.996 = 1.555 \text{ mg/L}$$

$$DO_0 = \frac{(550 \times 1.555) + (50 \times 1)}{550 + 50} = 1.508 \text{ mg/L} \#$$

(iii) Determine the location at which DO is the lowest. What is the DO at that location?

$$L_0 = \text{BOD at } x=20\text{km} = 17.72 \text{ mg/L}$$

$$DO = DO_{\text{sat}} - DO_0 = 8.4 - 1.508 = 6.902 \text{ mg/L}$$

$$x_c = \frac{u}{k_2 - k_1} \ln \left\{ \frac{k_2}{k_1} \left[1 - \left(\frac{k_2}{k_1} - 1 \right) \left(\frac{DO_0}{L_0} \right) \right] \right\} = \frac{8640}{0.173 - 0.15} \ln \left\{ \frac{0.173}{0.15} \left[1 - \left(\frac{0.173}{0.15} - 1 \right) \left(\frac{6.902}{17.72} \right) \right] \right\}$$

$$= 30456 \text{ m from point 2}$$

$$x = 30456 + 20000 = 50456 \text{ m from } x=0 \#$$

$$PO = DO_{\text{sat}} - (DO_{\text{sat}} - DO_0) e^{-\frac{k_2}{u}x} - \frac{k_1 L_0}{k_2 - k_1} \left(e^{-\frac{k_1}{u}x} - e^{-\frac{k_2}{u}x} \right)$$

$$= 8.4 - (8.4 - 1.508) e^{-\frac{0.173}{8640}(30456)} - \frac{0.15(17.72)}{0.173 - 0.15} \left(e^{-\frac{0.15(30456)}{8640}} - e^{-\frac{0.173}{8640}(30456)} \right)$$

$$= 4.655 - 5.304$$

$$= -0.649$$

$$DO = 0 \text{ mg/L} \#$$

(iv) What is the BOD at the location where DO is the lowest?

$$t = \frac{x}{u} = \frac{8164}{0.1} = 81640 \text{ s} = 0.945 \text{ day}$$

$$\text{BOD} = 17.72 e^{-0.15(0.945)} = 15.38 \text{ mg/L} \#$$

3. (a) An abandoned sump in a petrochemical plant contains polluted water, sediment and confined air. The predominant chemical in the sump, Y, is volatile with the following properties at the prevailing temperature of 20°C:

- P Vapor pressure = 0.076 atm
 S Aqueous solubility = 9.97×10^{-3} mol/L
 Octanol-water partition coefficient, $K_{ow} = 10^{2.33}$

$$\rho_s = 1.5 \times 10^3 \text{ kg/m}^3$$

$$f_{oc} = 0.05$$

The sump is sealed and there is no exchange of fluids with outside environment. It can be modeled as an environmental system comprising the overhead air (or vapor), water column and bottom sediment with a volumetric ratio $V_{Air}:V_w:V_{Solid}$ equals to 10:20:1. The bottom sediment has a density of $1.5 \times 10^3 \text{ kg/m}^3$ and contains 5% organic carbon. There is no other phase of Y existed in the system. The Y concentrations in these compartments are at equilibrium. Universal gas constant (R) is 0.082 atm-L/mol-K.

- (i) What is the Henry's Constant of Y at 20°C, expressed in the unit of atm-L/mol? Express its Henry's Constant in its dimensionless form (H_{cc}).

$$P = HC$$

$$H_{pc} = \frac{P}{C} = \frac{0.076}{9.97 \times 10^{-3}} = 7.62 \text{ atm L/mol} \#$$

$$H_{cc} = \frac{H_{pc}}{RT} = \frac{7.62 \text{ atm L/mol}}{0.082 \text{ atm L/mol K} (273+20 \text{ K})}$$

$$= 0.317 \#$$

- (ii) Estimate the distribution coefficient between sediment solid and water, K_d (in m^3/g), and its dimensionless form, K_p . For Y, its K_{ow} and K_{oc} (organic-carbon normalized partition coefficient) follows the linear relationship of $K_{oc} = 0.63 K_{ow}$, in which K_{oc} is in L/kg.

$$K_{oc} = 0.63 K_{ow} = 0.63 (10^{2.33}) = 134.69 \text{ L/kg} = 1.3469 \times 10^{-4} \text{ m}^3/\text{g}$$

$$K_d = f_{oc} K_{oc} = 0.05 (1.3469 \times 10^{-4}) = 6.73 \times 10^{-6} \text{ m}^3/\text{g} \#$$

$$\rho_s = 1.5 \times 10^3 \text{ kg/m}^3 = 1.5 \times 10^6 \text{ g/m}^3$$

$$K_p = K_d \rho_s = (6.73 \times 10^{-6}) \times (1.5 \times 10^6) = 10.1 \#$$

- (iii) Estimate the mass fractions of Y in air, water column and bottom sediment, respectively.

Air	10
water	20
Solid	1

$$\frac{1}{F_{total}} = 1 + \frac{V_w}{V_s} \frac{1}{K_p} + \frac{V_g}{V_s} \frac{H_{cc}}{K_p}$$

$$= 1 + \frac{20}{1} \frac{1}{10.1} + \frac{10}{1} \frac{0.317}{10.1}$$

$$= 3.29$$

$$F_{solid} = 0.304 \#$$

$$\frac{1}{F_w} = \frac{V_s K_p}{V_w} + 1 + \frac{V_g H_{cc}}{V_w}$$

$$= \frac{1(10.1)}{20} + 1 + \frac{10(0.317)}{20}$$

$$= 1.6635$$

$$F_w = 0.601 \#$$

(14 Marks)

$$\frac{1}{F_g} = \frac{V_s K_p}{V_g} \frac{1}{H_{cc}} + \frac{V_w}{V_g H_{cc}} + 1$$

$$= \frac{1}{10} \frac{10.1}{0.317} + \frac{20}{10(0.317)} + 1$$

$$= 10.495$$

$$F_g = 0.0953 \#$$

Check $F_s + F_w + F_g = 1$ ok ✓

4. A factory premises illegally discharges wastewater containing 2840 kg of ethanol at a constant flow rate of $0.2 \text{ m}^3/\text{s}$ into an impervious drain of 5 m width over a period of 30 min. The water upstream of the illegal discharge point has a flow rate of $0.8 \text{ m}^3/\text{s}$ and negligible ethanol concentration. The wastewater after entering the drain is well mixed with the drain water with respect to the flow cross section, and the mixed water forms a plug flow of ethanol-polluted plume downstream of the mixing point. The following are the properties of ethanol at the prevailing water temperature:

$m = 2840 \text{ kg}$ $Q_i = 0.2 \text{ m}^3/\text{s}$ width $W = 5 \text{ m}$ $t = 30 \text{ min}$ $Q_r = 0.8 \text{ m}^3/\text{s}$
 $D_m = \text{Molecular diffusivity in water} = 1.24 \times 10^{-9} \text{ m}^2/\text{s} = 1800 \text{ s}$
 Henry's constant (H_{cc}) = $2.7 \times 10^{-4} [-]$

- (a) Show that the concentration of ethanol in the wastewater is 7.9 kg/m^3 .

$$\text{Vol} = 0.2 \frac{\text{m}^3}{\text{s}} \times 1800 \text{ s} = 360 \text{ m}^3 \quad (1 \text{ Mark})$$

$$C = \frac{m}{V} = \frac{2840 \text{ kg}}{360 \text{ m}^3} = 7.9 \text{ kg/m}^3$$

- (b) Show that the concentration of ethanol at the mixing point is approximately 1.6 kg/m^3 .

$$1 \text{ s}, \text{ Vol in} = 0.2 \text{ m}^3 \quad (1 \text{ Mark})$$

$$m = CV = 7.9 \times 0.2 = 1.58 \text{ kg}$$

$$\text{Total vol} = \text{vol in} + \text{vol river} = 1 \text{ m}^3$$

$$C = \frac{m}{V} = \frac{1.58}{1} = 1.6 \text{ kg/m}^3 \quad \#$$

- (c) Given that the depth of combined flow is 1.0 m, show that the flow velocity of the ethanol-polluted water downstream of the spillage is 0.20 m/s .

$$\text{Total } Q = 0.2 + 0.8 = 1 \text{ m}^3/\text{s} \quad (1 \text{ Mark})$$

$$\text{Area} = 1 \times 5 = 5 \text{ m}^2$$

$$\text{Velocity} = \frac{1 \text{ m}^3/\text{s}}{5 \text{ m}^2} = 0.2 \text{ m/s} \quad \#$$

- (d) What is the length of the produced ethanol-polluted plume downstream of the mixing point as a result of the illegal discharge?

$$\text{Length} = 4\sigma = 4\sqrt{2Dt} = 4\sqrt{2(1.24 \times 10^{-9})(1800)} \quad (2 \text{ Mark})$$

$$= 8.451 \times 10^{-3} \text{ m}$$

length = velocity \times time
 $= 0.2 \times 1800$
 $= 360 \text{ m} \quad \#$

- (e) Using the O'Connor-Dobbins relationship, estimate the liquid-phase mass transfer coefficient, k_L (in m/s), for volatilization of ethanol while travelling in the drain.

$$k_L = \sqrt{Ds} = \sqrt{1.24 \times 10^{-9} \frac{0.2}{1}} = 1.575 \times 10^{-5} \text{ m/s} \quad \# \quad (2 \text{ Marks})$$

$s = \frac{\text{velocity}}{\text{depth}}$

- (f) Calculate the gas-phase mass transfer rate constant, k_{Ga} (in s^{-1}), given that k_L/k_G ratio is $1/500$. Prove that the ethanol volatilization from the plume is a gas-phase resistance-controlled process though the liquid-phase mass transfer process cannot be ignored in this case.

$$\frac{k_L}{k_G} = \frac{1}{500}, \quad k_G = 500k_L = 500(1.575 \times 10^{-5}) = 7.875 \times 10^{-3} \text{ m/s} \quad (6 \text{ Marks})$$

$$Q_{\text{TOT}} = 1 \text{ m}^3/\text{s}$$

$$\text{Vol} = 1 \text{ m}^3/\text{s} \times 1800 \text{ s} = 1800 \text{ m}^3 \quad \left| \quad a = \frac{\text{Area}}{\text{Vol}} = \frac{360 \times 5}{1800} = 1 \text{ m}^{-1}$$

$$k_G a = (7.875 \times 10^{-3})(1) = 7.875 \times 10^{-3} \text{ s}^{-1}$$

- (g) Calculate the concentration of ethanol in the drain water (in kg/m^3) after it travels over a distance of 20 km. It is estimated that ethanol in the plume can undergo first-order biochemical degradation with a rate constant of 0.25 day^{-1} . Compare the significance of volatilization versus natural degradation process for the fate of ethanol in the drain.

$$t = \frac{x}{v} = \frac{20000}{0.2} = 1 \times 10^5 \text{ s} = 1.157 \text{ day} \quad (7 \text{ Marks})$$

$$c = 1.6 e^{-0.25(1.157)} = 1.198 \text{ kg/m}^3 \#$$

5. (a) In a textile wastewater treatment plant, the main organic pollutant to be removed from the wastewater is dye. Ozonation is to be used since ozone can effectively degrade the dye and decolorize the wastewater. The dye concentration in wastewater, measured in term of total organic carbon (TOC), is 200 mg/L . It is estimated that 1 mg/L of dissolved ozone can degrade 1 mg/L of TOC in the wastewater. Ozone can be supplied to the water through ozonation system that injects ozone gas through nozzles immersed in water tank (the same phenomenon as aeration). However, the ozone solubility limit in the wastewater is 30 mg/L only. The excess ozone gas that cannot be dissolved in water becomes the off-gas that has to be destructed since it is hazardous. Based on your knowledge of reactor design and the limited ozone solubility, propose a conceptual reactor design for a continuous flow system with ozonation to remove at least 75% of the dye in the wastewater. Sketch a reactor diagram to illustrate at least the following information.

- Reactor type
- Reactor orientation (or elevation view) to show its length-to-diameter (length-to-depth) ratio
- Ozone injection point(s)

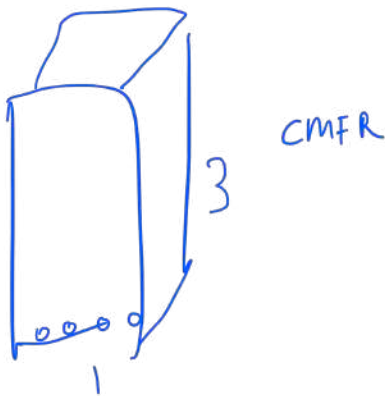
Note: You do not need to perform reactor design calculation, but your design should show conceptually that it can deliver sufficient amount of dissolved ozone to the water such that the treatment goal is achievable and economical.

$$\text{TOC} = 200 \text{ mg/L}$$

(6 Marks)

$$1 \text{ mg/L Ozone} \rightarrow 1 \text{ mg/L TOC}$$

$$\text{solubility} = 30 \text{ mg/L}$$



- (b) In a reactor design, what are the advantages of CMFR comparing with CMBR for a wastewater treatment?

No downtime (no need to wait in between batches) (4 Marks)

Easier to build