NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2020-2021

CV2011 – STRUCTURAL ANALYSIS I

April / May 2021 **Time Allowed: 2.5 hours**

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FOUR (4)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. An Appendix of **ONE (1)** page is attached to the Question Paper.
5. This is an Open-Book Examination with restriction. Only **ONE (1)** sheet of A4-size paper is allowed.
6. All answers must be written in the answer book provided. Answer each question beginning on the **FRESH** page of the answer book.
7. Avoid illegible handwriting. Your writing must be **CLEAR** and **READABLE**.

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1. A pin-jointed truss is subjected to four point loads as shown in Figure Q1.

   (a) Prove that the truss is overall statically determinate.

   (5 marks)

   (b) Calculate the reactions at the supports **A** and **B**.

   (5 marks)

   (c) Calculate the internal forces of members **FJ, HJ** and **HK**.

   (15 marks)

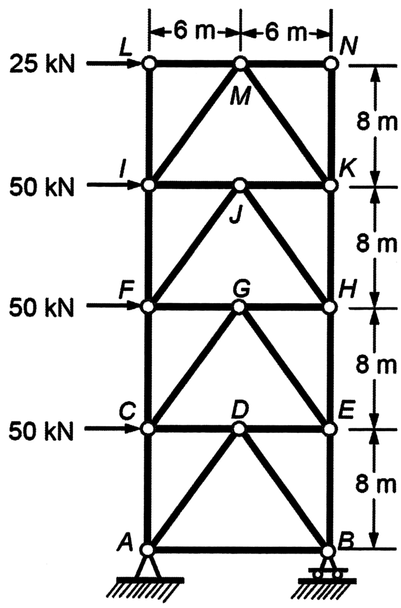


Figure Q1

2. The frame ABCDE shown in Figure Q2 is pin supported at A and roller supported at B. One uniformly distributed load of 15 kN/m is applied at the section CDE. One uniformly horizonal distributed load of 12 kN/m is applied at the section AC.

(a) Prove that the frame ABCDE is statically determinate.

(2 marks)

(b) Calculate the reactions at the supports A and B.

(5 marks)

(c) Draw the bending moment and shear force diagrams of the frame ABCDE. Indicate the values of the bending moments at points A, B, C, D and E.

(18 marks)

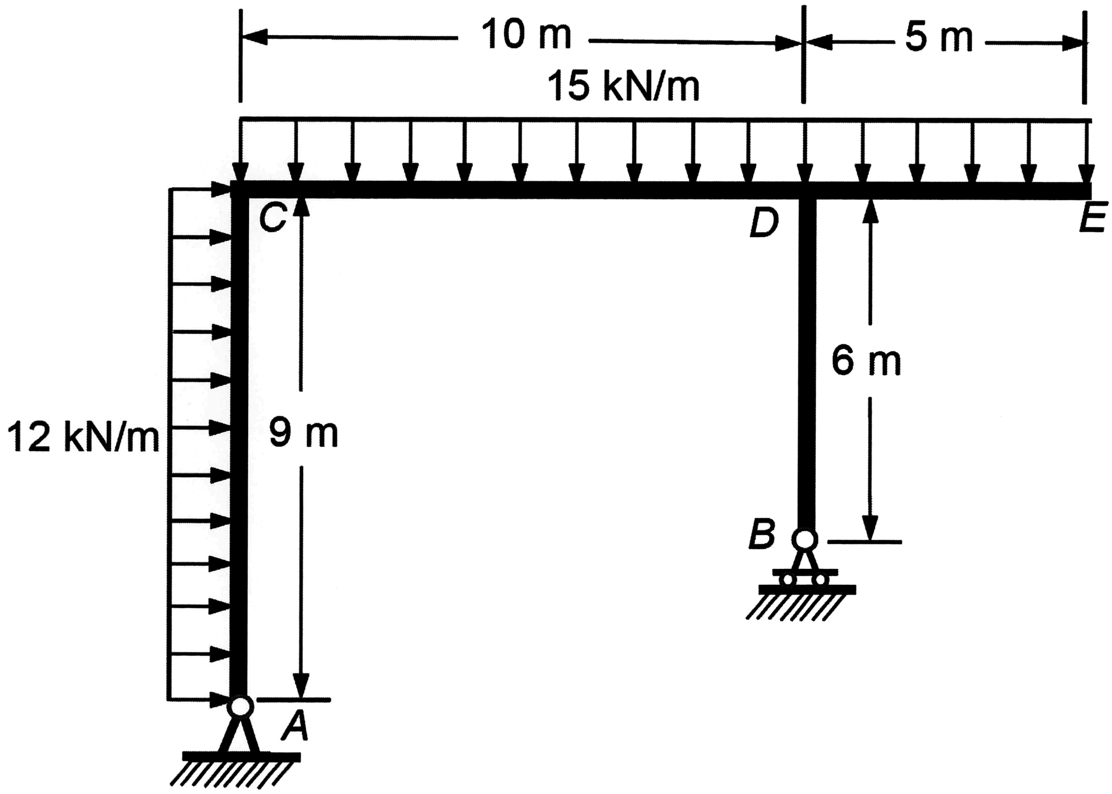


Figure Q2

3. Having a constant flexural rigidity *EI*, the simply supported beam *ABCDE* as shown in Figure Q3 is loaded by a moment M_0 at the mid-span *C*. Points *B* and *D* are the mid-points of segments *ABC* and *CDE*, respectively.

(a) Using the **Integration Method**, derive the equations of deflection curve for segments *ABC* and *CDE* in terms of the coordinates x_1 and x_2, respectively, and determine the vertical deflections at points *B* and *D*. For the entire beam, draw the bending moment diagram and sketch the deflected shape, showing their values at the critical points.

(15 marks)

(b) Using the **Moment Area Method**, determine the angle of rotation at support *A* and the coordinate of x_1 (in terms of *L*) where the maximum vertical deflection occurs in segment *ABC*.

(10 marks)

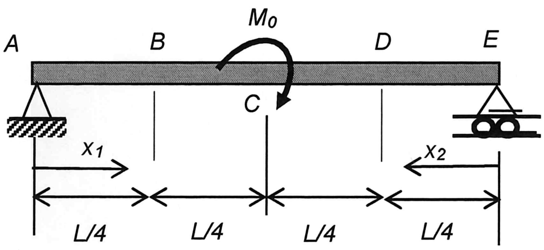


Figure Q3

4. With a constant flexural rigidity EI, the compound beam $ABCD$ as shown in Figure Q4 is loaded by a uniformly distributed load w over the span ABC. The span ABC is pin connected at joint C to a cantilever beam CD.

(a) Using the **Method of Virtual Work**, determine the vertical displacement of point B and the angle of rotation at support A.

(15 marks)

(b) For the entire compound beam, draw the bending moment diagram and sketch the deflected shape, showing their values at critical points.

(10 marks)

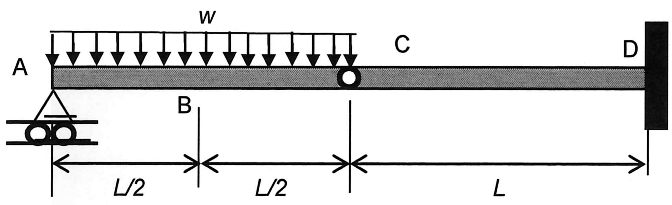
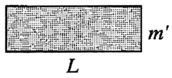
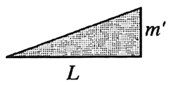
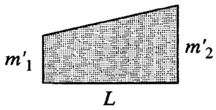
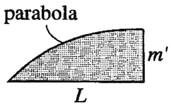
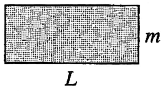
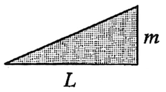
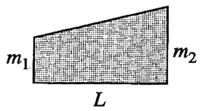
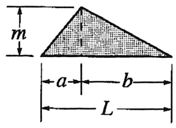
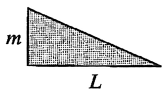


Figure Q4

END OF PAPER

Appendix: Values of Product Integrals $\int_0^L mm'dx$

$\int_0^L m\,m'\,dx$				
	$mm'L$	$\frac{1}{2}mm'L$	$\frac{1}{2}m(m'_1 + m'_2)L$	$\frac{2}{3}mm'L$
	$\frac{1}{2}mm'L$	$\frac{1}{3}mm'L$	$\frac{1}{6}m(m'_1 + 2m'_2)L$	$\frac{5}{12}mm'L$
	$\frac{1}{2}m'(m_1 + m_2)L$	$\frac{1}{6}m'(m_1 + 2m_2)L$	$\frac{1}{6}[m'_1(2m_1 + m_2) + m'_2(m_1 + 2m_2)]L$	$\frac{1}{12}[m'(3m_1 + 5m_2)]L$
	$\frac{1}{2}mm'L$	$\frac{1}{6}mm'(L + a)$	$\frac{1}{6}m_1[m'_1(L + b) + m_2(L + a)]$	$\frac{1}{12}mm'\left(3 + \frac{3a}{L} - \frac{a^2}{L^2}\right)L$
	$\frac{1}{2}mm'L$	$\frac{1}{6}mm'L$	$\frac{1}{6}m(2m'_1 + m'_2)L$	$\frac{1}{4}mm'L$

Lee Jing Xuan

1. A pin-jointed truss is subjected to four point loads as shown in Figure Q1.

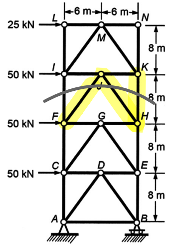
   (a) Prove that the truss is overall statically determinate. (5 marks)

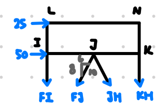
   (b) Calculate the reactions at the supports **A** and **B**. (5 marks)

   (c) Calculate the internal forces of members **FJ, HJ** and **HK**. (15 marks)

a.
$m = 25$
$j = 14$
$r = 3$

$25 = 2(14) - 3$ //





At J

$\Sigma F_y = 0,$

$FJ = -JH \quad -(1)$

$\circlearrowright^{+} \Sigma M_L = 0,$

$$50(8) + 8 \times \frac{6JH}{10} = 6 \times \frac{8FJ}{10} + 8 \times \frac{6FJ}{10} + 6 \times \frac{8JH}{10} + 12KH$$

$$400 = \frac{48}{5} FJ + 12KH$$

$$12KH = 400 - \frac{48}{5} FJ \quad -(2)$$

$\circlearrowright^{+} \Sigma M_I = 0,$

$$-6 \times \frac{8FJ}{10} - 6 \times \frac{8JH}{10} - 25 \times 8 - 12KH = 0$$

$$-\frac{24}{5} FJ - \frac{24}{5} JH - 200 - 12KH = 0 \quad -(3)$$

Sub 2 into 3,

$$-\frac{24}{5} FJ - \frac{24}{5} JH - 200 - 400 + \frac{48}{5} FJ = 0 \quad -(4)$$

Sub 1 into 4,

$$\frac{24}{5} JH - \frac{24}{5} JH - 600 + \frac{48}{5} FJ = 0$$

$\therefore FJ = 62.5 \text{ kN (T)}$ //

$JH = 62.5 \text{ kN (C)}$ //

$KH = \frac{50}{3} \text{ kN (C)}$ //

Lee Jing Xuan

2. The frame ABCDE shown in Figure Q2 is pin supported at A and roller supported at B. One uniformly distributed load of 15 kN/m is applied at the section CDE. One uniformly horizonal distributed load of 12 kN/m is applied at the section AC.

(a) Prove that the frame ABCDE is statically determinate. (2 marks)

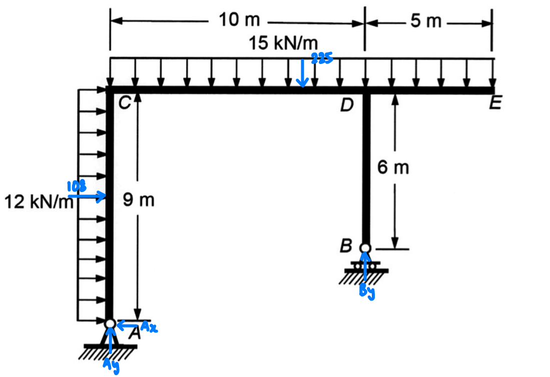
$r = 3$, $n = 1$

$r = 3n$

$3 = 3(1)$ //

(b) Calculate the reactions at the supports A and B. (5 marks)

(c) Draw the bending moment and shear force diagrams of the frame ABCDE. Indicate the values of the bending moments at points A, B, C, D and E. (18 marks)



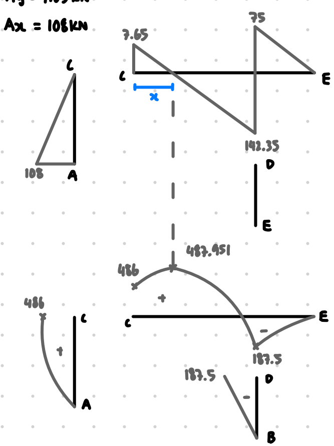
$\circlearrowright^{+} \Sigma M_A = 0,$

$10B_y - 225 \times \frac{15}{2} - 108 \times \frac{9}{2} = 0$

$B_y = 217.35\text{ kN}$

$A_y = 7.65\text{ kN}$

$A_x = 108\text{ kN}$



$7.65 - 15x = 0$

$x = 0.51$

$\circlearrowleft^{+} M_C = 9A_x - 108 \times \frac{9}{2} = 486$

$\circlearrowleft^{+} M_x = 0.51A_y + 9A_x - 108 \times \frac{9}{2} - 15 \times 0.51 \times \frac{0.51}{2} = 487.95075$

$\circlearrowright^{+} M_D = -15 \times 5 \times \frac{5}{2} = -187.5$

Lee Jing Xuan

3. Having a constant flexural rigidity EI, the simply supported beam $ABCDE$ as shown in Figure Q3 is loaded by a moment M_0 at the mid-span C. Points B and D are the mid-points of segments ABC and CDE, respectively.

   (a) Using the **Integration Method**, derive the equations of deflection curve for segments ABC and CDE in terms of the coordinates x_1 and x_2, respectively, and determine the vertical deflections at points B and D. For the entire beam, draw the bending moment diagram and sketch the deflected shape, showing their values at the critical points.

   (15 marks)

   (b) Using the **Moment Area Method**, determine the angle of rotation at support A and the coordinate of x_1 (in terms of L) where the maximum vertical deflection occurs in segment ABC.

   (10 marks)

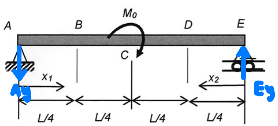


Figure Q3

1.25

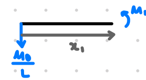
$\circlearrowleft \Sigma M_A = 0,$

$LE_y = M_0$

$E_y = \frac{M_0}{L}$

$A_y = \frac{M_0}{L}$

a.



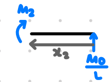
$\circlearrowleft \Sigma M = 0$

$M_1 = -\frac{M_0 x_1}{L}$

$EIV_1'' = -\frac{M_0 x_1}{L}$

$EIV_1' = -\frac{M_0 x_1^2}{2L} + C_1$

$EIV_1 = -\frac{M_0 x_1^3}{6L} + C_1 x_1 + C_2$



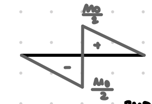
$\circlearrowleft \Sigma M = 0,$

$-M_2 + \frac{M_0 x_2}{L} = 0$

$EIV_2'' = \frac{M_0 x_2}{L}$

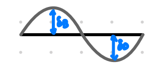
$EIV_2' = \frac{M_0 x_2^2}{2L} + C_3$

$EIV_2 = \frac{M_0 x_2^3}{6L} + C_3 x_2 + C_4$



BMD

$\circlearrowleft \Sigma M_C = -A_y \times \frac{L}{2} = -\frac{M_0}{2}$



Deflection

BC

At $x_1 = 0$, $V_1 = 0$

$\therefore C_2 = 0$

At $x_2 = 0$, $V_2 = 0$

$\therefore C_4 = 0$

CC

At $x_1 = x_2 = \frac{L}{2}$, $V_1 = V_2$

$-\frac{M_0(L/2)^3}{6L} + C_1 L/2 = \frac{M_0(L/2)^3}{6L} + C_3 L/2$

$-\frac{M_0 L^2}{48} + C_1 L/2 = \frac{M_0 L^2}{48} + C_3 L/2$

$C_1 = \frac{M_0 L}{12} + C_3$

At $x_1 = x_2 = \frac{L}{2}$, $V_1' = -V_2'$

$-\frac{M_0(L/2)^2}{2L} + C_1 = -\frac{M_0(L/2)^2}{2L} - C_3$

$C_1 = -C_3$

$-C_3 = \frac{M_0 L}{12} + C_3$

$C_3 = -\frac{M_0 L}{24}$

$C_1 = \frac{M_0 L}{24}$

$\therefore EIV_1 = -\frac{M_0 x_1^3}{6L} + \frac{M_0 L x_1}{24}$ //

$EIV_2 = \frac{M_0 x_2^3}{6L} - \frac{M_0 L x_2}{24}$ //

At $x_1 = L/4$,

$EIV_1 = -\frac{M_0(L/4)^3}{6L} + \frac{M_0 L(L/4)}{24}$

$EIV_1 = \frac{M_0 L^2}{128}$

$\delta_B = \frac{M_0 L^2}{128EI}$ (↑) //

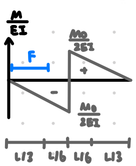
At $x_2 = L/4$,

$EIV_2 = \frac{M_0(L/4)^3}{6L} - \frac{M_0 L(L/4)}{24}$

$EIV_2 = -\frac{M_0 L^2}{128}$

$\delta_D = \frac{M_0 L^2}{128EI}$ (↑) //

b.



$t_{E/A} = V_E' - V_A' - (x_E - x_A)\theta_A = \frac{1}{2} \times \frac{M_0}{2EI} \times \frac{L}{2} \times \frac{L}{3} + \frac{1}{2} \times -\frac{M_0}{2EI} \times \frac{L}{2} \times \frac{2L}{3}$

$-L\theta_A = -\frac{M_0 L^2}{24EI}$

$\theta_A = \frac{M_0 L}{24EI}$ (↻) //

Let F be a point at max deflection

$\theta_{F/A} = \theta_F - \theta_A = \frac{1}{2} \times \left(\frac{M_0}{2EI} \div \frac{L}{2}\right) x \times x$

$-\theta_A = \frac{M_0 x^2}{2EIL}$

$-\frac{M_0 L}{24EI} = \frac{M_0 x^2}{2EIL}$

$-\frac{L^2}{24} = \frac{x^2}{2}$

$x = \sqrt{\frac{L^2}{12}}$

$\approx 0.29L$ //

4. With a constant flexural rigidity EI, the compound beam $ABCD$ as shown in Figure Q4 is loaded by a uniformly distributed load w over the span ABC. The span ABC is pin connected at joint C to a cantilever beam CD.

(a) Using the **Method of Virtual Work**, determine the vertical displacement of point B and the angle of rotation at support A.

(15 marks)

(b) For the entire compound beam, draw the bending moment diagram and sketch the deflected shape, showing their values at critical points.

(10 marks)

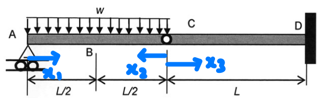
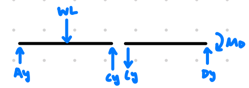


Figure Q4

a.



Actual load

$\circlearrowright \Sigma M_A = 0$

$WL \times \frac{L}{2} = C_y L$

$C_y = \frac{WL}{2}$

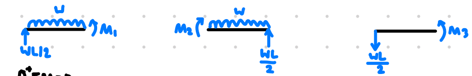
$A_y = \frac{WL}{2}$

$\circlearrowright \Sigma M_D = 0$

$-M_D + C_y L = 0$

$M_D = \frac{WL^2}{2}$

$D_y = \frac{WL}{2}$



$\circlearrowright \Sigma M = 0$

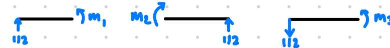
$M_1 + \frac{Wx_1^2}{2} = \frac{WLx_1}{2}$

$M_1 = \frac{WLx_1}{2} - \frac{Wx_1^2}{2}$

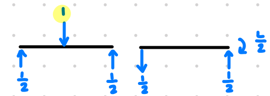
$\frac{WLx_2}{2} = \frac{Wx_2^2}{2} + M_2$

$M_2 = \frac{WLx_2}{2} - \frac{Wx_2^2}{2}$

$M_3 = -\frac{WLx_3}{2}$



$m_1 = \frac{x_1}{2}$ $\quad m_2 = \frac{x_2}{2}$ $\quad m_3 = -\frac{x_3}{2}$



Virtual load

$\circlearrowright \Sigma M_A = 0,$

$B_y L = \frac{L}{2}$

$B_y = \frac{1}{2}$

$\circlearrowright \Sigma M_D = 0$

$-M_D + \frac{1}{2}L = 0$

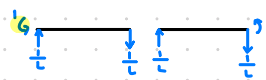
$M_D = \frac{L}{2}$

$m_{1\theta} + 1 = \frac{x_1}{L}$

$m_{1\theta} = \frac{x_1}{L} - 1$

$m_{2\theta} = -\frac{x_2}{L}$

$m_{3\theta} = \frac{x_3}{L}$



Virtual Moment

$\circlearrowright \Sigma M_A = 0$

$1 = B_y L$

$B_y = \frac{1}{L}$

$$1 \cdot \delta_B = \Sigma \int \frac{Mm}{EI} dx = \int_0^{L/2} \left(\frac{WLx_1}{2} - \frac{Wx_1^2}{2}\right)\left(\frac{x_1}{2}\right) dx + \int_0^{L/2} \left(\frac{WLx_2}{2} - \frac{Wx_2^2}{2}\right)\left(\frac{x_2}{2}\right) dx + \int_0^L -\frac{WLx_3}{2} \times -\frac{x_3}{2} dx$$

$$= \int_0^{L/2} \frac{WLx_1^2}{4} - \frac{Wx_1^3}{4} dx + \int_0^{L/2} \frac{WLx_2^2}{4} - \frac{Wx_2^3}{4} dx + \int_0^L \frac{WLx_3^2}{4} dx$$

$$= \left[\frac{WLx_1^3}{12} - \frac{Wx_1^4}{16}\right]_0^{L/2} + \left[\frac{WLx_2^3}{12} - \frac{Wx_2^4}{16}\right]_0^{L/2} + \left[\frac{WLx_3^3}{12}\right]_0^L$$

$$= \frac{5WL^4}{768} + \frac{5WL^4}{768} + \frac{WL^4}{12}$$

$$= \frac{37WL^4}{384EI} (\downarrow)$$

$$1 \cdot \theta_A = \Sigma \int \frac{Mm_\theta}{EI} dx = \int_0^{L/2} \left(\frac{WLx_1}{2} - \frac{Wx_1^2}{2}\right)\left(\frac{x_1}{L} - 1\right) dx + \int_0^{L/2} \left(\frac{WLx_2}{2} - \frac{Wx_2^2}{2}\right)\left(-\frac{x_2}{L}\right) dx + \int_0^L -\frac{WLx_3}{2} \times \frac{x_3}{L} dx$$

$$= \int_0^{L/2} \frac{Wx_1^2}{2} - \frac{WLx_1}{2} - \frac{Wx_1^3}{2L} + \frac{Wx_1^2}{2} dx + \int_0^{L/2} -\frac{Wx_2^2}{2} + \frac{Wx_2^3}{2L} dx + \int_0^L -\frac{Wx_3^2}{2} dx$$

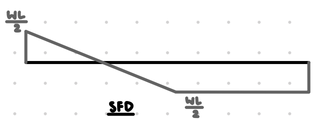
$$= \int_0^{L/2} Wx_1^2 - \frac{WLx_1}{2} - \frac{Wx_1^3}{2L} dx + \int_0^{L/2} -\frac{Wx_2^2}{2} + \frac{Wx_2^3}{2L} dx + \int_0^L -\frac{Wx_3^2}{2} dx$$

$$= \left[\frac{Wx_1^3}{3} - \frac{WLx_1^2}{4} - \frac{Wx_1^4}{8L}\right]_0^{L/2} + \left[-\frac{Wx_2^3}{6} + \frac{Wx_2^4}{8L}\right]_0^{L/2} + \left[-\frac{Wx_3^3}{6}\right]_0^L$$

$$= -\frac{11WL^3}{384} - \frac{5WL^3}{384} - \frac{WL^3}{6}$$

$$\theta_A = -\frac{5WL^3}{24EI} (\circlearrowright)$$

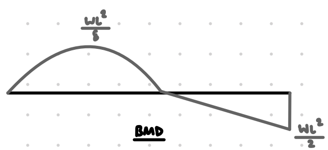
b.



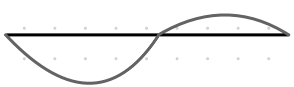
SFD

$\circlearrowright M_B = A_y \cdot \frac{L}{2} - W \cdot \frac{L}{2} \cdot \frac{L}{4}$

$= \frac{WL^2}{8}$



BMD



4. With a constant flexural rigidity EI, the compound beam $ABCD$ as shown in Figure Q4 is loaded by a uniformly distributed load w over the span ABC. The span ABC is pin connected at joint C to a cantilever beam CD.

(a) Using the **Method of Virtual Work**, determine the vertical displacement of point B and the angle of rotation at support A.

(15 marks)

(b) For the entire compound beam, draw the bending moment diagram and sketch the deflected shape, showing their values at critical points.

(10 marks)

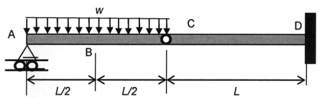


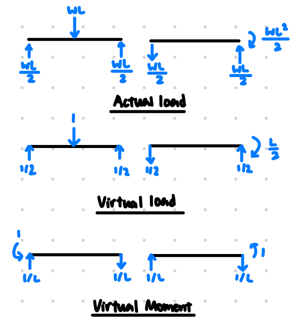
Figure Q4

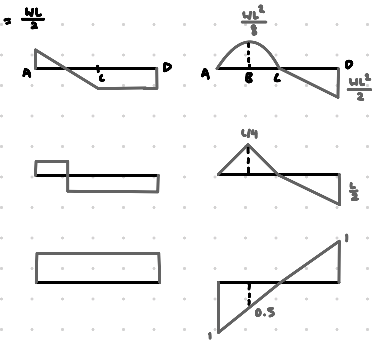
Using table IV

$$\frac{WL^2}{2} = B_y L$$

$$B_y = \frac{WL}{2}$$

$$\circlearrowright \Sigma M_B = \frac{L}{2} A_y - \frac{WL^2}{8} = \frac{WL^2}{8}$$





$$\circlearrowright \Sigma M_B = \left(\frac{1}{L} \times \frac{L}{2}\right) - 1 = -0.5$$

a. 1. $\delta_B = \int_0^L \frac{mm'}{EI} dx = \left(\frac{5}{12} \times \frac{WL^2}{8} \times \frac{L}{4} \times \frac{L}{2}\right) + \left(\frac{5}{12} \times \frac{WL^2}{8} \times \frac{L}{4} \times \frac{L}{2}\right) + \left(\frac{1}{3} \times \frac{-WL^2}{2} \times -\frac{L}{2} \times L\right)$

$$= \frac{5WL^4}{786EI} + \frac{5WL^4}{768EI} + \frac{WL^4}{12EI}$$

$$= \frac{37WL^4}{384EI} (\downarrow) //$$

$$\theta_A = \frac{1}{12}\left[\frac{WL^2}{8}(3 \times -1 + 5 \times -0.5)\right]\frac{L}{2} + \left(\frac{5}{12} \times \frac{WL^2}{8} \times -0.5 \times \frac{L}{2}\right) + \left(\frac{1}{3} \times -\frac{WL^2}{2} \times 1 \times L\right)$$

$$= -\frac{11WL^3}{384EI} - \frac{5WL^3}{192EI} - \frac{WL^3}{6EI}$$

$$= -\frac{5WL^3}{24EI} (\circlearrowleft) //$$